

Illustrations of Some Implemented Examples with Adaptive-Timing, Electronic-Stopping and Electron-Phonon Coupling Model in TurboGAP with Details of Input and Validations

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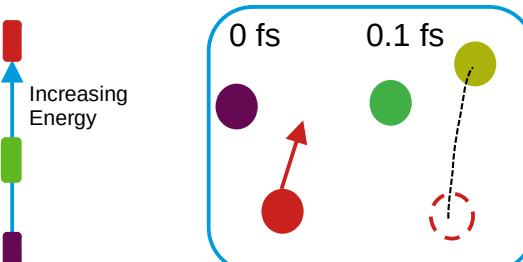
<https://github.com/mcaroba/turbogap/blob/stopping/Illustrations-Details-of-Inputs-Validations.pdf>

To select a time step (unit of time is fs):

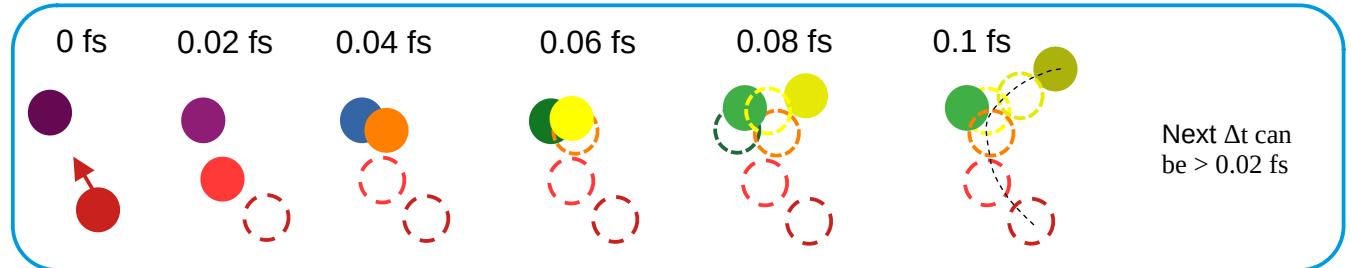
The basic relation $\Delta x = v\Delta t$ is suitable in normal situations. In radiation cascades the atoms experience very strong collisions where this time step could be very long.

Then, we will actually need $\Delta x \leq 0.0001 \text{ \AA}$ to see the exact trajectories and energy sharing by the atoms during cascade progress.

We use a x_{\max} criterion to allow maximum possible distance an atom can move in a time step and also a e_{\max} criterion to allow maximum possible change in K.E of an atom.



Fixed time step



Adaptive time step – with smaller Δt at high E and larger time steps at low E

The displacement of atoms in cascades take place through binary elastic collisions. The nuclear energy loss is important in damage calculations.

But there is electronic energy loss too, at all energies.

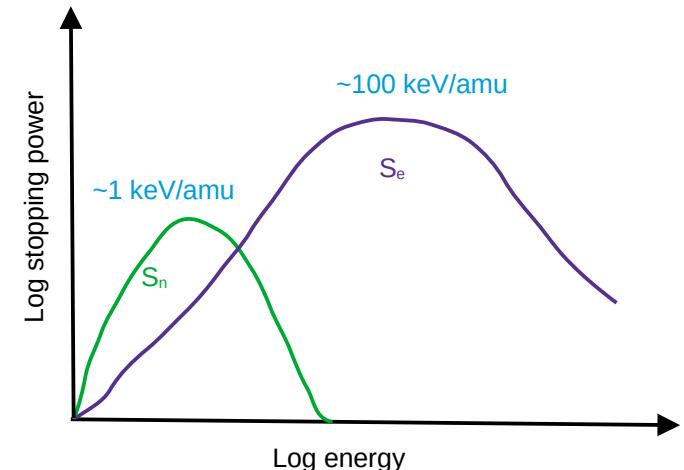
Only damage energy part of K.E of PKA is responsible for damage creation.

The damage energy depends on K.E of PKA and also on the types of the elements (PKA and the target) through the characteristic electronic energy loss.

In the past, we used to do MD with $T_{\text{dam}} = E_{\text{PKA}} - E(\text{el})$ as the PKA energy, calculated analytically from energy partition theory or from the data from SRIM software.

But advances allow for real-time EEL calculations while simulating a cascade in MD. There are a few approaches possible.

- S_e (in units eV/Å) is obtained as a function of E_{ion} from SRIM-2013.
- User provides a lower cut off energy (few tens of eV, usually) and the name of stopping data file.



Modify the forces as

$$\vec{F} = \vec{F}^0 - S_e \frac{\vec{v}}{v}$$

Adaptive time

For the user ::

Simple input keywords starting with the word 'adapt':

```
adaptive_time = .true.  
adapt_tstep_interval = 1  
adapt_tmin = 1.0e-07  
adapt_tmax = 0.1  
adapt_xmax = 1.0e-4  
adapt_emax = 50.0
```

Electronic stopping

For the user ::

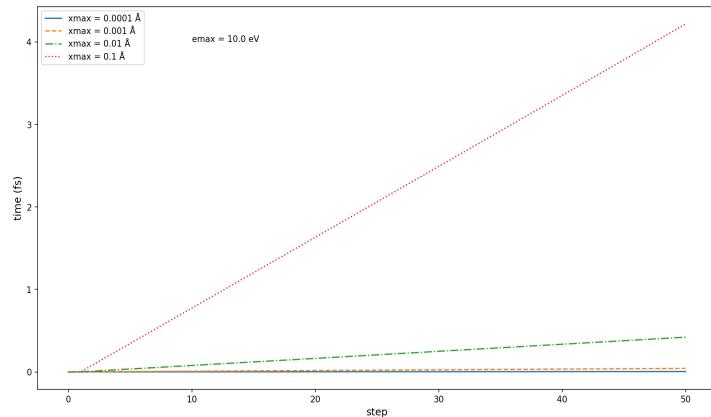
Simple input keywords

```
electronic_stopping = .true.  
eel_cut = 5.0  
eel_freq_out = 10  
estop_filename = 'stopping-data-file.txt'
```

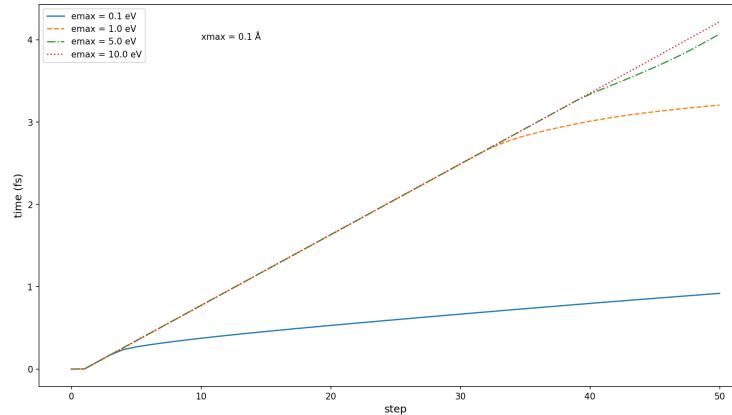
[Note: all illustrations here are shown by using a general purpose Si-GAP potential without short distance repulsive stiffening.]

Sensitivity of adaptive timing to maximum distance and energy criteria

Sensitivity to xmax



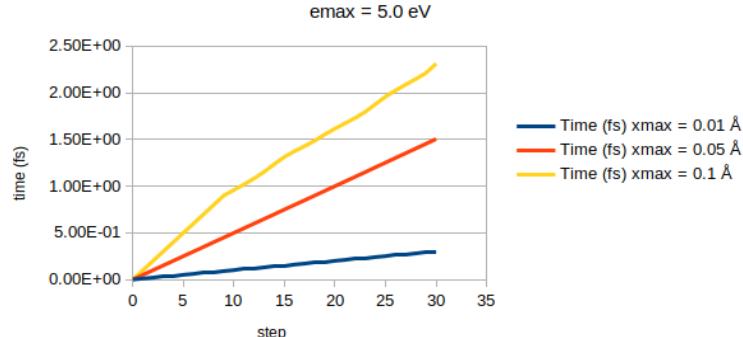
Sensitivity to emax



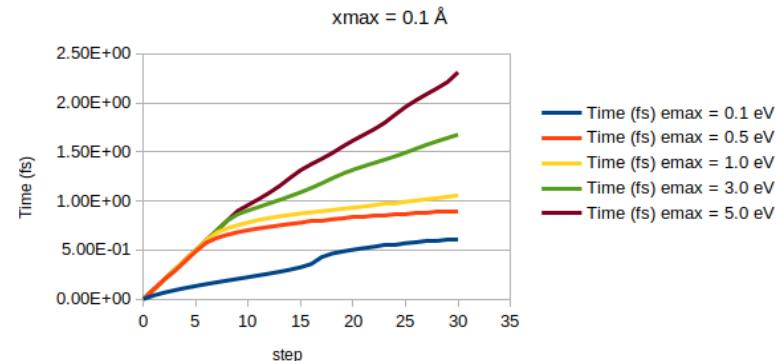
Sensitivity of time steps to xmax and emax in a binary collision simulation

Two Si atoms at $\mathbf{r}_1 = (10.0, 10.0, 10.0)$ and $\mathbf{r}_2 = (13.0, 10.0, 10.0)$ with $v_{1x} = 1.0 \text{ \AA/fs}$, $v_{1y} = 0.0$, $v_{1z} = 0.0$ and $v_{2x} = -1.0 \text{ \AA/fs}$, $v_{2y} = 0.0$, $v_{2z} = 0.0$ using Si (not stiffened) GAP potential.

Binary Collision using TurboGAP



Binary Collision using TurboGAP



Some Simulations with Fixed and Adaptive time steps

Silicon with 4096 atoms – one atom is given ~100 eV energy

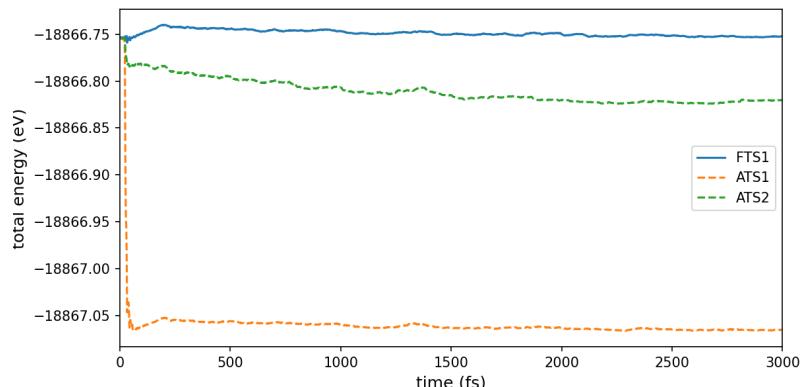
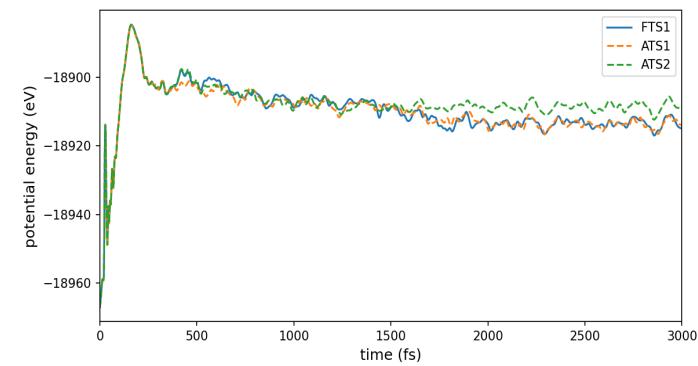
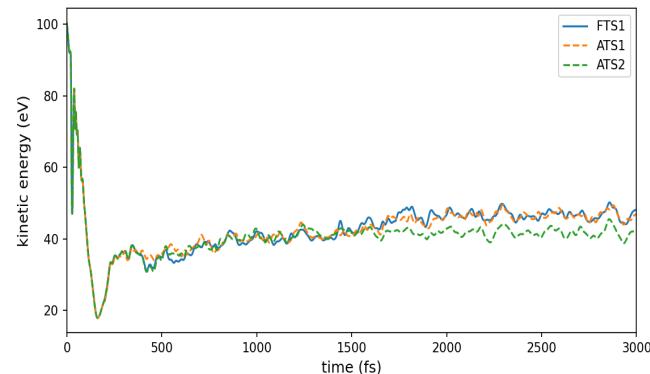
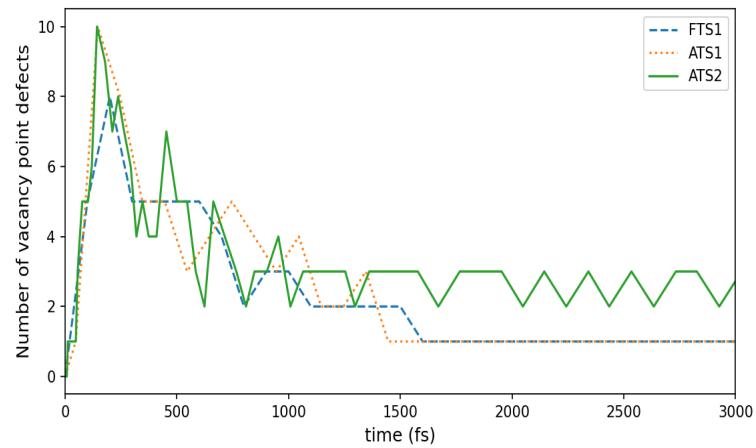
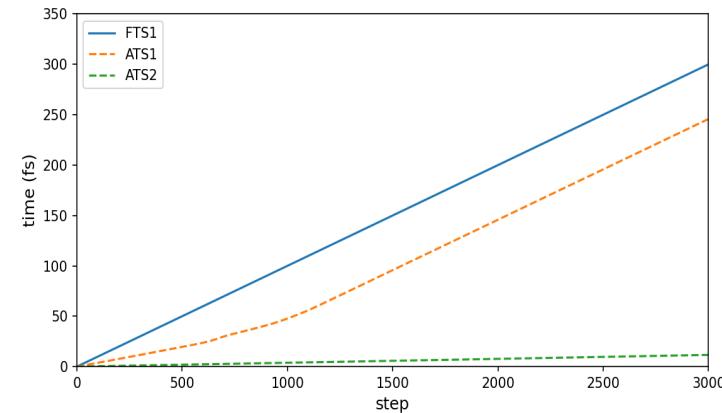
Fixed time step
 $= 0.1 \text{ fs} \rightarrow \text{FTS1}$

ATS1

`adapt_tstep_interval = 1`
`adapt_tmin = 1.0E-07`
`adapt_tmax = 1.0E-01`
`adapt_xmax = 1.0E-02`
`adapt_emax = 30.0`

ATS2

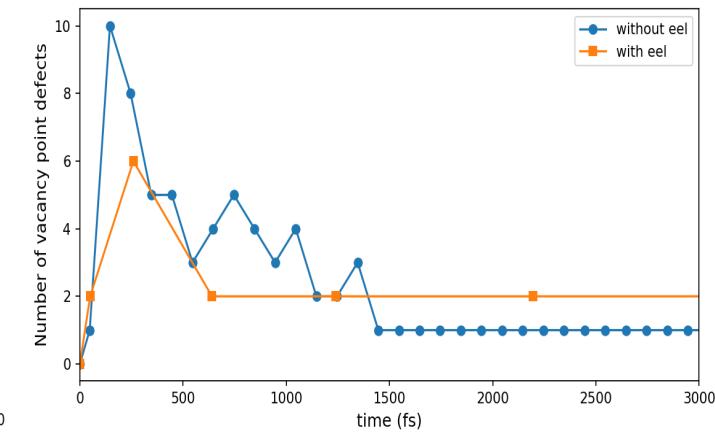
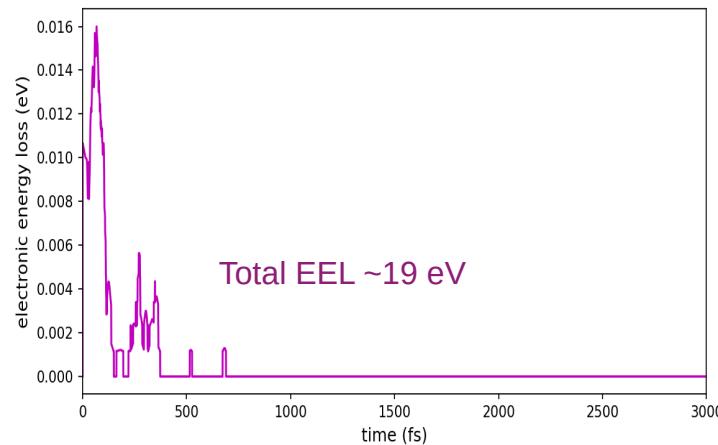
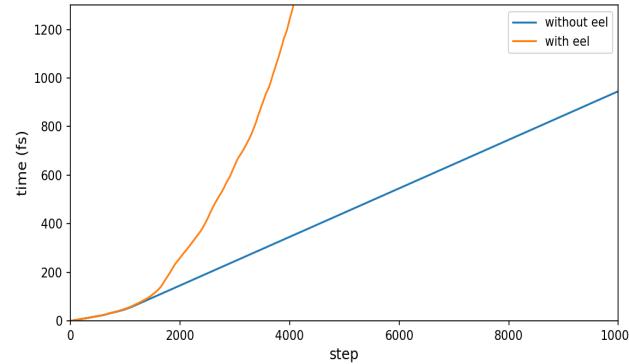
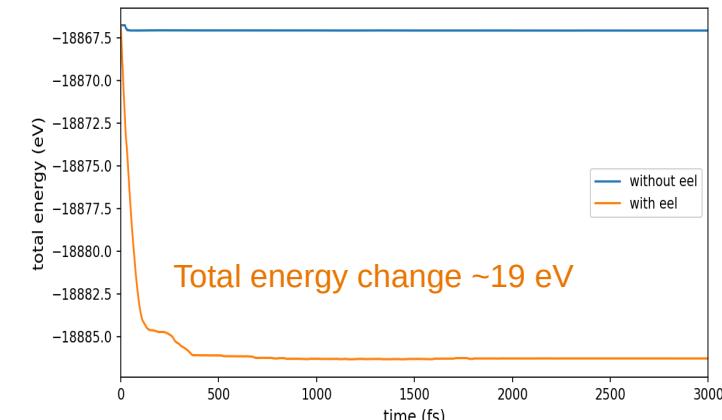
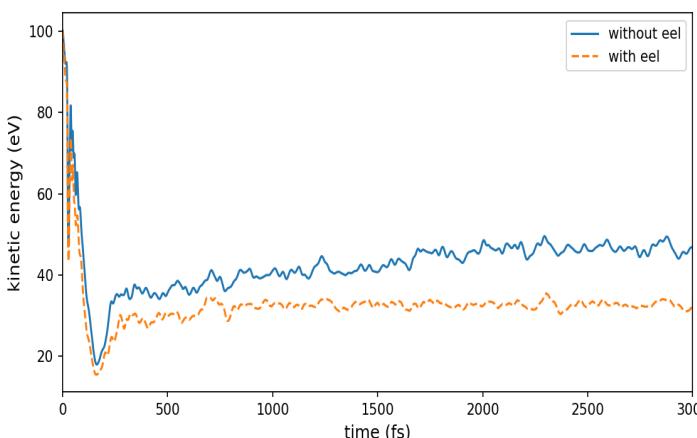
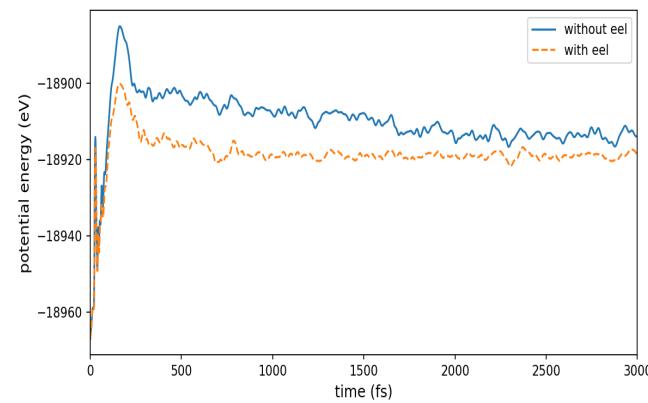
`adapt_tstep_interval = 1`
`adapt_tmin = 1.0E-07`
`adapt_tmax = 1.0E-01`
`adapt_xmax = 1.0E-03`
`adapt_emax = 30.0`



Simulations with Electronic Stopping using stopping data from SRIM-2013

Silicon with 4096 atoms – one atom is given ~100 eV energy

eel_cut = 10.0 eV,
 tstep_interval = 1,
 tmin = 1.0E-07 fs, tmax = 1.0 fs,
 xmax = 0.01Å, emax = 30.0 eV



If we do not account EEL, then we allow more defect creation and also more defect annealing. The defects may also be morphologically different from reality. [Note: the numbers of defects or exact values of the quantities shown here are not important, only the general trends are shown by using a general purpose potential without short distance repulsive stiffening.]

Electronic stopping and electron-phonon coupling

During the initial non-equilibrium stages of a radiation event, electronic stopping is dominant and at the later stages to equilibrium between the lattice and the electrons, the coupling between electrons and lattice phonons become significant.

Both aspects of energy dissipation between the atoms and electrons are important.

The previous method described simply acts as a friction term

- to only reduce the energy of the atoms,
- to arbitrarily low values determined by the cut off energy specified and
- assumes that electronic stopping at low velocities is linear in velocity.

However,

- electronic stopping and e-ph coupling both must be accounted within same theory,
- a Langevin drag force can act between atoms and electronic heat bath establishing thermal equilibrium, not only just reducing the atom energy to arbitrary values,
- fixing the low energy cut off to arbitrary values may not be correct,
- at low velocities the electronic stopping power is not linear in velocity and
- the drag force connected to a stochastic force through fluctuation-dissipation theorem within the Langevin dynamical theory can model the phenomena more accurately.

The inputs to be given to implement this electron-phonon coupling model are described here. There are two choices:
<1> less input keywords with accompanying text file for electronic temperature mesh and
<2> using a few more input keywords, but no additional file.

Choice <1>

```
nonadiabatic_processes = .true.  
model_eph = 1  
eph_fdm_option = 1  
eph_friction_option = 1  
eph_random_option = 1  
eph_betafile = 'Si_PRB2021_constant.beta'  
eph_Tinfile = 'T_input.fdm'  
eph_md_last_step = 0  
eph_md_prev_time = 0.0  
eph_E_prev_time = 0.0  
eph_freq_Tout = 10  
eph_freq_mesh_Tout = 1000  
eph_Toutfile = 'T-Si-Ta0Te50.out'
```

For Choice <1> Sample T_input.fdm

```
#  
# 3 lines of comments  
#  
2 2 2 1  
0.0 16.29  
0.0 16.29  
0.0 16.29  
i j k T_e S_e rho_e C_e K_e flag T_dyn_flag  
1 1 1 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
2 1 1 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
1 2 1 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
2 2 1 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
1 1 2 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
2 1 2 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
1 2 2 50.0 1.0 1.0 3.5E-06 0.1248 0 0  
2 2 2 50.0 1.0 1.0 3.5E-06 0.1248 0 0
```

Choice <2>

```
nonadiabatic_processes = .true.  
model_eph = 1  
eph_fdm_option = 1  
eph_friction_option = 1  
eph_random_option = 1  
eph_betafile = 'Si_PRB2021_constant.beta'  
box_limits = 0.0 16.29 0.0 16.29 0.0 16.29  
eph_rho_e = 1.0  
eph_C_e = 3.5E-06  
eph_kappa_e = 0.1248  
eph_Ti_e = 50.0  
eph_gsx = 2  
eph_gsy = 2  
eph_gsz = 2  
eph_fdm_steps = 1  
eph_md_last_step = 0  
eph_md_prev_time = 0.0  
eph_E_prev_time = 0.0  
eph_freq_Tout = 10  
eph_freq_mesh_Tout = 1000  
eph_Toutfile = 'T-Si-Ta0Te50.out'
```

Some simulations by using the e-ph coupling model in TurboGAP are shown in the following few pages.

Variations of atomic and electronic temperatures

216 Si atoms in a box of dimensions
16.29 Å on each side.

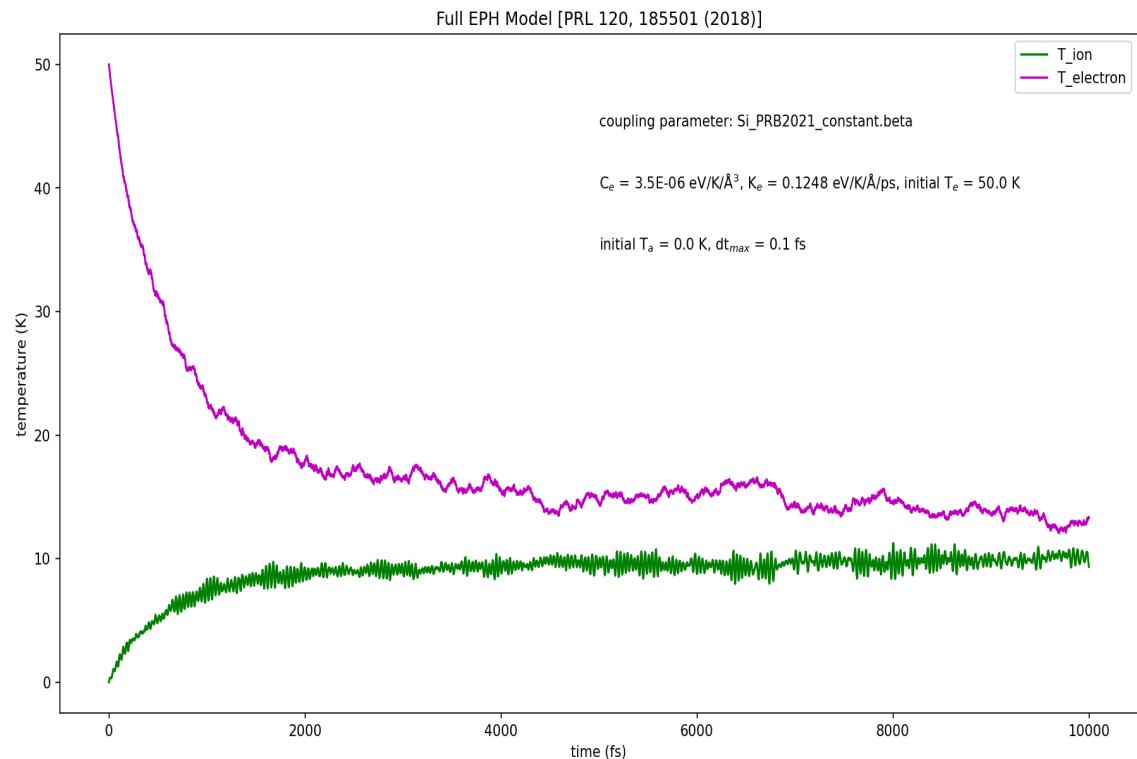
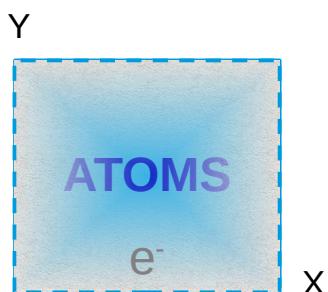
Initial atom temperature = 0.0 K

Initial electronic temperature = 50.0 K

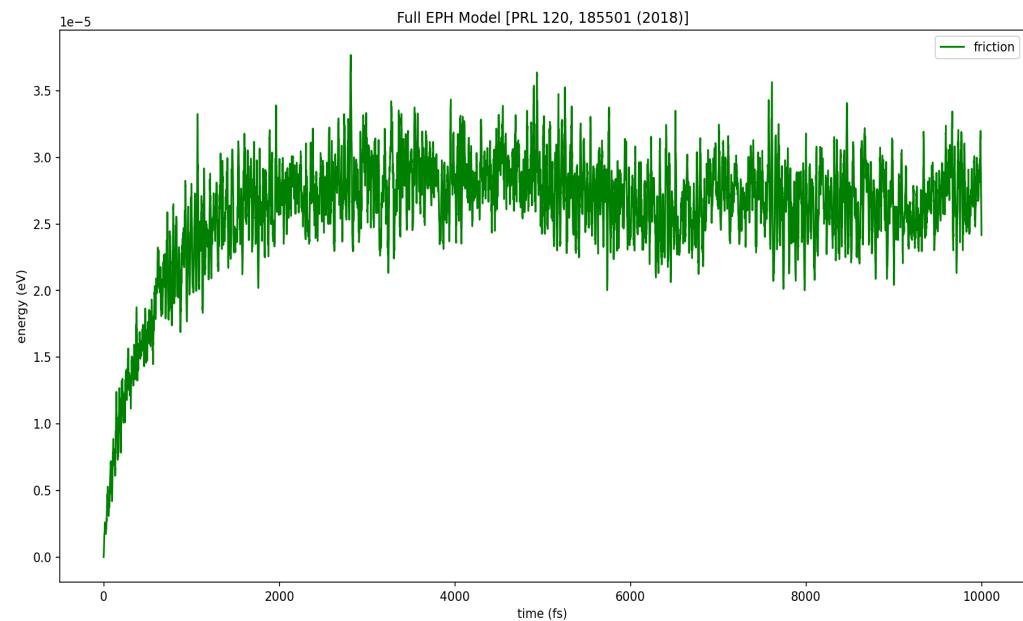
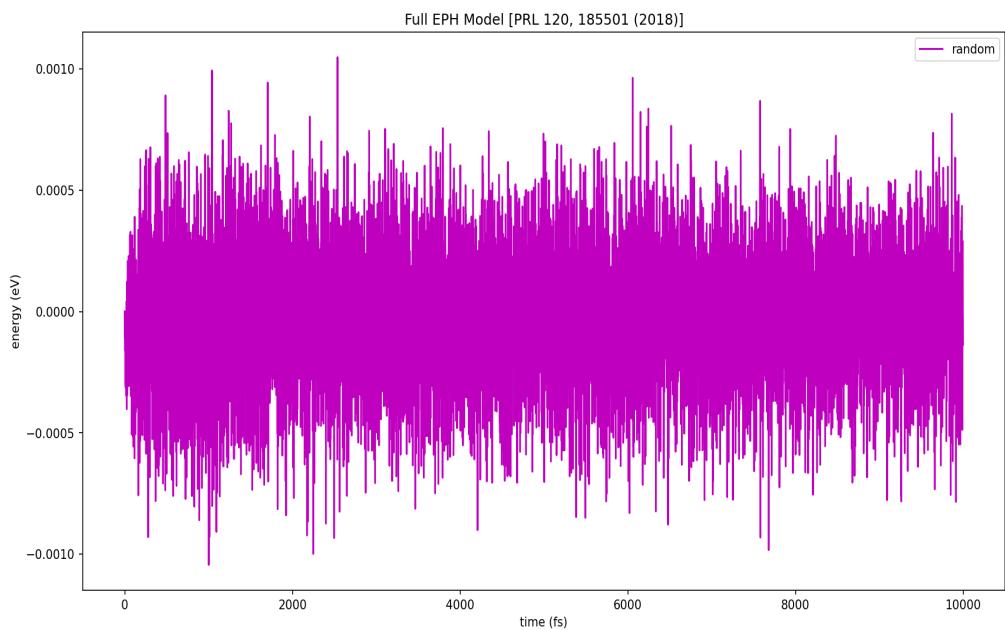
Coupling parameter – constant case
T. Jarrin et al. Phys. Rev. B 104, 195203

Electronic parameters:
 $C_e = 3.5 \times 10^{-6} \text{ eV/K/Å}^3$
 $K_e = 0.1248 \text{ eV/K/Å/ps}$

Electronic mesh:
Same size as atom box.
 $n_x = 1, n_y = 1, n_z = 1$.

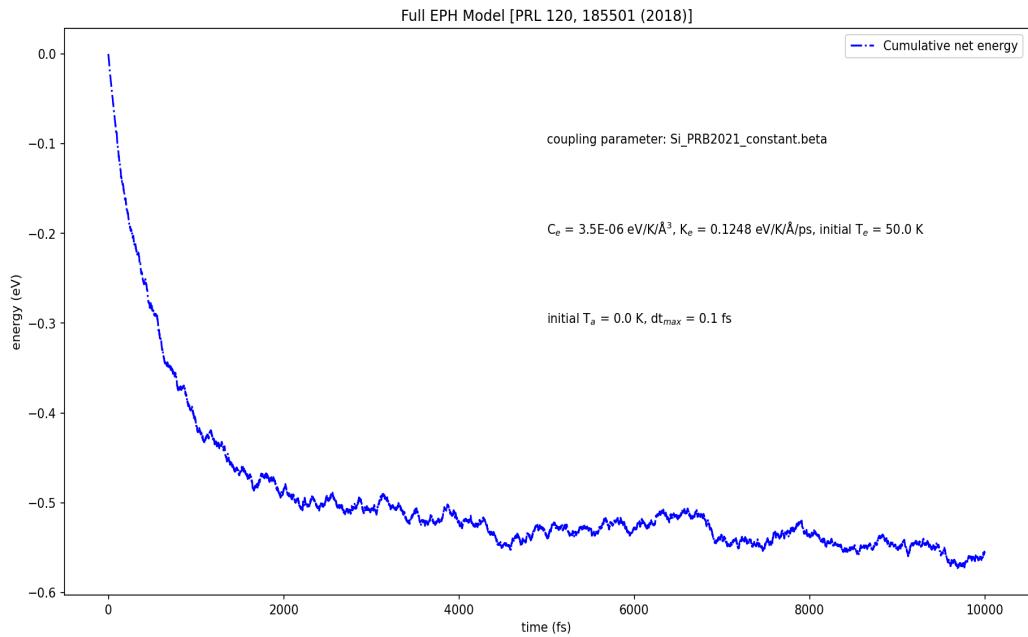


Variations of energies

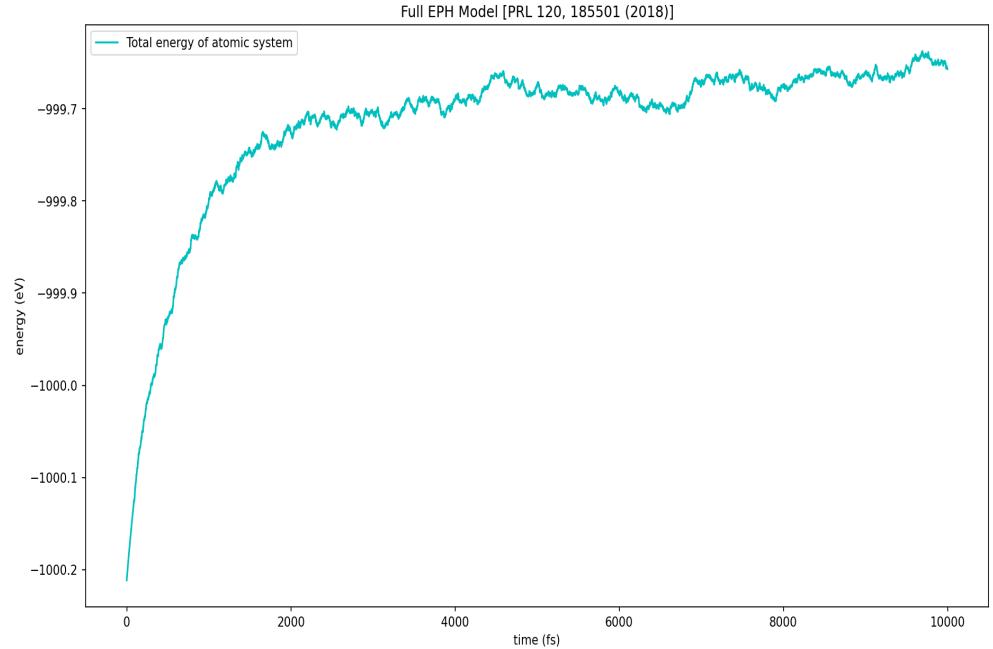


Variations of energies

Electronic system



Atomic system

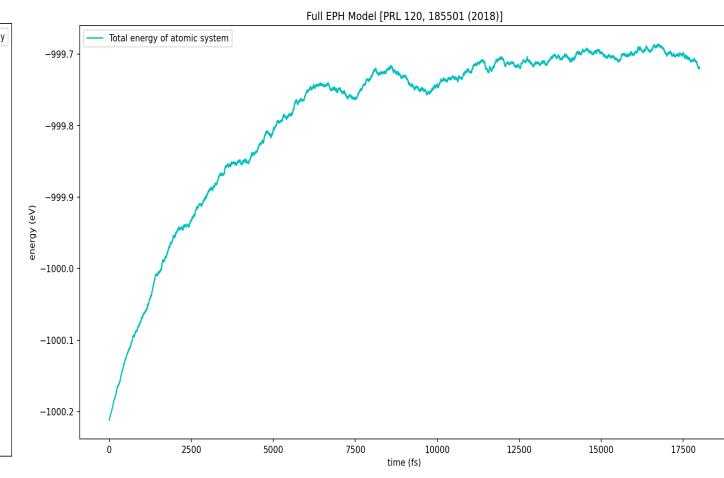
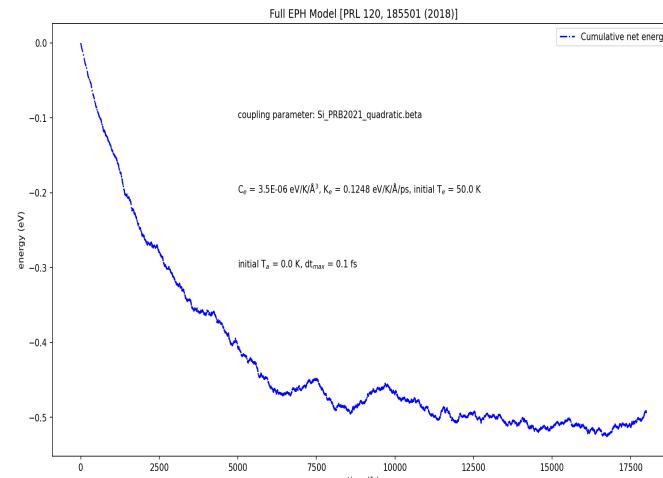
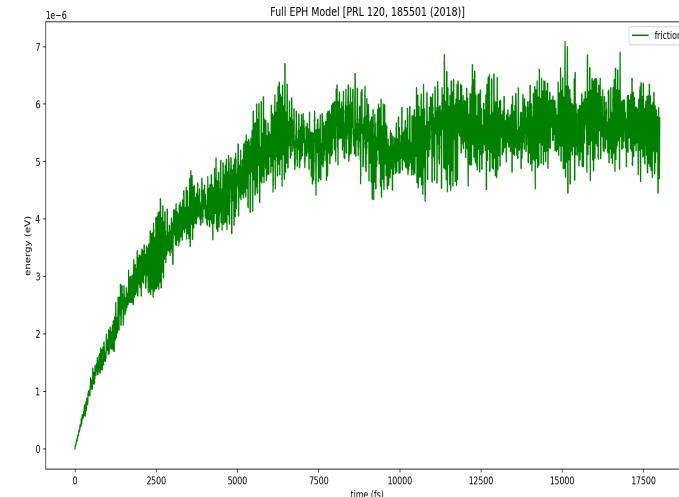
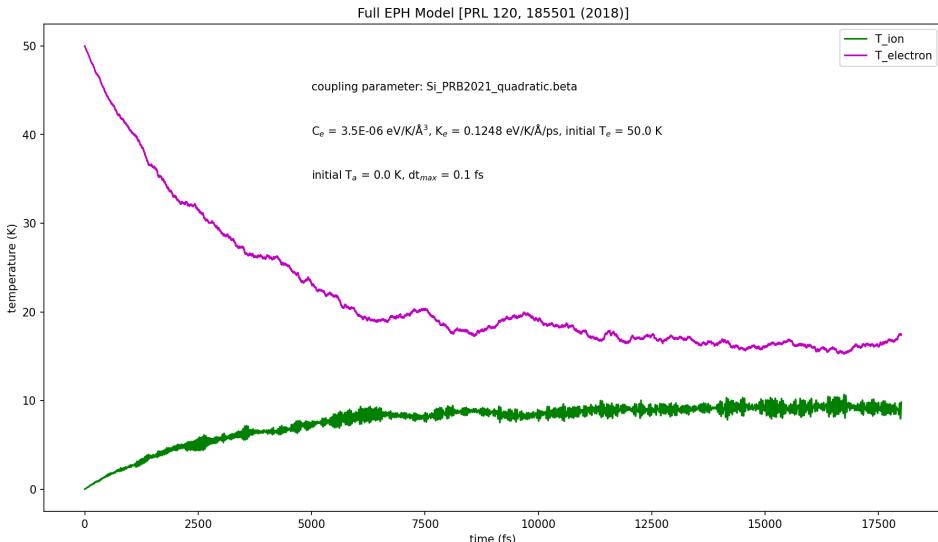


Energy is transferred from electronic to atomic system

Different coupling parameter will show some difference in rates to equilibrium.

There will also be some differences in the energies dissipated through friction and random forces

Coupling parameter – quadratic case T. Jarrin et al. Phys. Rev. B 104, 195203



Variations of atomic and electronic temperatures

The electronic mesh can be coarse or fine

216 Si atoms in a box of dimensions
16.29 Å on each side.

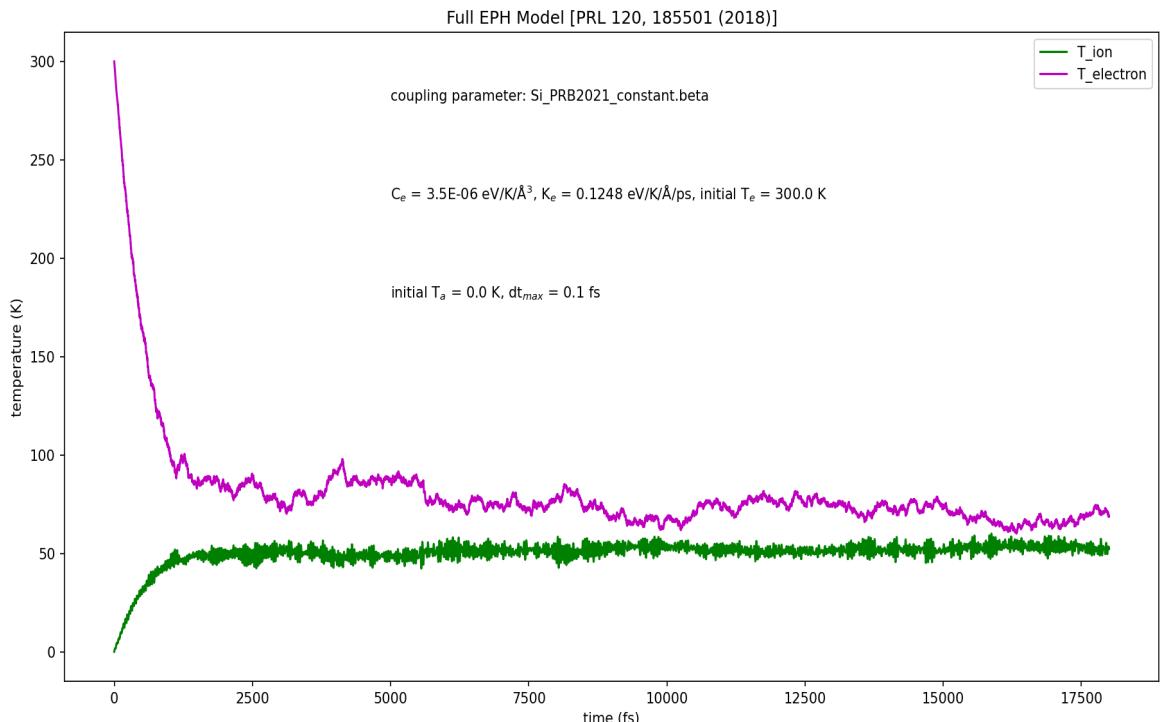
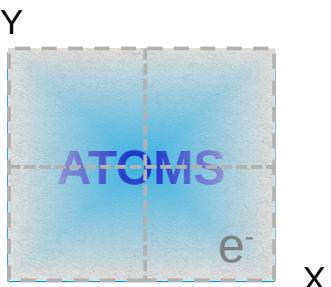
Initial atom temperature = 0.0 K

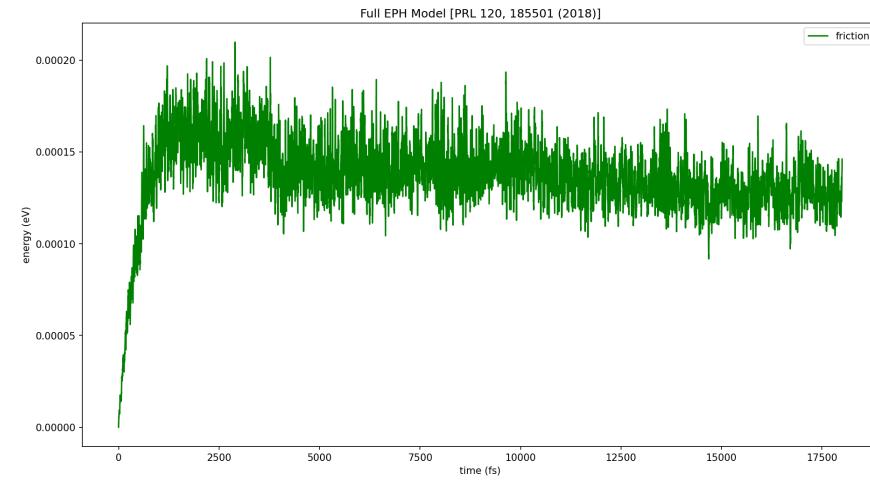
Initial electronic temperature = 300.0 K

Coupling parameter - constant case
T. Jarrin et al. Phys. Rev. B 104, 195203

Electronic parameters:
 $C_e = 3.5 \times 10^{-6} \text{ eV/K/Å}^3$
 $K_e = 0.1248 \text{ eV/K/Å/ps}$

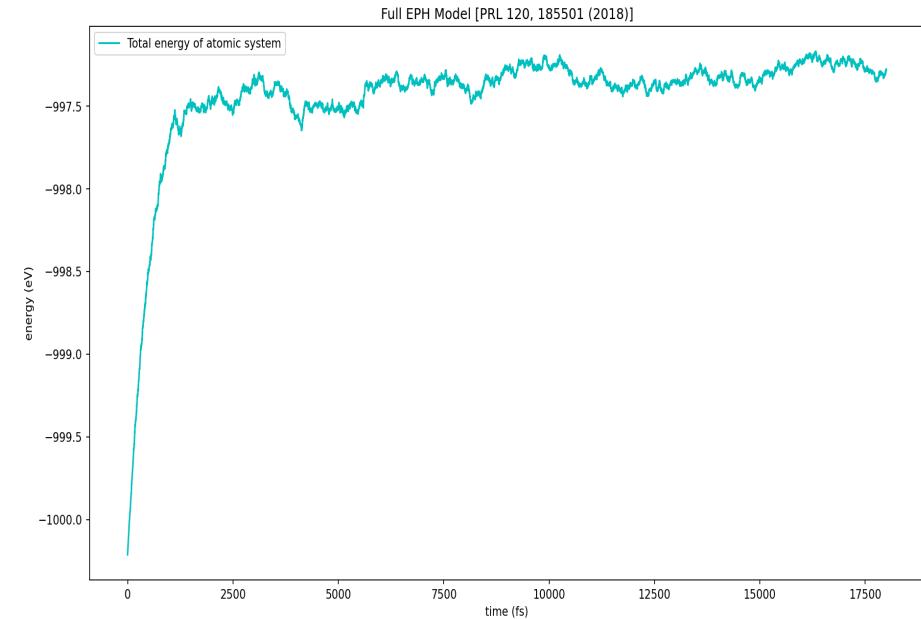
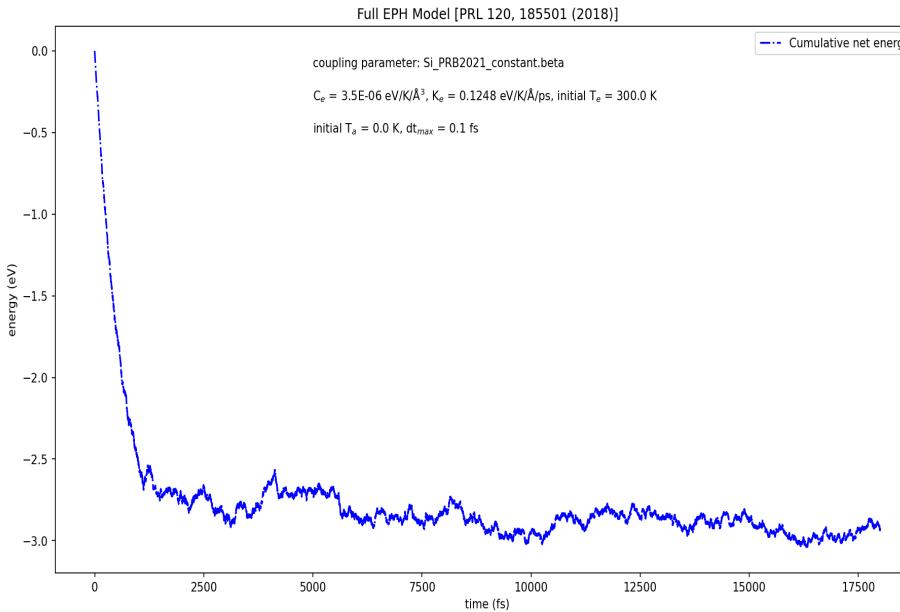
Electronic mesh:
Same size as atom box.
 $n_x = 2, n_y = 2, n_z = 2$.





Variations of energies

Energy is transferred from the electrons to the atoms



Along with the Coupling parameter, Electronic parameters also have effect on the Temperatures at equilibrium and the Energies dissipated

216 Si atoms in a box of dimensions 16.29 Å on each side.

Initial atom temperature = 0.0 K

Initial electronic temperature = 50.0 K

Coupling parameter – constant case
T. Jarrin et al. Phys. Rev. B 104, 195203

Electronic parameters:

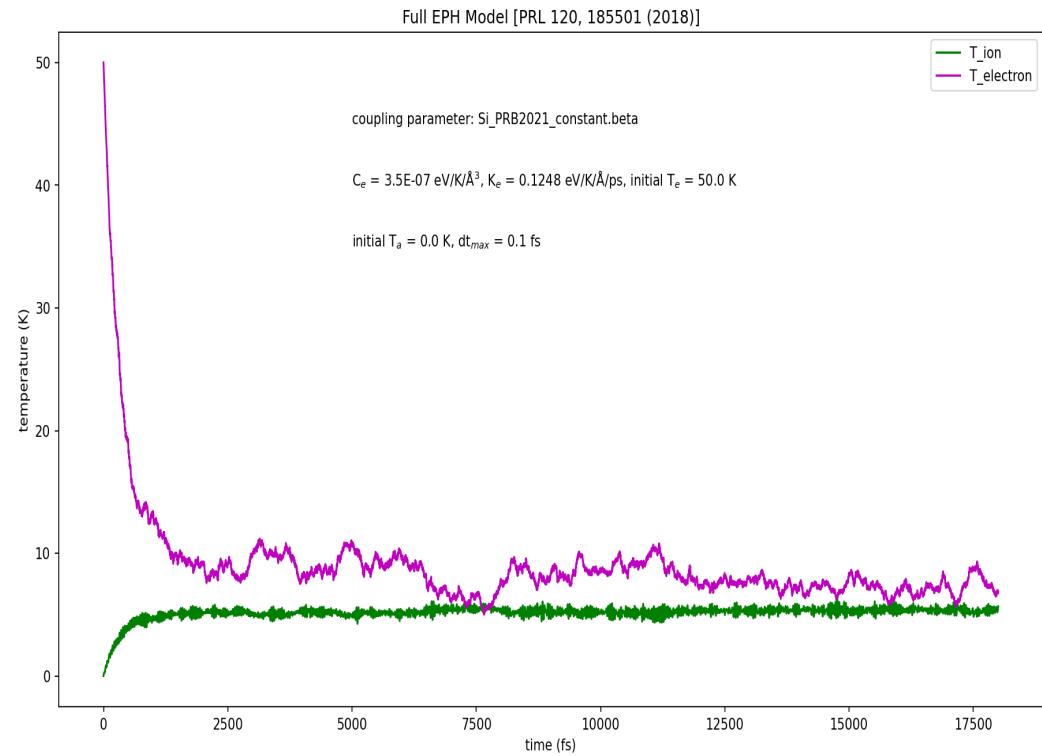
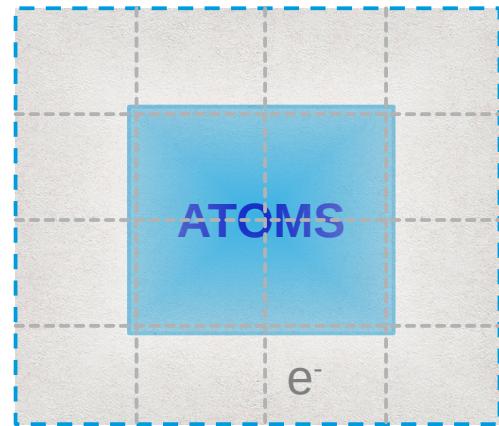
$C_e = 3.5 \times 10^{-7} \text{ eV/K/Å}^3$

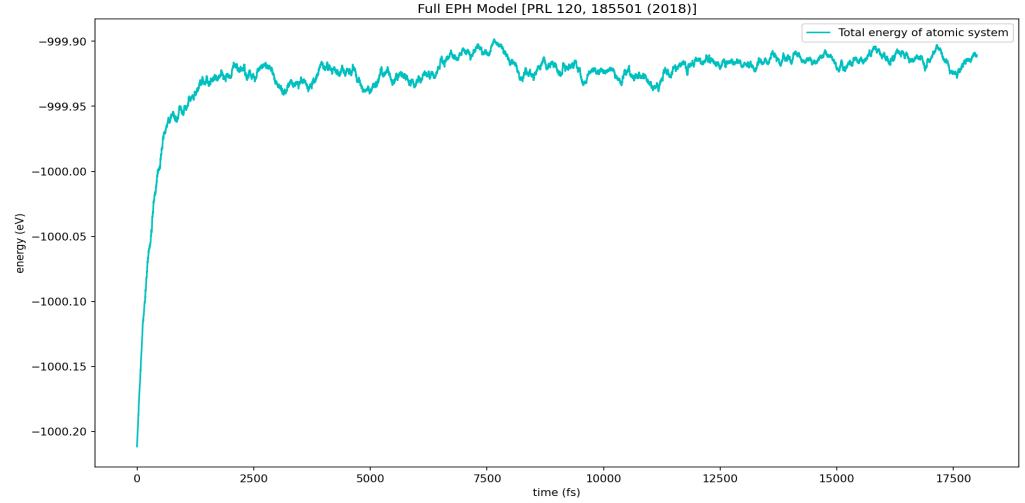
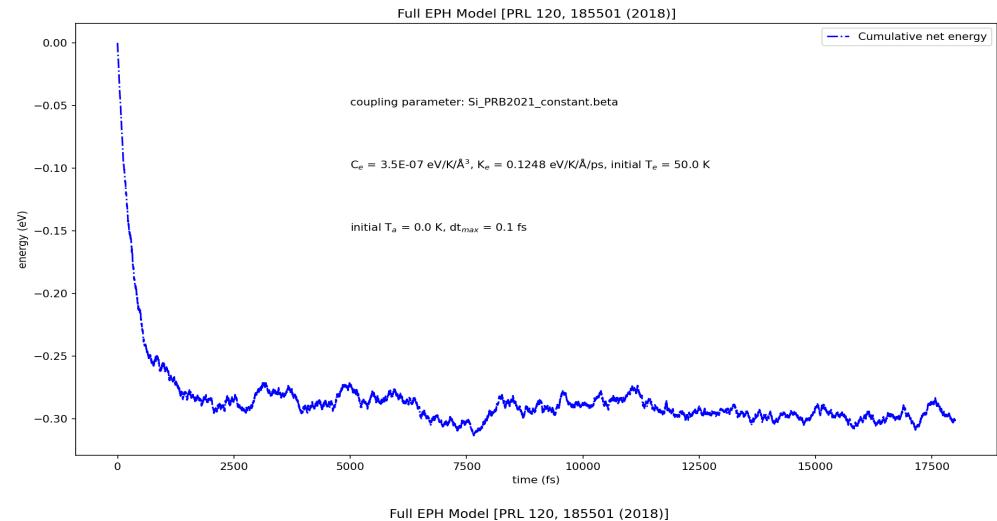
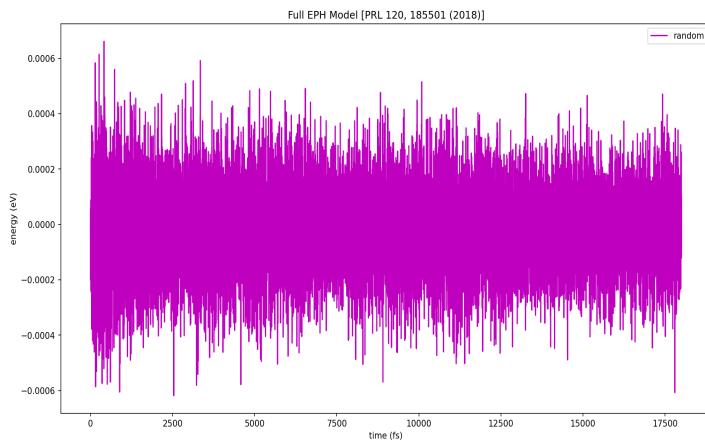
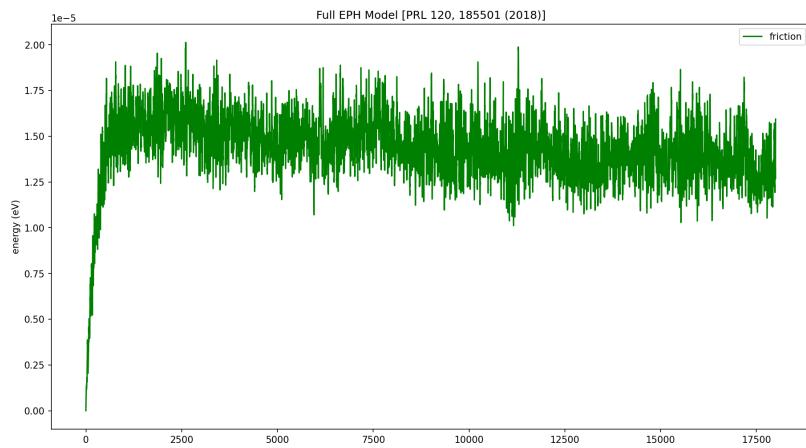
$K_e = 0.1248 \text{ eV/K/Å/ps}$

Electronic mesh:

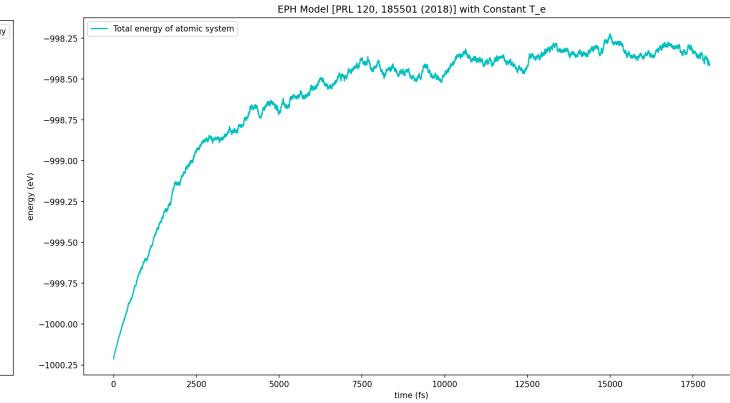
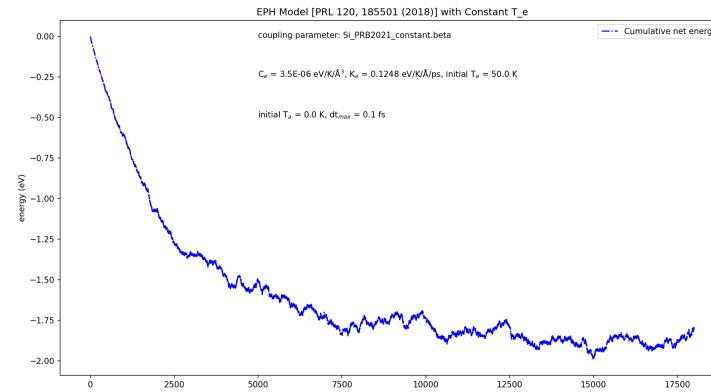
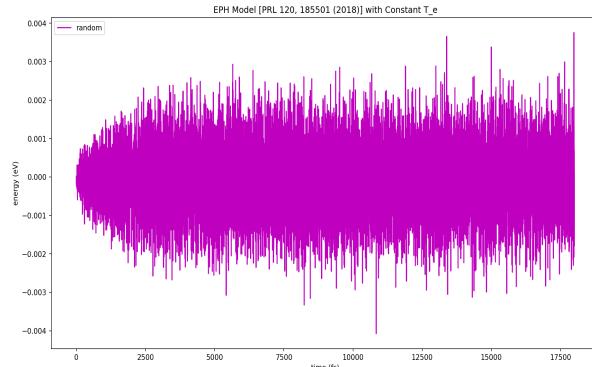
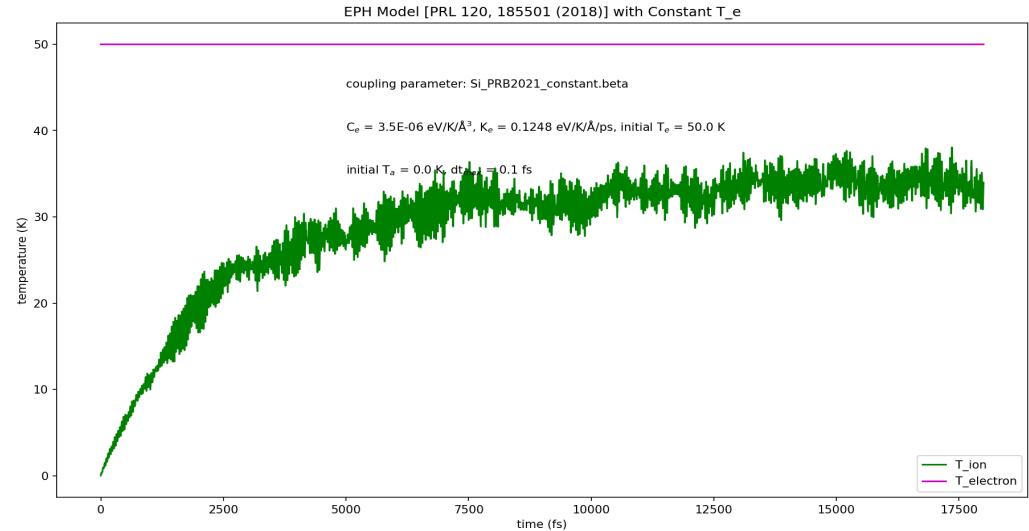
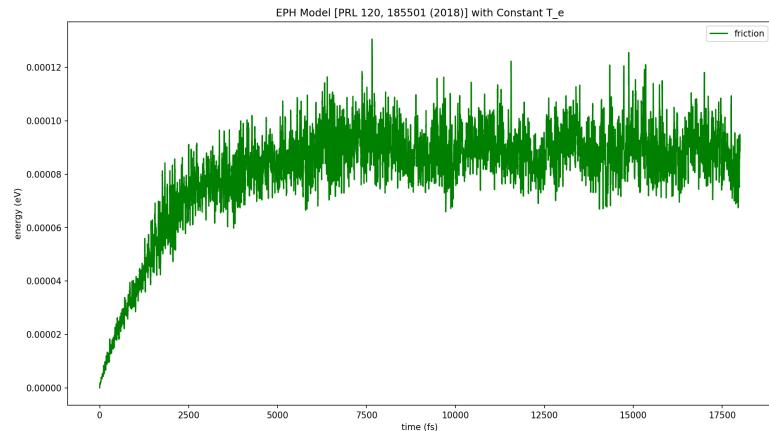
Larger than atom box
(-17.0 to +17.0 on each side).

$nx = 4, ny = 4, nz = 4$.

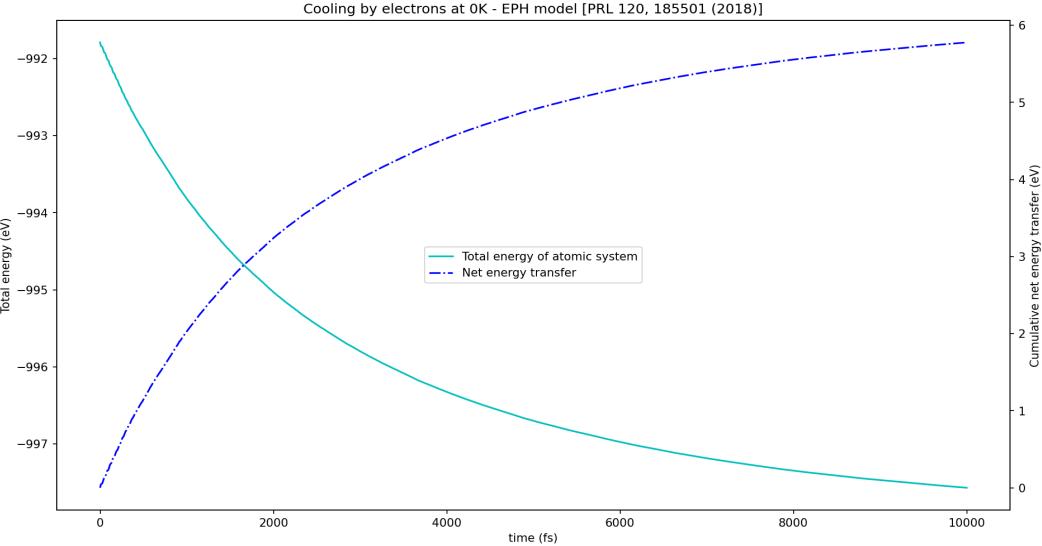
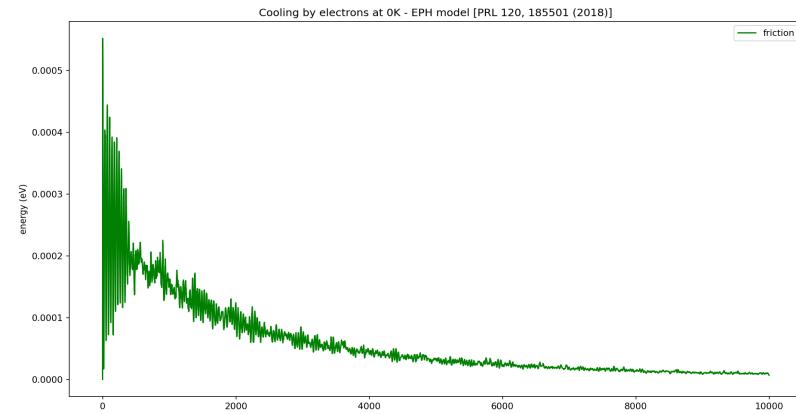
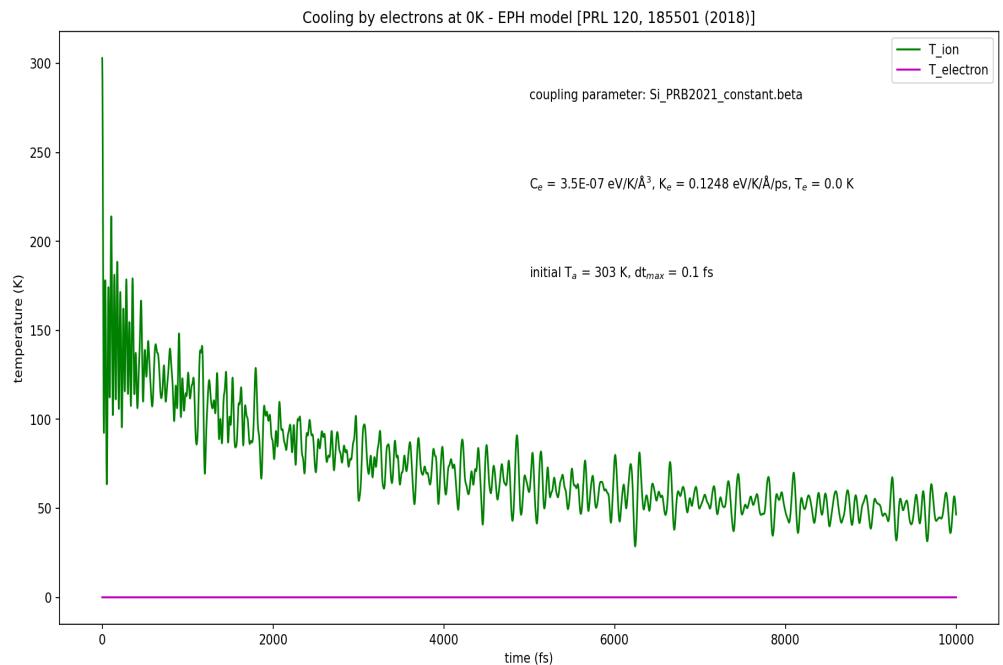




The electronic temperature is kept constant at 50 K and not updated



The system is cooled by friction forces only when the electrons are held at 0 K temperature



TurboGAP Simulations of 0.1 keV PKA with the EPH Model

EPH Model using Only Friction with 0.1 keV PKA

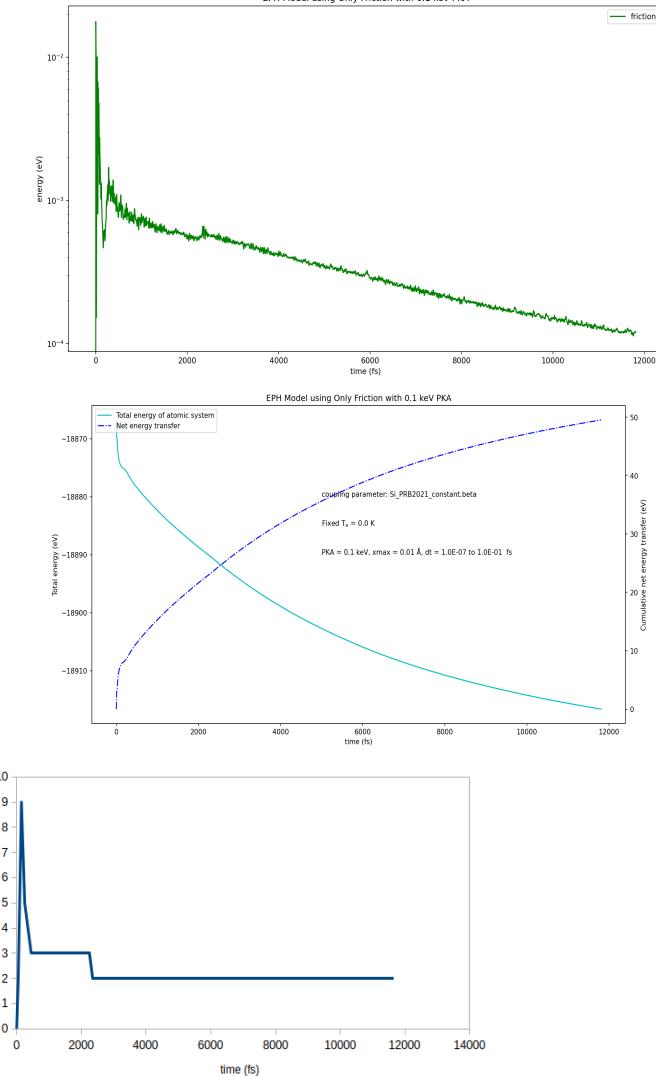
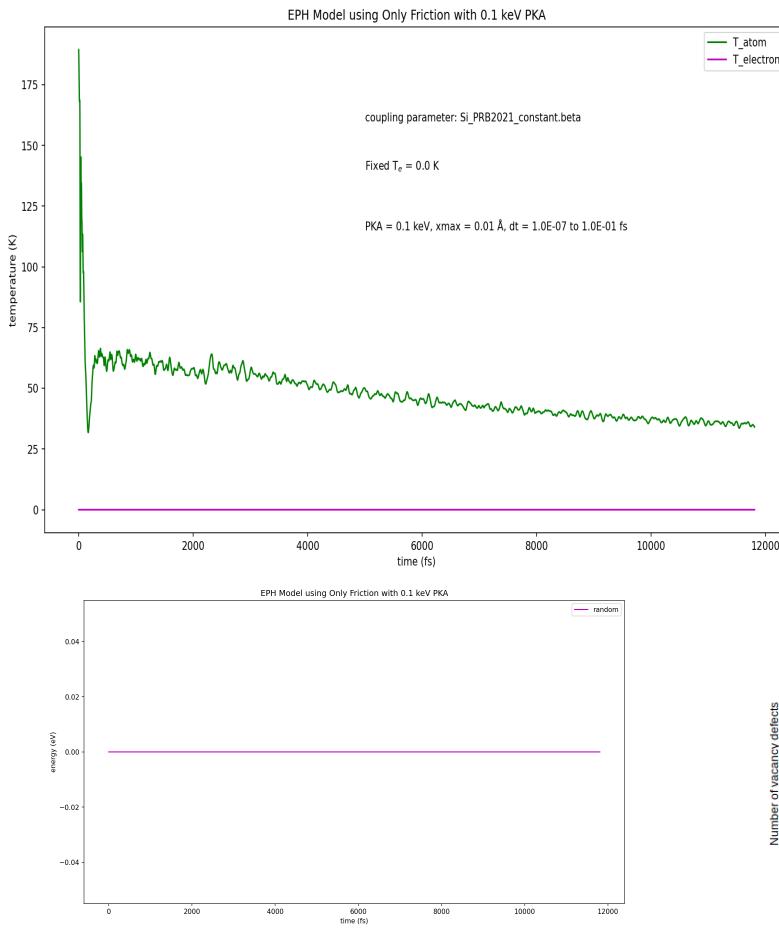
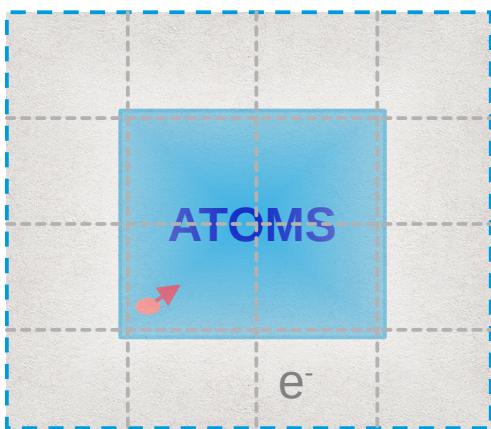
Only friction forces are applied

4096 Si atoms in a box of dimensions 43.44 \AA on each side.

Initial atom temperature $\sim 190 \text{ K}$ due to a 0.1 keV PKA

Initial and fixed electronic temperature = 0.0 K

Coupling parameter – constant case
T. Jarrin et al. Phys. Rev. B 104, 195203



TurboGAP Simulations of 0.1 keV PKA with the EPH Model

4096 Si atoms in a box of dimensions 43.44 Å on each side.

Initial atom temperature ~ 190 K due to a 0.1 keV PKA

Initial electronic temperature = 0.0 K

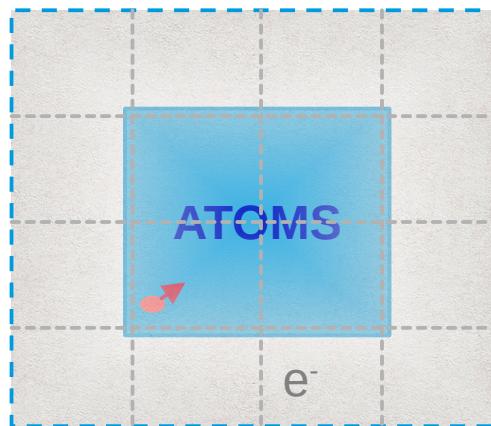
Coupling parameter – constant case
T. Jarrin et al. Phys. Rev. B 104, 195203

Electronic parameters:
 $C_e = 3.5 \times 10^{-6} \text{ eV/K/Å}^3$
 $K_e = 0.1248 \text{ eV/K/Å/ps}$

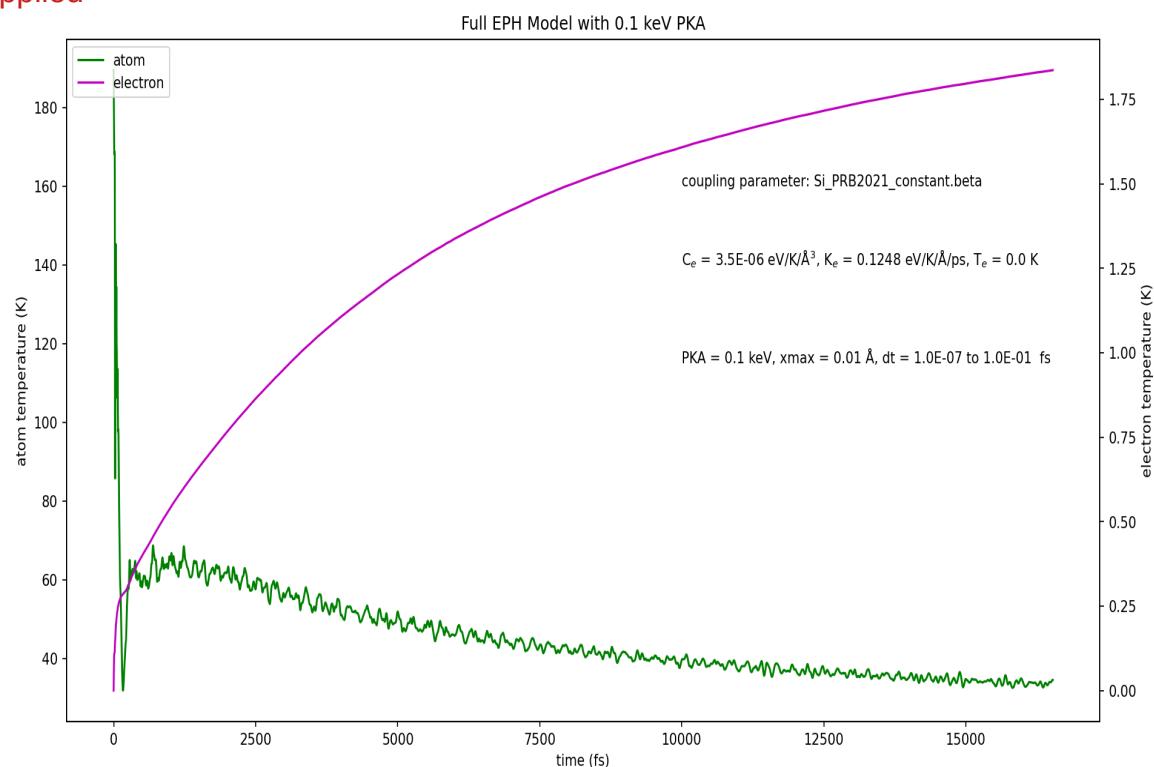
Electronic mesh:

Larger than atom box
(-100.0 to +100.0 on each side).

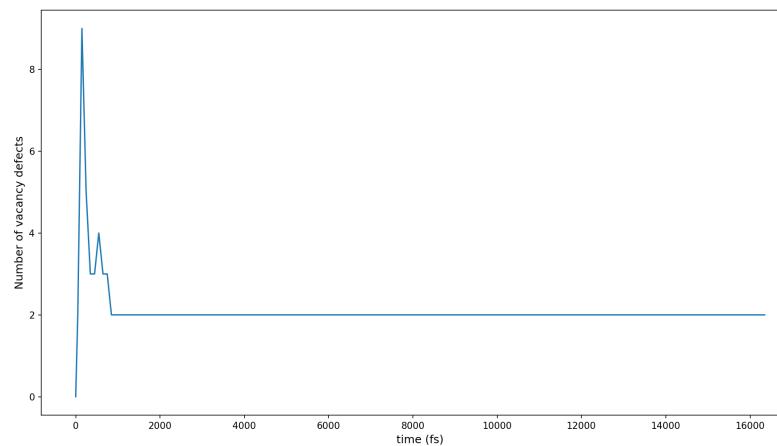
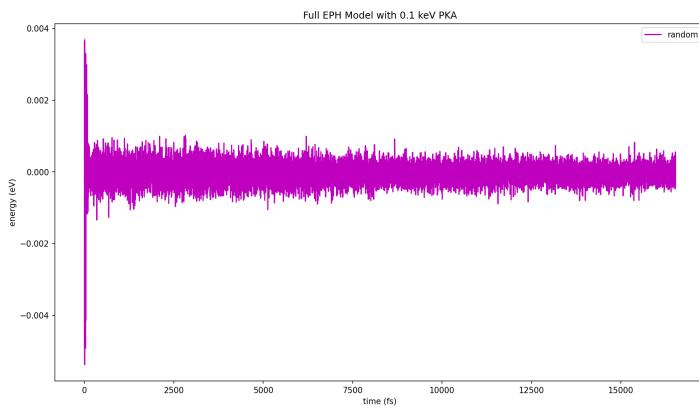
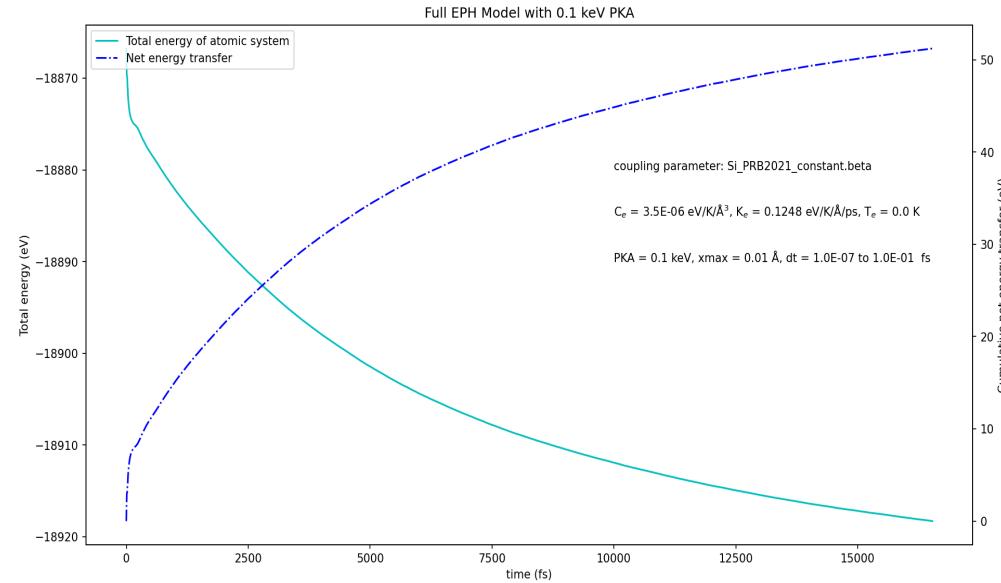
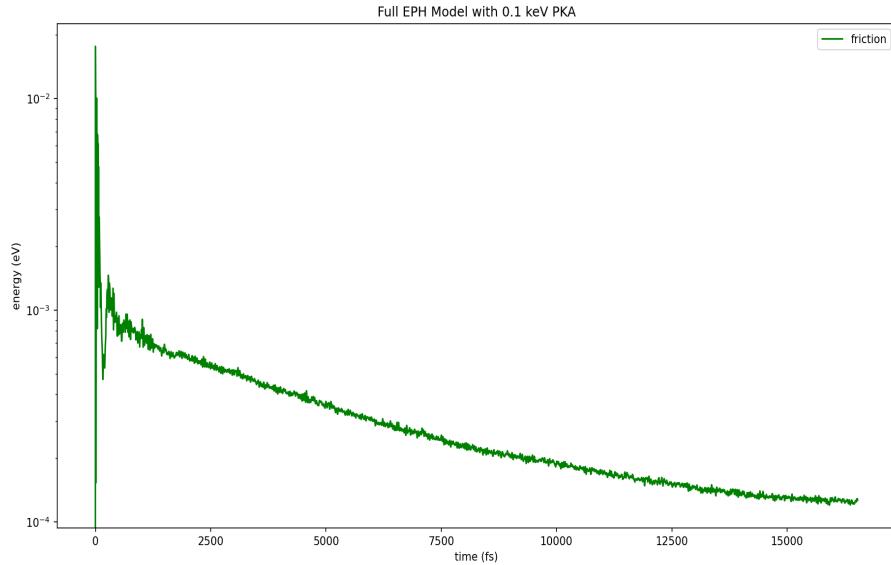
$nx = 4, ny = 4, nz = 4$.



Full model is applied



TurboGAP Simulations of 0.1 keV PKA with the EPH Model



Details of input in Adaptive time, Electronic stopping and Non-adiabatic processes

1. Adaptive time

```
adaptive_time = .true.  
adapt_tstep_interval = 1  
adapt_tmin = 1.0E-07  
adapt_tmax = 1.0E-01  
adapt_xmax = 1.0e-3  
adapt_emax = 50.0
```

The adaptive time steps can be switched off by writing *adaptive_time* = *.false.*, and then the following five lines of input will not be used. In order to apply this algorithm only after a certain number of time steps, the particular value should be specified to *adapt_tstep_interval*. A value of 1 means applying the algorithm at every time step and a value of 10 means applying it after every 10 time steps. The next two input lines are given to provide the maximum (*adapt_tmax*) and minimum (*adapt_tmin*) limits a time step can take. The current time step is determined primarily based on the value of maximum possible displacement for an atom that is provided in *adapt_xmax*. A value of 0.001 here implies that an atom moves maximum of 0.001 Å in a time step. Similarly, the maximum amount of kinetic energy that can be transferred in a time step is provided through the input keyword *adapt_emax*. A time step is chosen such that the distance that may be moved by any of the atoms is less than the maximum allowed displacement and the energy transfer is less than the maximum allowed value. If the allowed maximum displacement prompts for a time step that breaks the maximum energy transfer criterion, then the time step based on the latter condition is chosen. This implies that the smallest time step is always chosen. So, a reasonable value of at least one of the two criteria must be provided to do the simulation using a reasonable time step.

Default values:

```
adaptive_time = .false., adapt_tstep_interval = 1, adapt_tmin = 0.001, adapt_tmax = 1.0,  
adapt_xmax = 0.1, adapt_emax = 10.0.
```

2. Electronic stopping based on stopping data from SRIM-2013

```
electronic_stopping = .true.  
eel_cut = 1.0  
eel_freq_out = 10  
estop_filename = 'stopping-data-file'
```

The execution of this part can be switched off by writing *electronic_stopping* = *.true.*. When it is executed the supposed electronic energy losses of the atoms are subtracted from the atoms as if

frictional forces are acting on them depending on their velocities. It is based on the electronic stopping data from SRIM-2013 that is provided. The energy is subtracted from an atom till its energy is less than a cut-off value, which is provided through the input *eel_cut*. So, a value of 1.0 here means when an atom has energy higher than 1.0 eV electronic energy loss is calculated for it and the forces on the atom is reduced due to friction. The values of electronic energy loss with the progress of the simulation time are printed to an output file at certain interval of time steps. This frequency of output energy data can be provided through the input keyword *eel_freq_out*. The output file from this calculation will contain the per-time step and cumulative electronic energy losses with time in a file called ElectronicEnergyLoss.txt. The input stopping data is provided in a text file, the name of which is mentioned by using the input keyword *estop_filename*. The data for stopping in the text file should be provided such that it covers the value of energy given in *eel_cut*. For multiple elements in a target material, the stopping powers of the elements should be arranged in columns in order as they appear in the ‘species’ input. The text file must contain data in the following format:

1st line – Any useful information
 2nd line – number of rows of data points, N
 3rd line – energy unit (eV) and symbols of elements (must be) in order as in input file
 4th line onwards – energy and electronic stopping values (eV/Ang) are provided as follows
 4th line – $E_1 \quad (dE/dx)_1(\text{element 1}) \quad (dE/dx)_1(\text{element 2}) \quad (dE/dx)_1(\text{element 3}) \quad \dots$
 5th line – $E_2 \quad (dE/dx)_2(\text{element 1}) \quad (dE/dx)_2(\text{element 2}) \quad (dE/dx)_2(\text{element 3}) \quad \dots$

 Nth line – $E_N \quad (dE/dx)_N(\text{element 1}) \quad (dE/dx)_N(\text{element 2}) \quad (dE/dx)_N(\text{element 3}) \quad \dots$

Default values:

electronic_stopping = .false., eel_cut = 1.0, eel_freq_out = 1, estop_filename = 'NULL'.

3. Non-adiabatic processes

```
nonadiabatic_processes = .true.
eph_fdm_option = 1
eph_friction_option = 1
eph_random_option = 1
eph_betafile = 'beta-rho-data-file'
eph_Tinfile = 'input-fdm-electron-mesh-file'
box_limits = 0.0 28.0 0.0 28.0 0.0 28.0
eph_rho_e = 1.0
eph_C_e = 1.0E-06
eph_kappa_e = 0.8
eph_Ti_e = 5.0
eph_gsx = 4
```

```

eph_gsy = 4
eph_gsz = 4
eph_fdm_steps = 1
eph_md_last_step = 0
eph_md_prev_time = 0.0
eph_E_prev_time = 0.0
eph_freq_Tout = 100
eph_freq_mesh_Tout = 100000
eph_Toutfile = 'output-electron-mesh-fdm-file'

```

This calculation option can be switched off by writing *nonadiabatic_processes* = *.true.*, then none of the following lines of input will be used. The electron-phonon coupling that governs the energy dissipation during a radiation cascade event is described by the e-ph model [PhysRevLett.120.185501, PhysRevB.99.174302]. This model is now implemented in TurboGAP. The output data for energy transfer and average electronic temperature can be taken at some desired frequency which is specified through input keyword *eph_freq_Tout* and keeping it equal to the frequency of the original thermo-output (in thermo.log file) can be useful. The values of electronic parameters for the mesh may be needed only at some larger intervals (or may be the last one only) because this file will be generally large, so *eph_freq_mesh_Tout* input keyword can be set accordingly. The per-time step friction energy, random energy and the cumulative net energy (due to both friction and random energy exchanges) and the electronic temperature are provided in the output file eph-EnergySharingData.txt. When a simulation has to be restarted from a previous one then the *eph_md_last_step* input keyword can be set with value of previous number of steps already run to keep track of the number steps. By default it is equal to 0. Likewise, the input keywords *eph_md_prev_time* and *eph_E_prev_time* are then set to the MD time and the net cumulative energy transferred values corresponding to the last step of the previous run. If a restarted simulation is being run and the value in *eph_md_last_step* input keyword is set to the value of last step of previous simulation (say, N), then the output file for the mesh parameters shows values for the Nth + present_step. The output quantities in the file eph-EnergySharingData.txt are also updated by addition to their last values from the previous run. Note, the actual MD time in the thermo.log file has to be manually added from previous data in case of restarted runs. The name of the file for getting this mesh parameters output is provided through input keyword *eph_Toutfile*. By default its value is ‘NULL’. The data for electron-ion coupling parameter obtained from TD-DFT simulations is provided in a text file using the input keyword *eph_betafile*. By default its value is ‘NULL’. The input keyword *eph_Tinfile* is used to provide the text file name where several of the input parameters are provided. By default its value is ‘NULL’. If this file is not provided then the input keywords in the next nine lines of input are required. Either the data provided through text file in

eph_Tinfile or the following nine input keywords will define the mesh for electronic heat bath along with the required parameters. The format of this parameter file is as follows:

1st line – any comments
2nd line – any comments
3rd line – any comments
4th line – gsx gsy gsz number_of_fdm_steps
5th line – mesh_xlimit_low mesh_xlimit_high
6th line – mesh_ylimit_low mesh_ylimit_high
7th line – mesh_zlimit_low mesh_zlimit_high
8th line – i j k T_e S_e rho_e C_e K_e flag T_dyn_flag
9th line – values corresponding to column headers
10th line – values corresponding to column headers
....
....
Values are provided for the full mesh.

Alternative to providing the above data, the mesh can also be constructed for solving the heat diffusion equation with the following inputs:

box_limits = 0.0 28.0 0.0 28.0 0.0 28.0 make the boundaries of the mesh where the values are for xlow, xhi, ylow, yhi, zlow and zhi, respectively;
eph_rho_e = 1.0 value of rho_e as in data file;
eph_C_e = 1.0E-06 value of C_e as in data file;
eph_kappa_e = 0.8 value of K_e as in data file;
eph_Ti_e = 5.0 value of T_e as in data file;
eph_gsx = 4 number of small mesh boxes along x;
eph_gsy = 4 number of small mesh boxes along y;
eph_gsz = 4 number of small mesh boxes along z;
eph_fdm_steps = 1 same as the *number_of_fdm_steps* which is used to find the time step for solving heat diffusion equation.

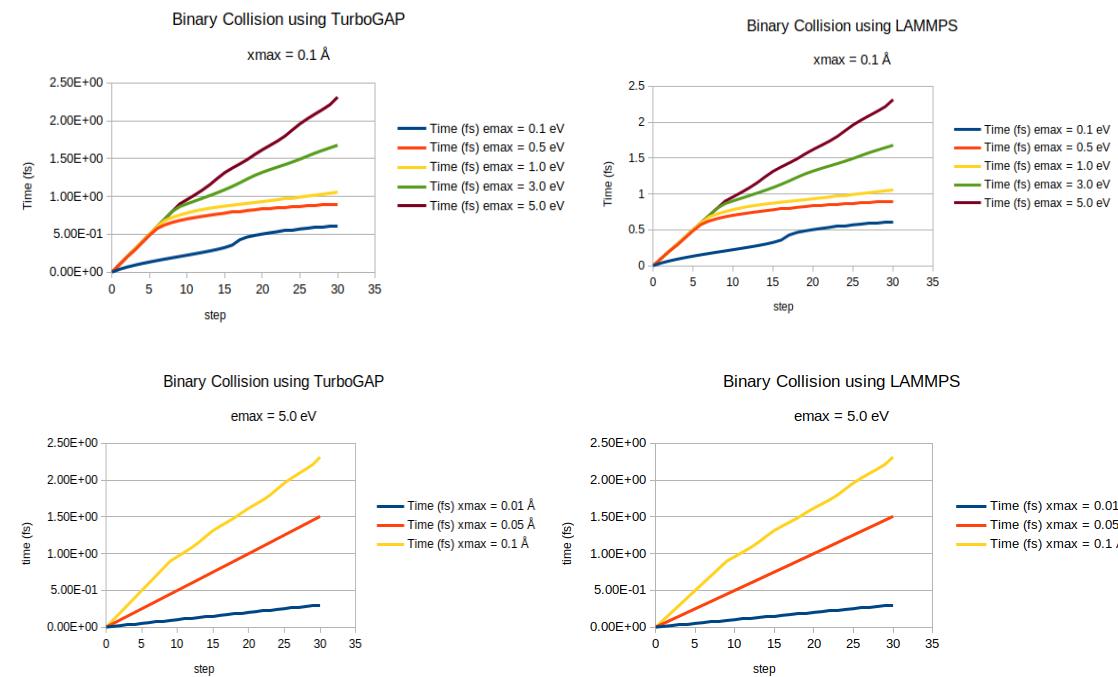
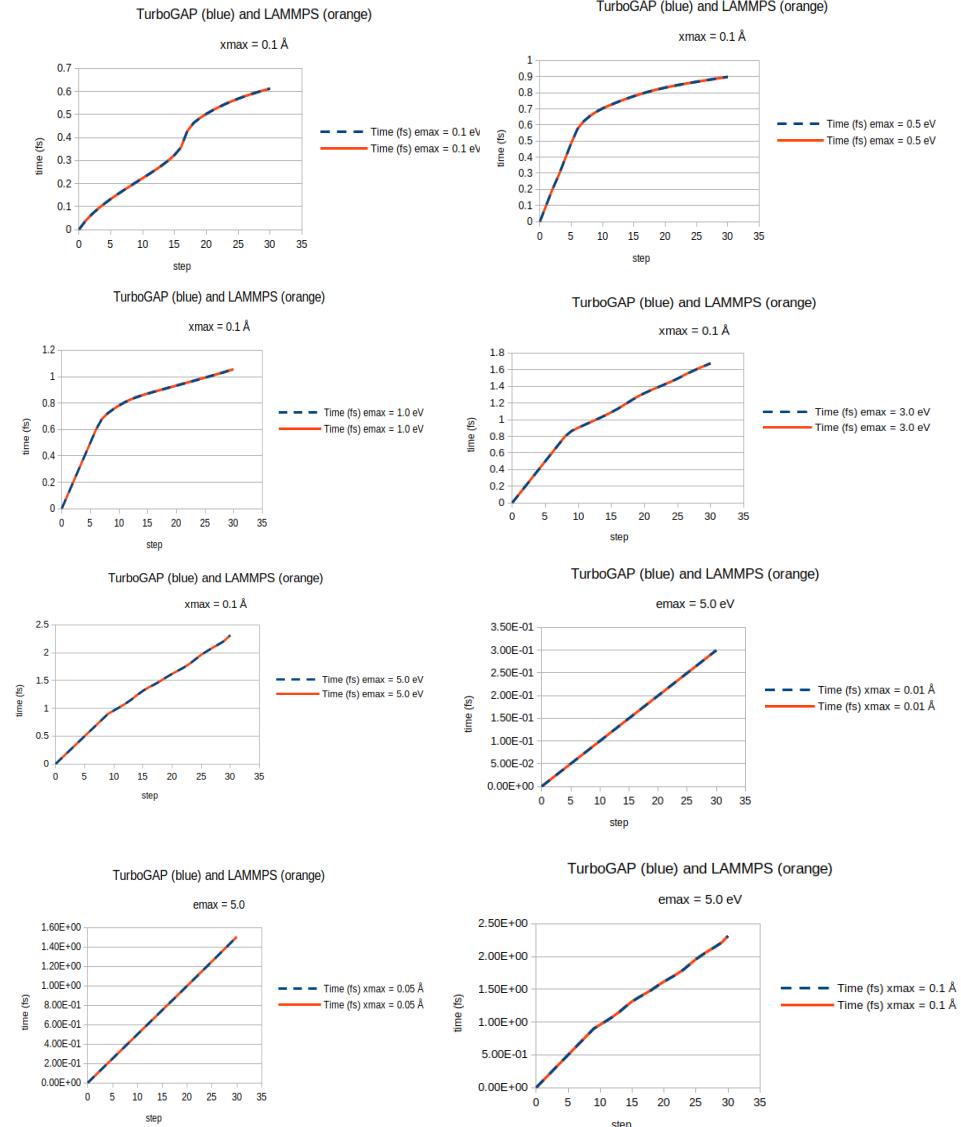
By default the full electron-phonon coupling along with electronic friction is implemented. But the user has some options for implementing only the parts of the model if required. There are three input keywords by which this can be done, viz., *eph_fdm_option*, *eph_friction_option*, *eph_random_option*. The finite difference method for solving heat diffusion equation for the electrons is governed by *eph_fdm_option*. The electronic energy loss from the atoms is governed by *eph_friction_option* and the mutual transfer of energy between the electrons and atoms (lattice phonon) through random forces is governed by *eph_random_option*. All of these three input keywords can take two values as input either 0 or 1, where 0 and 1 imply switch on and switch off,

respectively, of the process. If the *eph_fdm_option* is kept as ‘0’, then the updating of electronic temperature by solving the heat diffusion equation will not be done. If *eph_friction_option* is kept ‘0’ the frictional forces due to ion-electron interactions will not be calculated and if *eph_random_option* is kept ‘0’ the random forces of electron-phonon coupling will not be calculated. Since by default the full model is implemented, all these input keywords have default values set to ‘1’. So providing these inputs may be skipped unless any one of these needs to be turned off. Note that if both *eph_friction_option* and *eph_fdm_option* are turned off, then the *eph_fdm_option* should as well be turned off.

Default values:

```
nonadiabatic_processes = .false., eph_fdm_option = 1, eph_friction_option = 1, eph_random_option = 1, eph_md_last_step = 0, eph_freq_Tout = 1, eph_freq_mesh_Tout = 1, eph_fdm_steps = 1, eph_gsx = 1, eph_gsy = 1, eph_gsz = 1, model_eph = 1, eph_rho_e = 1.0, eph_C_e = 1.0, eph_kappa_e = 1.0, eph_Ti_e = 300.0, eph_E_prev_time = 0.0, eph_md_prev_time = 0.0, eph_Tinfile = 'NULL', eph_Toutfile = 'NULL', eph_betafile = 'NULL'.
```

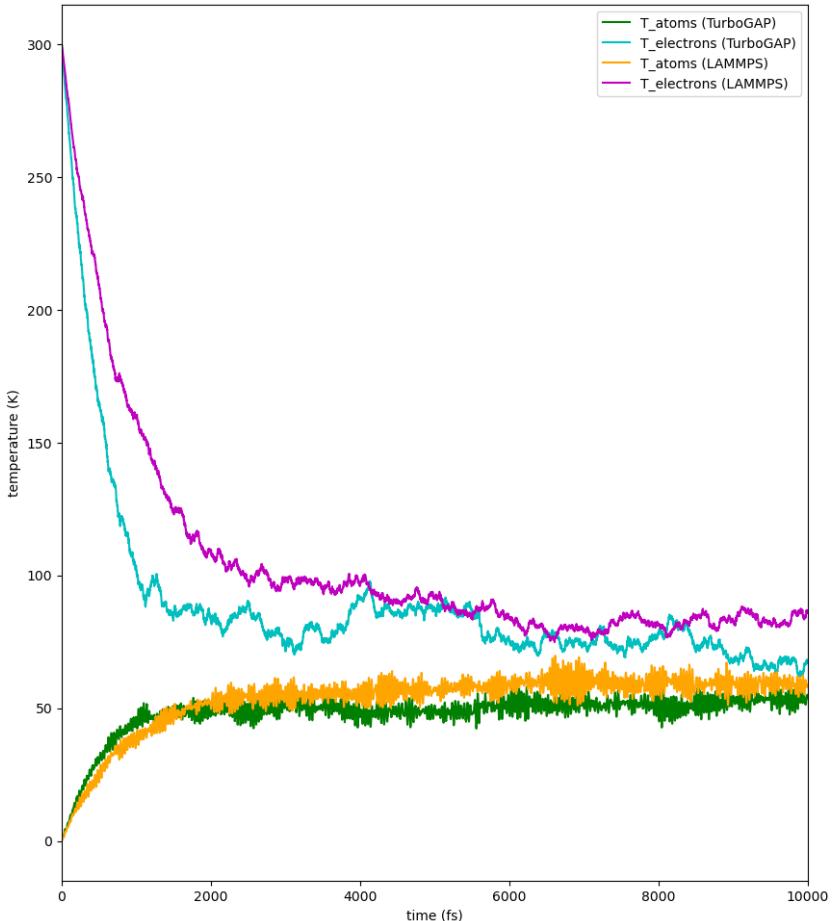
Binary Collision Simulations using TurboGAP and LAMMPS and sensitivity of time step to xmax and emax criterion



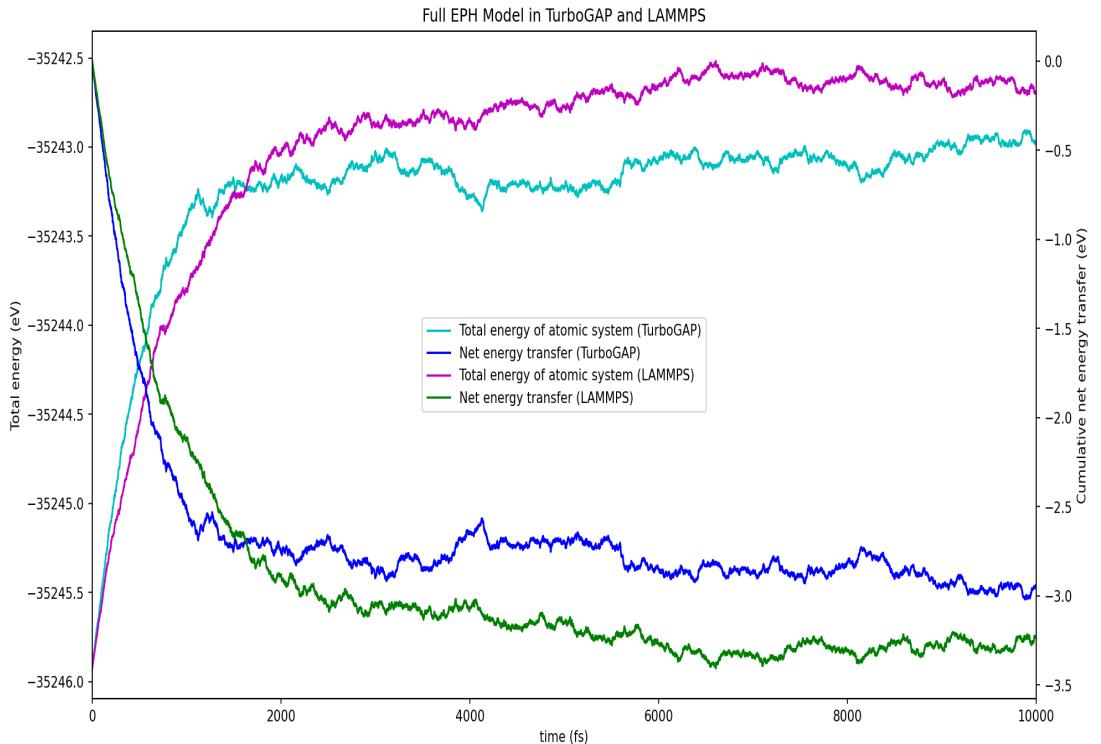
Two Si atoms
at $r_1 = (10.0, 10.0, 10.0)$ and $r_2 = (13.0, 10.0, 10.0)$
with $v_{1x} = 1.0 \text{ \AA/fs}$, $v_{1y} = 0.0$, $v_{1z} = 0.0$ and $v_{2x} = -1.0 \text{ \AA/fs}$, $v_{2y} = 0.0$, $v_{2z} = 0.0$
using Si (not stiffened) GAP potential.

Full EPH model in TurboGAP and LAMMPS

216 Si atoms in a box of dimensions 16.29 Å on each side.
Using Si-GAP potential.



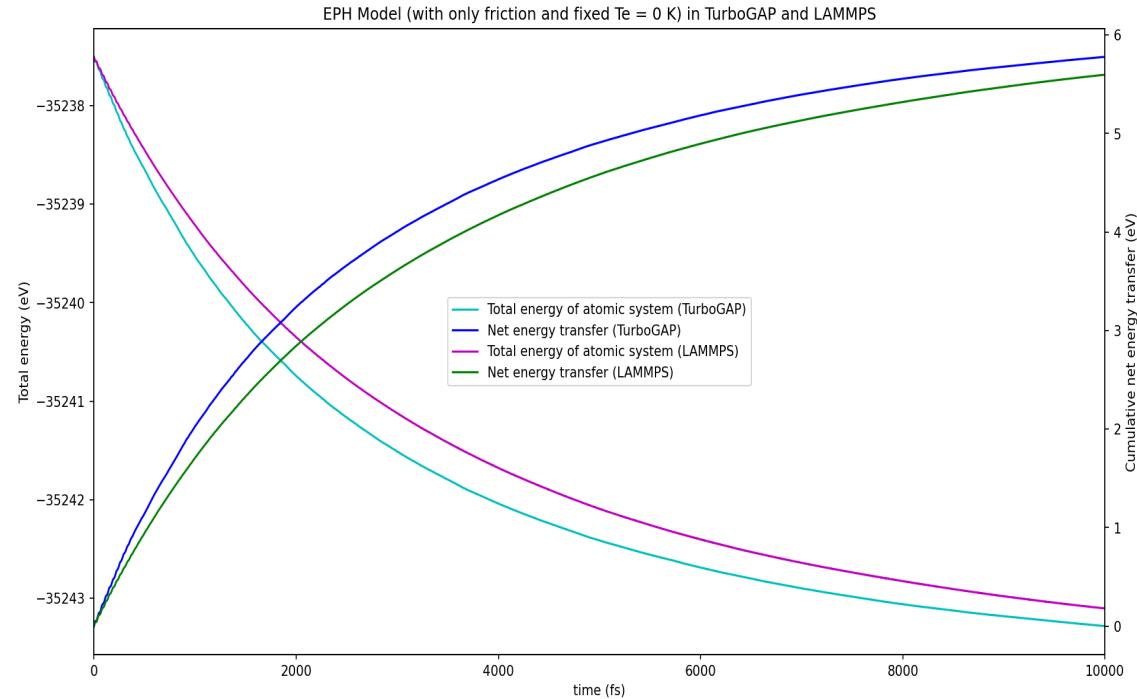
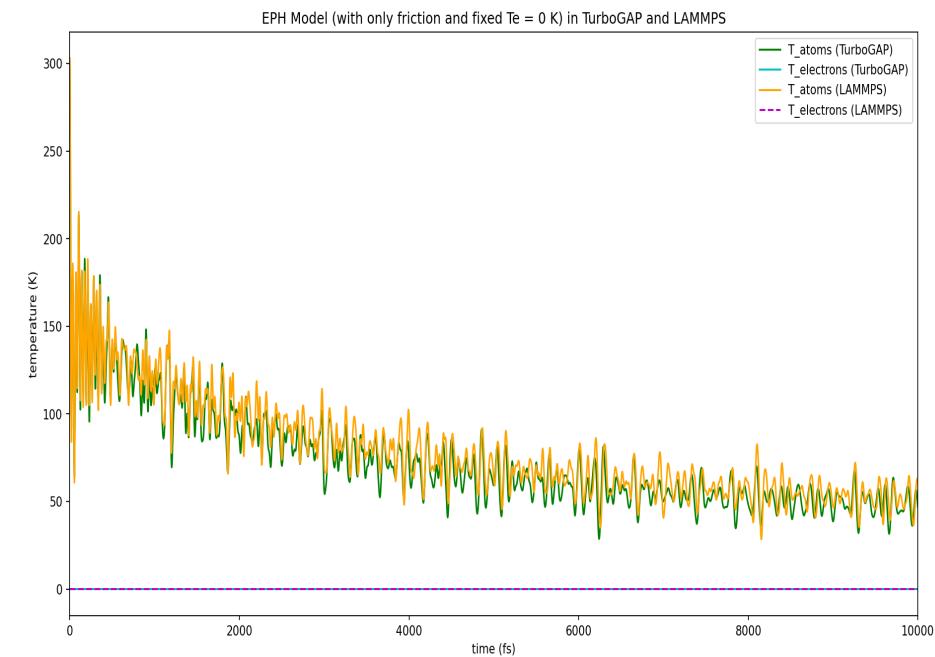
T_a = 0.0 K, T_e = 300.0 K
Coupling parameter – constant case
T. Jarrin et al. Phys. Rev. B 104, 195203
C_e = 3.5*10⁻⁶ eV/K/Å³
K_e = 0.1248 eV/K/Å/ps



EPH model (with only friction and fixed Te = 0.0 K) in TurboGAP and LAMMPS

216 Si atoms in a box of dimensions 16.29 Å on each side.
Using Si-GAP potential.

T_a = 303 K, constant Te = 0.0 K
Coupling parameter – constant case of T. Jarrin et al. Phys. Rev. B 104, 195203
 $C_e = 3.5 \times 10^{-7} \text{ eV/K/Å}^3$
 $K_e = 0.1248 \text{ eV/K/Å/ps}$



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