Homework 1

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Problem 1

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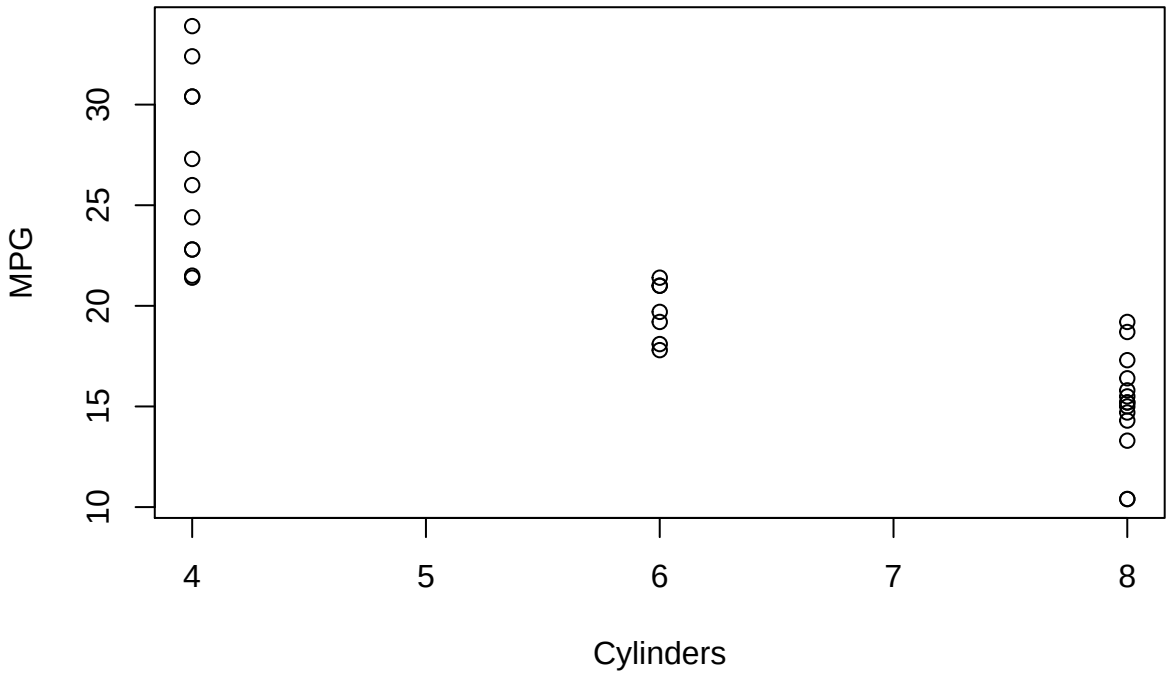
Problem 2

(a)

```
myCars <- mtcars
plot(mpg ~ cyl, myCars, xlab="Cylinders",ylab="MPG")
```



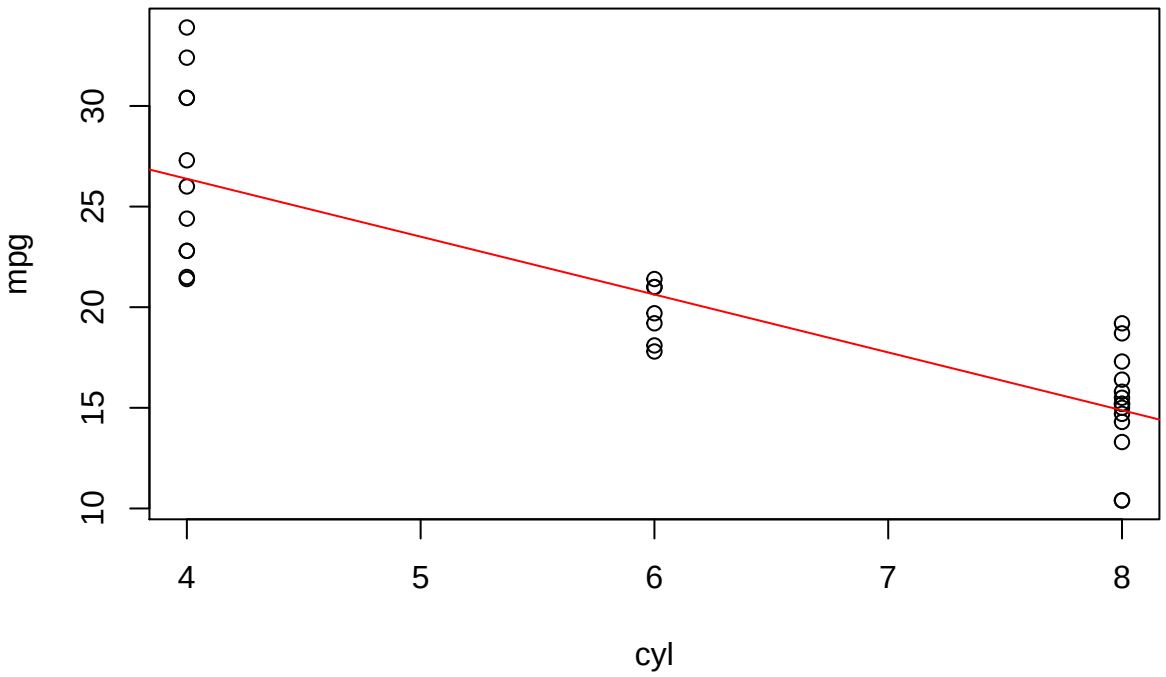
From above, we see that a linear model approximates the data well.

```
myOLS <- lm(mpg ~ cyl, myCars)
summary(myOLS)
```

```
##
## Call:
## lm(formula = mpg ~ cyl, data = myCars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.9814 -2.1185  0.2217  1.0717  7.5186
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.8846     2.0738   18.27  < 2e-16 ***
## cyl          -2.8758     0.3224   -8.92 6.11e-10 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.206 on 30 degrees of freedom
## Multiple R-squared:  0.7262, Adjusted R-squared:  0.7171
## F-statistic: 79.56 on 1 and 30 DF,  p-value: 6.113e-10
```

```
plot(mpg~cyl,myCars)
abline(myOLS,col="red")
```



We see that our parameter estimates are $\hat{\beta}_0 = 37.88$ and $\hat{\beta}_1 = -2.88$.

b) From above, we see our model is of the form

$$\hat{Y} = 37.88 - 2.88X$$

where Y denotes mpg and X denotes the number of cylinders of the vehicle. Thus, we see that an additional cylinder is usually associated with a drop of 2.88 mpg for the vehicle.

c) Adding vehicle weight, we get

```
myOLS2 <- lm(mpg ~ cyl + wt, myCars)
summary(myOLS2)
```

```
##
## Call:
## lm(formula = mpg ~ cyl + wt, data = myCars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.2893 -1.5512 -0.4684  1.5743  6.1004
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  39.6863     1.7150  23.141  < 2e-16 ***
## cyl          -1.5078     0.4147  -3.636 0.001064 **
## wt           -3.1910     0.7569  -4.216 0.000222 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.568 on 29 degrees of freedom
## Multiple R-squared:  0.8302, Adjusted R-squared:  0.8185
## F-statistic: 70.91 on 2 and 29 DF,  p-value: 6.809e-12
```

This suggests the following modified population regression function

$$\hat{Y} = 39.69 - 1.51X_1 - 3.19X_2$$

where X_1 denotes the number of cylinders and X_2 denotes the vehicle weight. Thus, we can see that, when we take vehicle weight into account, the effect of having extra cylinders plays less of a role in predicting mpg (though it is still negatively associated with mpg). Further, the results also suggest that vehicle weight as a larger negative effect on mpg (that is, the heavier vehicles have reduced mpg). Further, the adjusted r-squared value increased from 0.72 to 0.81, suggesting that this model accounts for more of the variability in the data, and is thus a better fit.

d) With the interaction term weight x cylinders, we have

```
myOLS3 <- lm(mpg ~ cyl + wt + (cyl*wt), myCars)
summary(myOLS3)
```

```
##
## Call:
## lm(formula = mpg ~ cyl + wt + (cyl * wt), data = myCars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.2288 -1.3495 -0.5042  1.4647  5.2344
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  54.3068     6.1275   8.863 1.29e-09 ***
## cyl          -3.8032     1.0050  -3.784 0.000747 ***
## wt           -8.6556     2.3201  -3.731 0.000861 ***
## cyl:wt        0.8084     0.3273   2.470 0.019882 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.368 on 28 degrees of freedom
## Multiple R-squared:  0.8606, Adjusted R-squared:  0.8457
## F-statistic: 57.62 on 3 and 28 DF,  p-value: 4.231e-12
```

This suggests the following model, where X_1 denotes the number of cylinders, X_2 denotes the vehicle weight, and X_3 denotes the interaction term.

$$\hat{Y} = 54.30 - 3.80X_1 - 8.66X_2 + 0.81X_3$$

This model has a much larger intercept than the previous model, and greatly reduced coefficients on X_1 and X_2 (though the sign on each is the same). Further, the adjusted R-squared slightly increased, suggesting that this model may provide a better fit. The interaction term suggests that the relationship between weight and vehicle mpg is different based on the amount of cylinders a car has.

Problem 3

(a)

```r
wages <- read.csv("/Users/Michael/Downloads/wage_data.csv")
age2 <- wages$age^2
myOLS4 <- lm(wage ~ age + age2, wages)
summary(myOLS4)
```

```
##
## Call:
## lm(formula = wage ~ age + age2, data = wages)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -99.126 -24.309  -5.017  15.494 205.621
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -10.425224   8.189780  -1.273    0.203
## age           5.294030   0.388689  13.620   <2e-16 ***
## age2         -0.053005   0.004432 -11.960   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 39.99 on 2997 degrees of freedom
## Multiple R-squared:  0.08209,    Adjusted R-squared:  0.08147
## F-statistic:   134 on 2 and 2997 DF,  p-value: < 2.2e-16
```

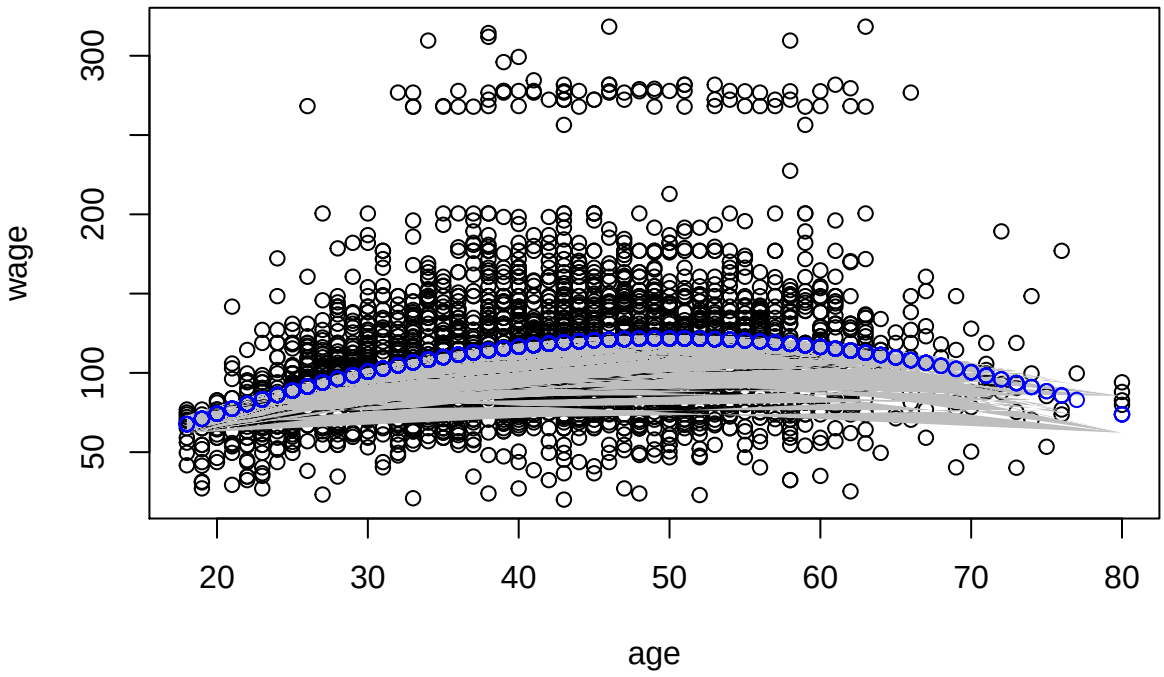
We get the following regression model (where X_1 represents age, and X_2 age squared).

$$\hat{Y} = -10.43 + 5.29X_1 - 0.05X_2$$

This model yields an adjusted r-squared of 0.08, suggesting that it is not a great fit. While the intercept term is not interpretable, we see in that age is positively correlated with wage. Specifically, for every year older one individual is than another, we expect an increase in wages by 5.3 (thousand) dollars.

(b)

```r
myPredict <- predict(myOLS4, type="response",se.fit=TRUE)
plot(wage~age,wages)
x <- wages$age
myL <- myPredict$fit-1.96*myPredict$se.fit
myU <- myPredict$fit+1.96*myPredict$se.fit
polygon(c(x,rev(x)),c(myL,rev(myU)),border=NA,col="grey")
points(wages$age,myPredict$fit,col="blue")
```



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(c) The our model doesn't seem to be a great fit for the data. The data is stratified, with a large amount of ~$260k wages which our model doesn't account for. Further, the data is widely scattered around each age value, suggesting that perhaps we need additional features to more accurately predict wage. By fitting a polynomial regression, we are asserting that wage increases, then decreases (or vice versa), which is not an unreasonable initial guess. However, the data doesn't lend itself particularly well to a quadratic upon further inspection.

(d) Statistically, a linear model is of the form $Y = \beta_0 + \beta_1 X$ while a polynomial regression is of the form $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + ... + \beta_n X^n$. Because of the higher order terms, a polynomial model can "bend", and is more flexible than a linear model. However, increased the order too much can lead to overfitting.