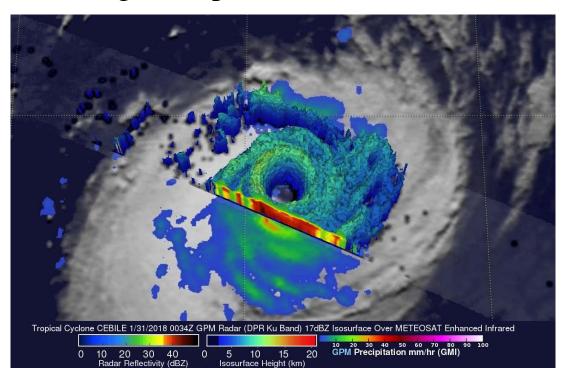
SECOND YEAR: 2604 Intermediate Applied Physics PLANETARY REMOTE SENSING

Lecture 7: radar basics

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http://www.metoffice.gov.uk/public/weather/observation/rainfall-radar



Purpose of lecture

Radar basics

The purpose of lecture is the following:

- > To explain the basic principle of radars
- > To introduce the radar equation for single and distributed targets

Next two radar lectures: we will provide examples for atmospheric radars, altimeters and SARs.

Radio Detection and Ranging

RADAR - RAdio Detection And Ranging LIDAR - Light Detection And Ranging SODAR - SOund Detection And Ranging

Radar is a remote sensing technique: Capable of gathering information about objects located at remote distances from the sensing device.



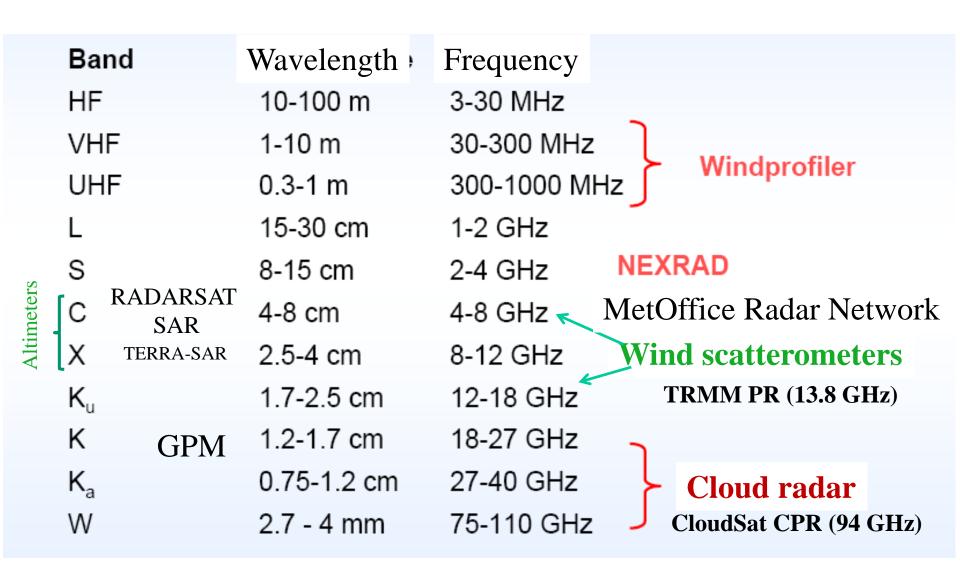
Two distinguishing characteristics:

1. Employs EM waves that fall into the microwave portion of the electromagnetic spectrum

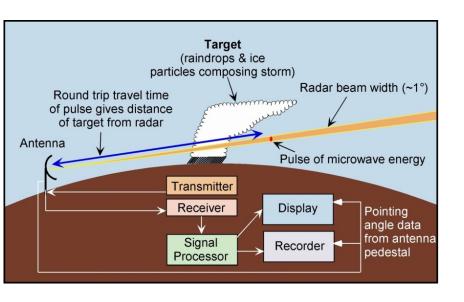
 $(1 \text{ mm} < \lambda < 75 \text{ cm})$

2. Active technique: radiation is emitted by radar – radiation scattered by objects is detected by radar.

Radar wavelength



How a Pulsed Radar Works

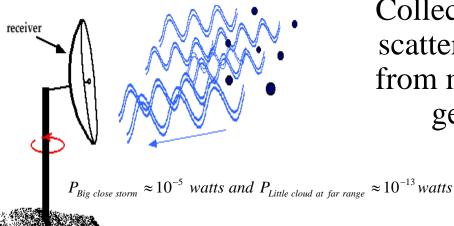




Radar transmits a wave of electromagnetic energy (pulse)

- -Energy scatters in all directions
- -A *very* small portion is reflected back from the target (raindrop)

 $P_{t} \approx kW \ or \ higher$



Collectively, the energy is scattered back to the radar from millions of droplet to generate the radar reflectivity

dB terminology

Common ways to express power (basic unit: watts):

$$dB = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$

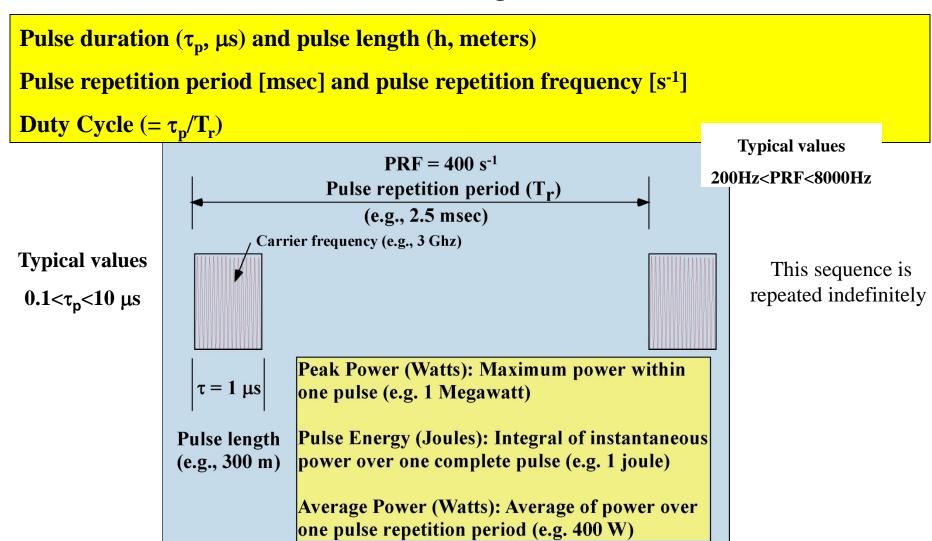
decibels (e.g. SNR is expressed in dB)

$$dBm = 10\log_{10}\left(\frac{P_1}{1 \ mW}\right)$$

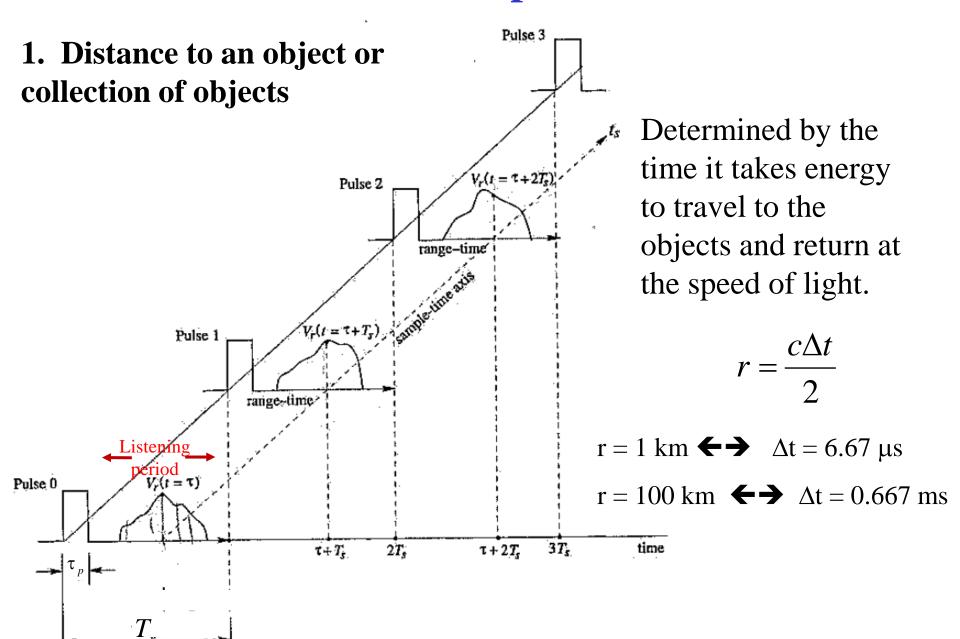
$$dBW = 10\log_{10}\left(\frac{P_1}{1W}\right)$$

How a Pulsed Radar Works

Meteorological radars send out <u>pulses</u> of energy with relatively long periods of "listening" between pulses. Pulses are required, rather than continuous waves, to determine the distance to the target.

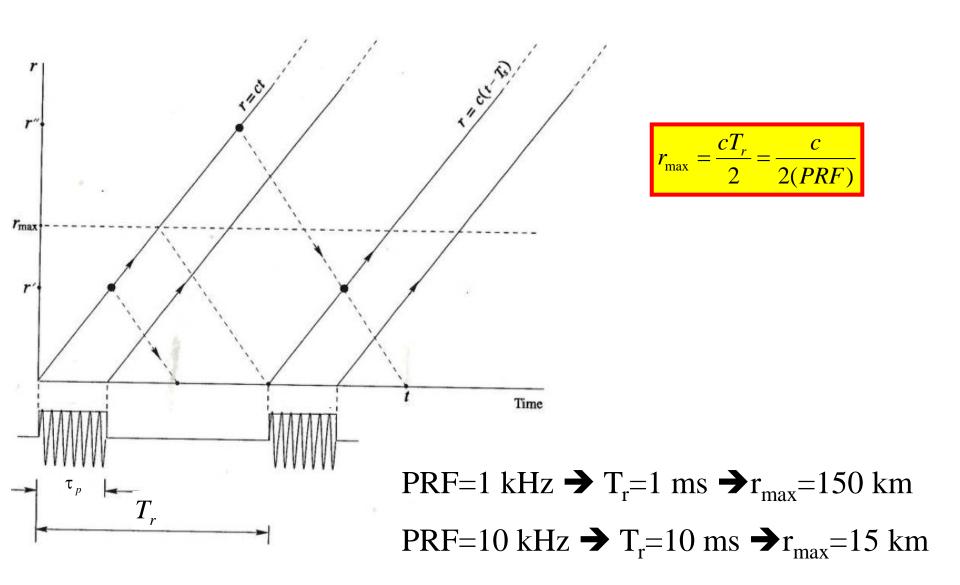


What does a conventional pulsed radar measure?



Maximum unambiguous range

Maximum Unambiguous Range (r_{max}): The maximum distance that an object can be located such that a pulse arriving at the object can return to the radar before another pulse is emitted.



Resolution along the direction of the beam: distributed targets



The back of the pulse at "a" will arrive at "b" at the same time that radiation scattered from objects at the front end of the pulse at "c" will arrive back at "b".

When energy arrives back at the radar, an instantaneous sample will include all radiation scattered between locations b and c: the sample volume is half the pulse length (h/2).

$$\Delta r = \frac{h}{2} = \frac{c\tau_p}{2}$$

$$\tau_p=3.3 \mu s \rightarrow \Delta r=500 \text{ m}$$
 $\tau_p=0.6 \mu s \rightarrow \Delta r=100 \text{ m}$

Radar equation for an isolated target

Measurement of the echo power received from a target provides useful information about it.

The radar equation provides a relationship between the received power, the characteristics of the target, and characteristics of the radar itself.

Steps in deriving the radar equation for an isolated target:

- 1) Determine the radiated power per unit area (the power flux density) incident on the target (key quantity= antenna gain)
- 2) Determine the power flux density scattered back toward the radar (key quantity=radar cross section)
- 3) Determine the amount of power collected by the antenna (key quantity=antenna effective area).

Isotropic antenna

Consider an isotropic antenna = an antenna that transmits radiation equally in all directions

Power flux density $(S, watts/m^2)$ at radius r from an isotropic antenna

$$S_{isotropic} = \frac{P_t}{4\pi r^2} \tag{1}$$

where P_t is the transmitted power.

In a radar system we want to identify the angular position of the object

→ antennas that focus radiation in a particular direction are needed

Gain function

The gain is the ratio of the power flux density at radius r, azimuth θ , and elevation ϕ for a directional antenna, to the power flux density for an isotropic antenna radiating the same total power.

$$G(\theta, \phi) = \frac{S_{inc}(\theta, \phi)}{S_{isotropic}}$$

So from (1)

$$\left| S_{inc}(\theta, \phi) = \frac{G(\theta, \phi)P_t}{4\pi r^2} \right|$$

Gain-beamwidth-antenna size relationship

The width of the main beam is proportional to wavelength and inversely proportional to the antenna aperture

$$\Theta_{3dB} = \underbrace{1.22 \frac{\lambda}{D}}_{in \ radiants} = \underbrace{70^{\circ} \frac{\lambda}{D}}_{in \ deg \ rees}$$

(3-dB beamwidth of dish antenna)

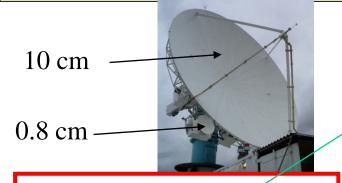
$$3dB \equiv factor 2$$

$$\lambda = 10 \, cm; \quad D = 3 \, m \quad \Rightarrow \Theta_{3dB} = 2.33^{\circ}$$

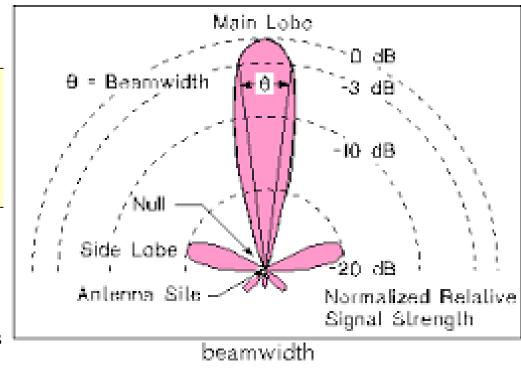
$$\lambda = 10 \, cm; \quad D = 6 \, m \quad \Rightarrow \Theta_{3dB} = 1.17^{\circ}$$

$$\lambda = 8 mm$$
; $D = 0.5 m \Rightarrow \Theta_{3dB} = 1.12^{\circ}$

Large wavelength radars = big antenna Small wavelength radars = small antenna for same beam width (i.e. same resolution)



Circular antennas



 $G_{\text{max}} = \frac{16}{\left(\Theta_{3dB}\Phi_{3dB}\right)} = \frac{16}{\left(\Theta_{3dB}\right)^2}$

In radiants!!!!

Example

Compute the incident power density at 100 km range in the direction corresponding to the peak of the main lobe for an antenna with D=3m for a wavelength of 10 cm and for a transmitted power of 10⁵Watts

$$\lambda = 10 \, cm; \quad D = 3 \, m \quad \Rightarrow \Theta_{3dB} = 1.22 \frac{\lambda}{D} = 4.07 \times 10^{-2} \, rad$$

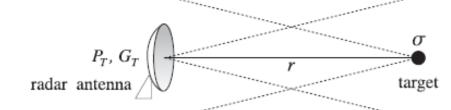
$$G_{\text{max}} = \frac{16}{(\Theta_{3dB})^2} = 9674.7 = 39.8 dB$$
Target is at 100 km range

$$S_{inc} = \frac{GP_t}{4\pi r^2} = \frac{9674.7 \times 10^3}{4\pi \times (10^5)^2} \approx 8 \times 10^{-3} W/m^2$$

Incident Power Flux Density = $8 \times 10^{-3} \text{ Watts/m}^2 \Rightarrow E = 2.45 \text{ V/m} \Rightarrow E_{rms} = 1.7 \text{ V/m}$ Note that

 $E_{rms}=6 \text{ V/m} \leftarrow 0.1 \text{ W/m}^2 \text{ suggested value for health in the EU } [0.1 < f < 300 \text{ GHz}]$

Radar cross section



σ provides a measure of the effective area of the target and thus of the re-radiated power as a function of the impinging flux

Fig. 14.11.1 Radar antenna and target.

$$\sigma_{back} \equiv \frac{P_{backscattered\ by\ t \arg et}}{S_{inc\@\ t \arg et}} = 4\pi r^2 \left(\frac{S_{rec\@\ radar}}{S_{inc\@\ t \arg et}}\right)$$

$$S_{inc@target} = \frac{GP_t}{4\pi r^2}$$
 Recall from object

Recall from before the power flux density incident on an object

$$S_{rec @ radar} = \frac{\sigma_{back} G P_t}{16\pi^2 r^4}$$

Some typical values:

Gain = 10,000 (40 dB) Transmitted Power = 100,000 Watts Target is at 100 km range Radar cross section = 1 m²

Power Flux Density at the antenna = $6.3 \times 10^{-14} \text{ Watts/m}^2!!$

Factors affecting radar cross section (RCS)

In general, the radar cross section of an object depends on:

- 1) Object's shape
- 2) Size (in relation to the radar wavelength)
- 3) Complex dielectric constant and conductivity of the material (related to substances ability to absorb/scatter energy)
- 4) Viewing geometry
- 5) Polarization of incident wave

Example: radar cross section (RCS) of an aircraft

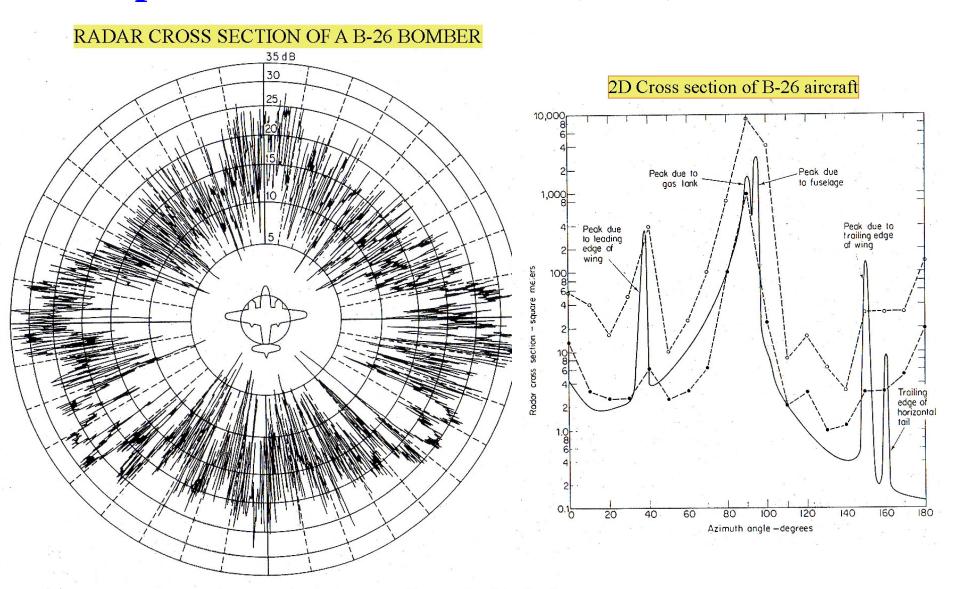


Figure 2.16 Experimental cross section of the B-26 two-engine bomber at 10-cm wavelength as a function of azimuth angle. (From Ridenour, 28 courtesy McGraw-Hill Book Company, Inc.)

Antenna effective area

$$S_r = \frac{\sigma_{back} G P_t}{16\pi^2 r^4}$$
 Power flux density received at the radar

Power received at antenna:

$$P_r = A_e S_r = \frac{\sigma_{back} A_e G P_t}{16\pi^2 r^4}$$
 where A_e is the effective area of the antenna

area of the antenna

From antenna theory - Relationship between gain and effective area:

$$G(\theta,\phi) = \frac{4\pi A_e(\theta,\phi)}{\lambda^2} \Rightarrow A_e(\theta,\phi) = \frac{\lambda^2 G(\theta,\phi)}{4\pi}$$

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Radar equation for isolated target

Substituting for A_e:

$$P_r = \frac{\sigma_{back} \lambda^2 G^2 P_t}{64\pi^3 r^4}$$

Which we will write as:

$$P_{r} = \frac{1}{\left(4\pi\right)^{3}} \begin{bmatrix} P_{t}G^{2}(\theta, \varphi)\lambda^{2} \end{bmatrix} \begin{bmatrix} \sigma_{back} \\ r^{4} \end{bmatrix}$$
constant
radar characteristics
target characteristics

This is the radar equation for a <u>single isolated target</u> (e.g. an airplane, a ship, a bird, one raindrop, ...). From a calibrated radar we can get the object backscattering cross section.

What happens for distributed targets?

Atmospheric targets:

There are millions targets inside the scattering volume!!

Contributing volume

$$V_{c} \approx \pi \left(\frac{h}{2}\right) \left(\frac{r\Theta_{3dB}}{2}\right) \left(\frac{r\Phi_{3dB}}{2}\right) = \frac{\pi h r^{2}\Theta_{3dB}\Phi_{3dB}}{8} = \frac{\pi c\tau r^{2}\Theta_{3dB}\Phi_{3dB}}{8}$$
Radar equation for distributed targets:

Note that for a circular antenna $\Theta_{3dB} = \Phi_{3dB}$

$$\overline{P}_{r} = \frac{1}{64\pi^{3}} \left[P_{t}G^{2}\lambda^{2} \right] \left[\frac{\sum_{j} \sigma_{back, j}}{r^{4}} \right] \begin{array}{c} \text{scattering volume} \\ \text{(which is becoming bigger and bigger with increasing distance)} \end{array}$$

Sum over the radar

Radar backscattering coefficient or radar reflectivity

Taking the radar equation:

$$\overline{P}_{r} = \frac{1}{64\pi^{3}} \left[P_{t}G^{2}\lambda^{2} \right] \left[\frac{\sum_{j} \sigma_{back,j}}{r^{4}} \right] = \frac{1}{64\pi^{3}} \left[P_{t}G^{2}\lambda^{2} \right] \frac{V_{c}}{r^{4}} \left[\frac{\sum_{j} \sigma_{back,j}}{V_{c}} \right] = \frac{c}{512\pi^{2}} \left[P_{t}\tau G^{2}\lambda^{2}\Phi_{3dB}\Theta_{3dB} \right] \left[\frac{\eta_{avg}}{r^{2}} \right]$$

$$\text{where } V_{c} \text{ is}$$

$$\text{the contributing volume}$$

$$V_{c} = \frac{\pi c \tau r^{2}\Theta_{3dB}\Phi_{3dB}}{8}$$

Definition of the "radar backscattering coefficient or radar reflectivity", η

$$\eta \equiv \frac{\sum_{j} \sigma_{j}}{V_{c}} \quad \text{dim ension} \quad \frac{m^{2}}{m^{3}} = m^{-1} \quad \text{inverse length}$$

Radar equation for distributed targets

$$P_{r} = \frac{c}{512\pi^{2}} \left[P_{t}\tau G^{2}\lambda^{2}\Phi_{3dB}\Theta_{3dB} \right] \left[\frac{\eta}{r^{2}} \right]$$

The above equation applies for a uniform beam (e.g. with G constant within the 3dB beamwidth). For a Gaussian beam, a correction term

$$P_{r} = \frac{c}{\pi^{2} 1024 \ln(2)} \left[P_{t} \tau G^{2} \lambda^{2} \Phi_{3dB} \Theta_{3dB} \right] \left[\frac{\eta}{r^{2}} \right]$$

radar characteristics target characteristics

Isolated vs distributed targets

$$P_r = \frac{1}{64\pi^3} \left[P_t G^2 \lambda^2 \right] \left[\frac{\sigma_{back}}{r^4} \right]$$
 The returned power for a single target varies as r^4 .

$$P_r = \frac{c}{\pi^2 1024 \ln(2)} \left[P_t \tau G^2 \lambda^2 \Phi_{3dB} \Theta_{3dB} \right] \left[\frac{\eta_{avg}}{r^2} \right]$$
 The returned power for a distributed target varies as
$$P_r = C_{radar} P_t \left[\frac{\eta_{avg}}{r^2} \right]$$

Reason: As contributing volume grows with distance, more targets are added. Number of targets added is proportional to r^2 , which reduces the dependence of the returned power from r^{-4} to r^{-2} .

Hydrometeor backscattering cross sections

First Assumption: hydrometeor particles are all spheres (Mie theory)

Small raindrops and cloud droplets: Spherical

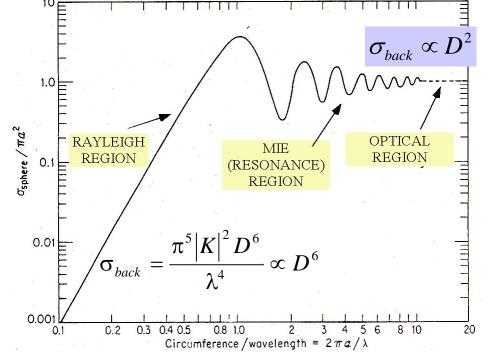
Large raindrops: Ellipsoids

Ice crystals: Different shapes

Graupel and rimed particles: Can be spherical

Hail: May or may not be spheres

Second assumption: The particles are sufficiently small compared to the wavelength of the impinging microwaves that the scattering can be described by Rayleigh Scattering Theory



Related to the dielectric property of the target (water/ice)

$$K = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{m^2 - 1}{m^2 + 2}$$

How small is small?

Note also $1/\lambda^4$

dependence

From the figure above, the radius of the particle, a, must be

$$a \le \frac{\lambda}{2\pi}$$

(~ 1/6 of the wavelength)

Validity of the Rayleigh Approximation for weather targets

Valid

 $\lambda = 10 \text{ cm}$ Raindrops: 0.01 – 0.5 cm (all rain)

Snowflakes: 0.01–3 cm (most snowflakes)

f=3 GHz Hailstones: 0.5-2.0 cm (small to moderate hail)

 $\lambda = 5$ cm Raindrops: 0.01 - 0.5 cm (all rain)

Snowflakes: 0.01–1 cm (small snowflakes)

f=6 GHz Hailstones: 0.5 – 0.75 cm (small hail)

 $\lambda = 3 \text{ cm}$ Raindrops: 0.01 - 0.5 cm (all rain)

Ice crystals: 0.01–0.5 cm (single crystals)

f=10 GHz Graupel: 0.1 -- 0.5 cm (graupel)

 $\lambda = 0.8 \text{ cm}$ Raindrops: 0.01 - 0.15 cm (cloud and drizzle drops)

f=35 GHz Ice crystals: 0.01–0.15 cm (single crystals)

What is K?

K is a complex number which accounts for the dielectric properties of the medium

$$K = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{m^2 - 1}{m^2 + 2} \quad \text{where} \quad \mathcal{E}_r = \frac{\mathcal{E}_1}{\mathcal{E}_0} \quad \frac{\text{Permittivity of medium}}{\text{Permittivity of vacuum}}$$

$$\text{Values of} \quad |K|^2 \quad \text{Water}$$

Temperature	$\lambda = 10 \text{ cm}$	$\lambda = 3.21 \text{ cm}$	$\lambda = 1.24 \text{ cm}$	$\lambda = 0.62 \text{ cm}$
20°C	0.9280	0.9275	0.9193	0.8926
10°C	0.9340	0.9282	0.9152	0.8726
0°C	0.9340	0.9300	0.9055	0.8312

 $|K|^2 \approx 0.93$ for water particles at low frequencies

Ice

 $|K|^2=0.176$ for ice particles with no strong dependence on T and frequency

→ice is producing scattering 5 times smaller for the same size

Radar reflectivity and radar equation for distributed targets

Reflected power
$$P_{r} = \frac{c}{\pi^{2} 1024 \ln(2)} \left[P_{t} \tau G^{2} \lambda^{2} \Phi_{3dB} \Theta_{3dB} \right] \left[\frac{\eta}{r^{2}} \right]$$

$$\eta = \frac{\sum_{j} \sigma_{back, j}}{V_{c}} = \frac{\pi^{5} |K|^{2}}{\lambda^{4}} \sum_{j} D_{j}^{6} \sum_{j} \left[\sum_{j} D_{j}^{6} \right] \times \left[\sum_{j}$$

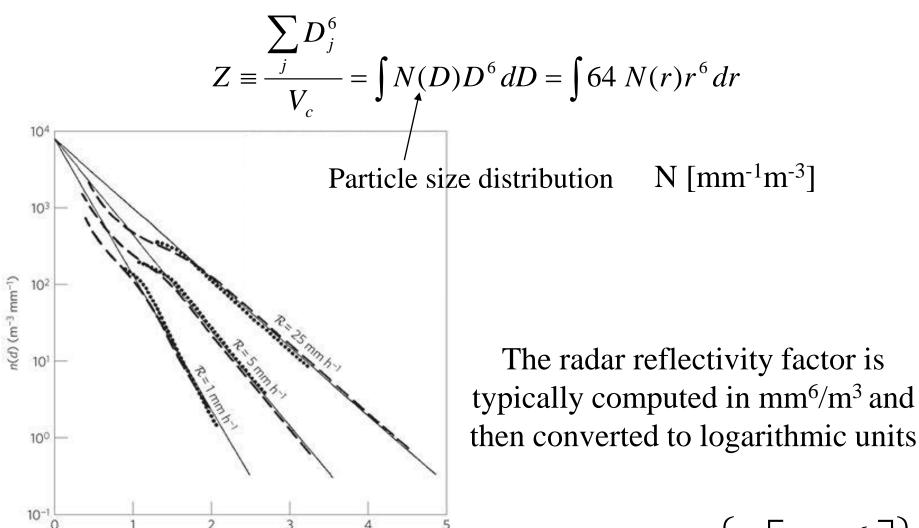
where the <u>radar reflectivity factor Z</u> fully characterize the radar backscattering of the targets within the scattering volume.

Radar constant [W/m]

$$P_{r} = \frac{c}{1024 \ln(2)} \frac{\pi^{3} |K|^{2}}{\lambda^{2}} \left[P_{t} \tau G^{2} \Phi_{3dB} \Theta_{3dB} \right] \frac{Z}{r^{2}} = C_{radar} \frac{Z}{r^{2}}$$

Z can be derived from measurements of P_r

Radar reflectivity factor



d (mm)

$$Z[dBZ] = 10\log_{10}\left\{Z\left[\frac{mm^6}{m^3}\right]\right\}$$

Example: Radar reflectivity of rain

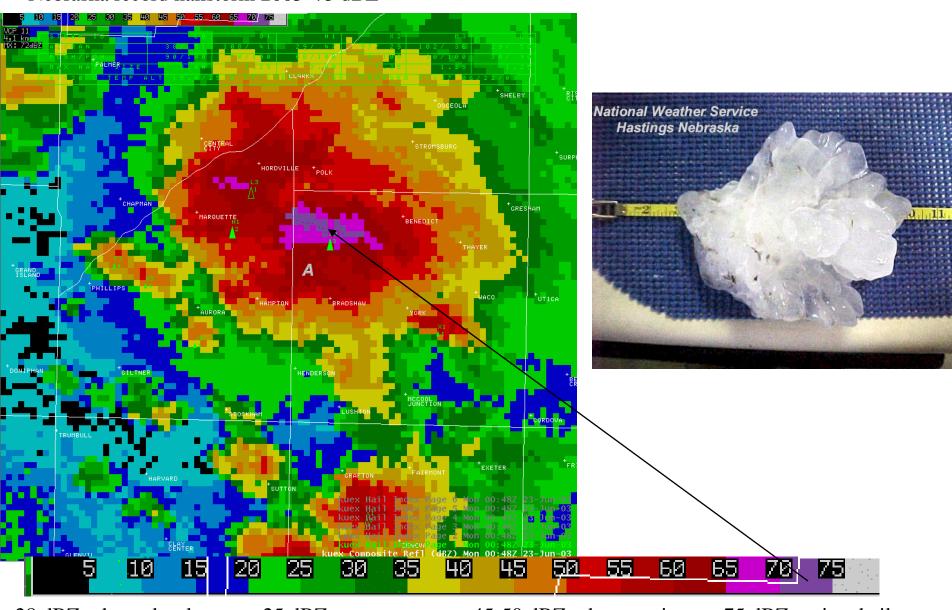
Diameter Range (mm)	Diameter D (at mid-range) (mm)	Measured Drop Concentration N(D) (m-3)	D6 (mm ⁶)	Contribution to Z , ΔZ (mm ⁶ /m ⁻³)	
0.5 - 0.7	0.6	6	0.05		
0.7 - 0.9	0.8	6	0.05		
		71	0.21	15	
0.9 - 1.1	1.0	280	1.0	280	
1.1 - 1.3	1.2	488	3.0	1464	
1.3 - 1.5	1.4	538	7.5	4030	
1.5 - 1.7	1.6	345	16.8	5800	
1.7 - 1.9	1.8	212	34	7200	
1.9 - 2.1	2.0	159	64	10180	
2.1 - 2.3	2.2	129	113	14580	
2.3 - 2.5	2.4	79	191	15100	
2.5 - 2.7	2.6	54	309	16690	
2.7 - 2.9	2.8	33	482	15900	
2.9 - 3.1	3.0	19	729	13830	
3.1 - 3.3	3.2	21	1073	22570	
3.3 - 3.5	3.4	9	1544	13900	
3.5 - 3.7	3.6	5	2180	10900	
3.7 - 3.9	3.8	8	3010	2480	
3.9 - 4.1	4.0	2	4096	8190	
4.1 - 4.3	4.2	0	5490		
4.3 - 4.5	4.4	1	7255	7255	
Tot	als	2459		170364	

These drops contribute most to the radar reflectivity

 $Z = 1.7 \times 10^5 \text{ mm}^6/\text{m}^3 = 52.3 \text{ dBZ}$

Range of radar reflectivity factor in weather echoes

Nebraska record hailstorm 2003 75 dBZ



-28 dBZ = haze droplets

25 dBZ = snow

45-50 dBZ = heavy rain

75 dBZ = giant hail

Relationship between the Reflectivity Factor and the Rain Rate

$$Z \equiv \frac{\sum_{j} D_{j}^{6}}{V_{c}} = \int_{0}^{D_{\text{max}}} N(D)D^{6} dD$$

6th moment of the particle size distribution

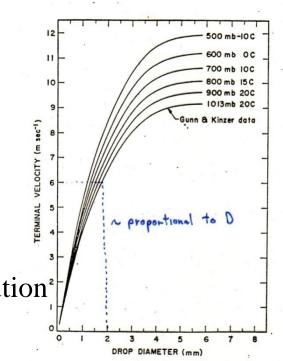
Precipitation rate (R): The volume of precipitation passing downward through a horizontal surface, per unit area, per unit time.

where v_i is the fall velocity of each drop j

$$R = \frac{\sum_{j} m_{j} v_{j}}{V_{c}} = \int_{0}^{D_{\text{max}}} \frac{\pi \rho_{w}}{6} N(D) D^{3} v(D) dD$$

since v proportional to D

 $R \propto \sum D^4$ 4th moment of the particle size distribution²



The Z-R conundrum

There have been hundreds of Z-R relationships published – here are just a few between 1947 and 1960 – there have been 4 more decades of new Z-R relationships to add to this table since!

Equation	Reference	Location	Remarks
$320R^{1.44}$	Wexler, R. (1947)	Washington, D.C.	8 rain intensities, each a mean of about
$ \begin{array}{c} 214R^{1.58} \\ 224R^{1.54} \\ 630R^{1.45} \\ 208R^{1.53} \\ 190R^{1.72} \end{array} $	Wexler (1948) Marshall, Langille,	Washington, D.C. Ynyslas, Great Britain Shoeburyness, England Hawaii Various locations	10 storms of same intensity 98 storms – original data 5 rainstorms 4 rainstorms 50 storms, orographic rain Various types of rain
220R ^{1.60}	and Palmer (1947) Marshall and Palmer (1948)	Various locations	Various types of rain
295R ^{1.612}	Hood (1950)	Canada	270 samples, 7 rainstorms; light rain
180R ^{1.55}	Boucher (1951)	Cambridge, Mass.	1-3 mm/hr; heavy thunderstorms 50 mm/hr 63 rain samples, widespread rain both
$127R^{2.87}$ $16.6R^{1.55}$	Higgs (1952)	Australia	uniform and variable; showers and thunderstorms Showers, 8 months of observation
$ \begin{array}{c} 31R^{1.71} \\ 290R^{1.41} \end{array} $	Blanchard (1953)	Hawaii	Orographic rain within cloud Orographic rain at cloud base
$ \begin{array}{c} 396R^{1.35} \\ 486R^{1.37} \\ 380R^{1.24} \end{array} $	Jones (1955)	Central Illinois	Nonorographic rain – thunderstorms 1,270 1-minute observations – all rains 560 1-minute observations – thunderstorms

MetOffice:
$$Z = 200 R^{1.6}$$

$$\begin{cases} R = 1mm/h \Rightarrow Z = 200mm^6/m^3 = 23 dBZ \\ R = 10mm/h \Rightarrow Z = 7962mm^6/m^3 = 39 dBZ \end{cases}$$

$162R^{1.16}$			opecua, 4 tanis
$ \begin{array}{c c} 102R \\ 215R^{1.34} \\ 350R^{1.42} \\ 310R^{1.34} \end{array} $	Atlas and Chmela (1957)	Lexington, Mass.	Stratiform rains, 16 April 1954 Stratiform rains, 23 April 1954 Stratiform rains, 27 April 1954 Stratiform rains, 28 April 1954
$220R^{1.54}$ $303R^{1.7}$	Sal'man (1957)	Near Leningrad, USSR	Showers and steady rain
$ \begin{array}{c} 303R^{11} \\ 405R^{1.49} \\ 289R^{1.59} \end{array} $	Shupiatskii (1957)	Near Moscow, USSR	$\begin{cases} \text{Various types of rain, } R < 7 \text{ mm/hr} \\ \text{Various types of rain, } 7 < R < 60 \text{ mm/hr} \end{cases}$
$109R^{1.64}$	Ramana Murty and	Kandia, India	Various types of rain, $R > 60$ mm/hr Orographic, Monsoon rains

Most of the uncertainties in the retrieval of rain rates comes from this fact.

219R 1.41 67.6R 1.94 66.5P 1.92 (1961)	Poona, India	☐ Prewarm frontal rain☐ Thunderstorms☐ Steady rains☐
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What do you need to know?

•	General	princi	ple of	a radar	system
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- Radar equation for single and distributed targets
- Radar backscattering cross section, the reflectivity and the radar reflectivity factor

Relationship between reflectivity and rain-rate

Questions

- Describe the basic principle of a radar system. Indicate the difference between a cloud and a rain radar. What are the key specifics in a radar system?
- Can you relate the antenna size to the antenna gain and to the antenna beamwidth?
- What is the chief factor limiting the accuracy of rainfall rate estimates from radar?
- Compute the radar reflectivity factor for a raindrop size distribution of the form

$$n(D) = n_0 \exp(-\Lambda D)$$
 $n_0 = 0.08 \, cm^{-4} = 8 \times 10^6 \, m^{-4}$

$$\Lambda[cm^{-1}] = \frac{41}{R^{0.21}}$$

where R is the rain rate in mm/h