SECOND YEAR: 2604
PLANETARY REMOTE SENSING 3

ELECTROMAGNETIC *RADIATION:*
a journey from the Sun to planet Earth

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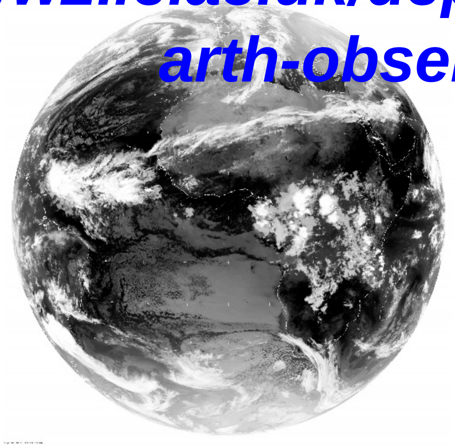
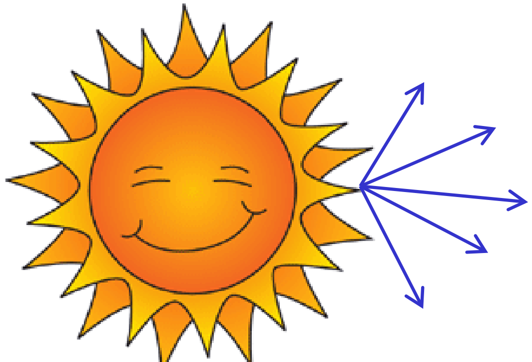
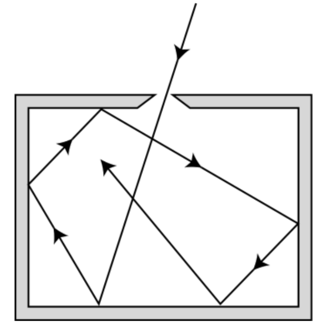


Radiation at sun surface



$B_\lambda(T=6000K)$

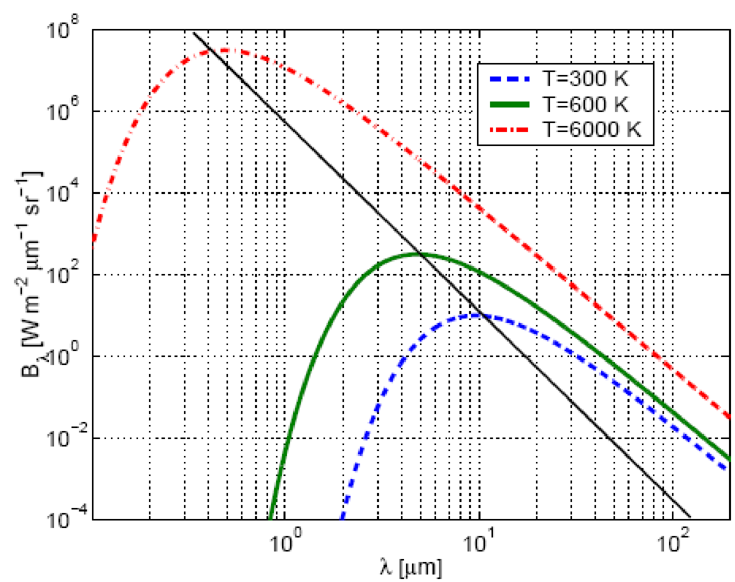


The Planck function describes the radiance, $B_\lambda(T)$, emitted by a <u>perfectly absorbing/emitting (black) body</u> in a small increment of wavelength $d\lambda$

At all wavelengths, B_λ (T), **is isotropic**, i.e. the same in all directions.

B_λ (T) depends **only on temperature at a given wavelength**.

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5\left\{e^{hc/k\lambda T} - 1\right\}}$$



Other forms of Planck's Law

A number of forms (inter-convert using $c = \lambda\nu$)

- **Standard Frequency (ν)**

$$B_\nu(T)\, d\nu = \frac{2h\nu^3}{c^2\{\exp(h\nu/kT) - 1\}}\, d\nu$$

Units of B are W m^{-2} sr^{-1} s^{-1} = W m^{-2} sr^{-1} Hz

Infra-red physicists tend to use the spectroscopic wavenumber ($\tilde{\nu}$ in cm^{-1})

Wavenumber $\tilde{\nu}$ form is the one you are expected to be familiar with when dealing with spectroscopy.

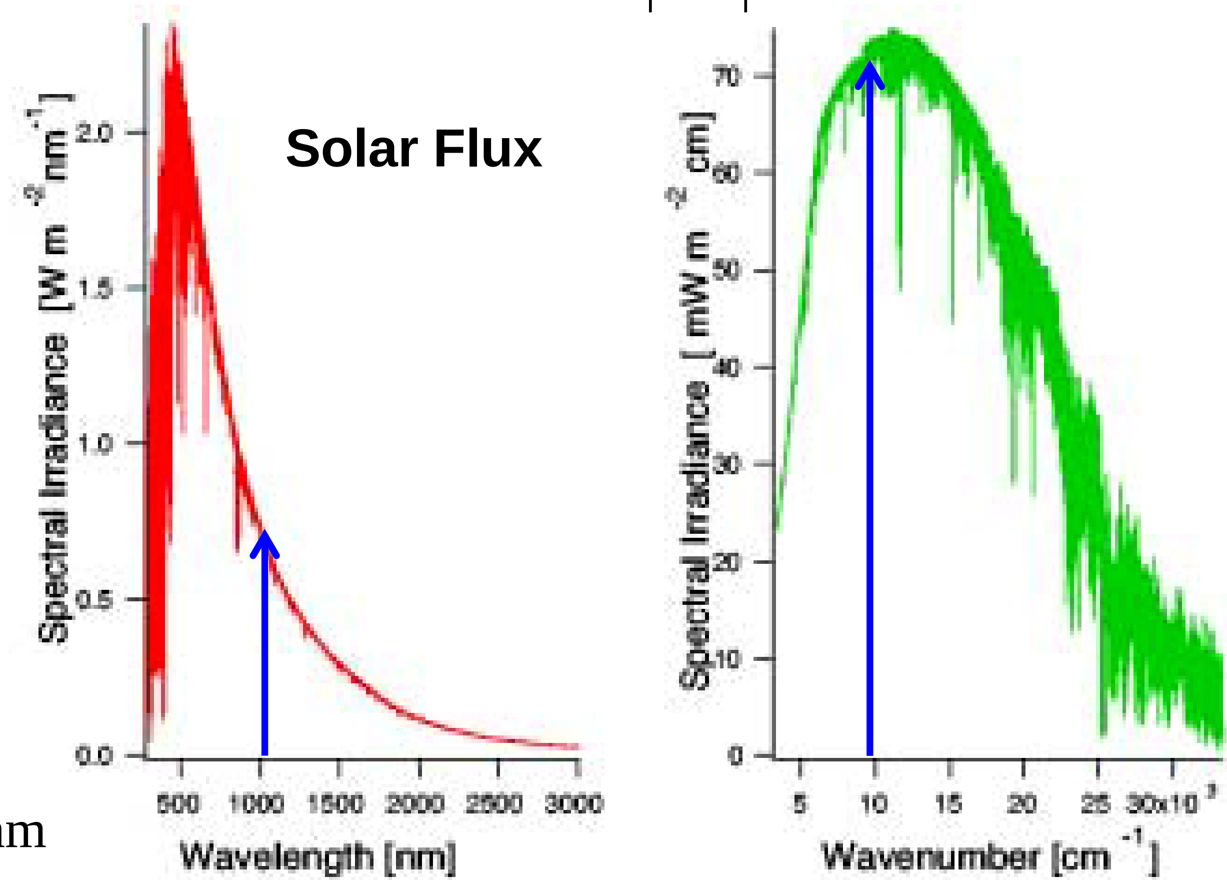
$$B_{\tilde{\nu}}(T) = \frac{2hc^2\tilde{\nu}^3}{\exp\left(\frac{hc\tilde{\nu}}{kT}\right) - 1}$$

Units of $B_{\tilde{\nu}}$ are W m^{-2} sr^{-1}/m^{-1} =W m^{-1} sr^{-1}

To pass from one to the other use the fact that B_λ (T) dλ is the same as B_ν (T) dν!

Spectral irradiance

$$F_\lambda d\lambda = F_{\tilde{\nu}} d\tilde{\nu} \quad \Rightarrow \quad F_\lambda = F_{\tilde{\nu}} \left| \frac{d\tilde{\nu}}{d\lambda} \right| = \frac{1}{\lambda^2} F_{\tilde{\nu}}$$



Example:

$\lambda = 1\mu m = 1000 nm$

$\tilde{\nu} = 10{,}000\ cm^{-1}$

$$F_{\lambda = 1\,\mu m} = 0.7 \left[\frac{W}{m^2\,nm} \right]; \qquad F_{\tilde{\nu} = 10000\,cm^{-1}} = 0.7 \times \left(10^3\right)^2 = 7 \times 10^5 \left[\frac{W\,nm}{m^2} \right]$$

$$F_{\tilde{\nu} = 10000\,cm^{-1}} = 7 \times 10^5 \times 10^3 \times 10^{-7} = 70 \left[\frac{mW\,cm}{m^2} \right]$$

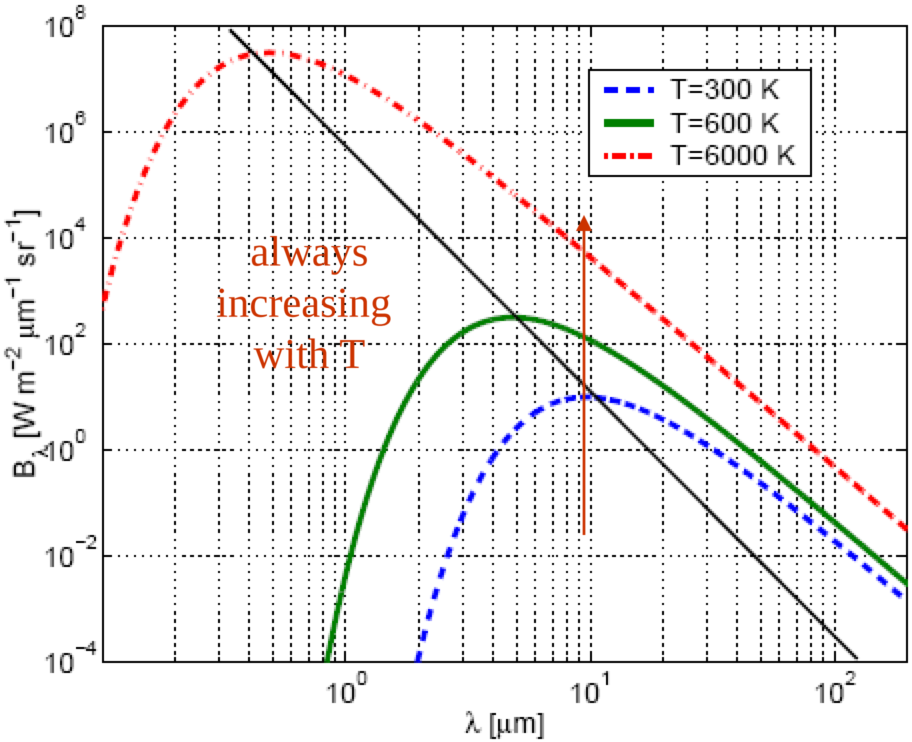
$1\,cm = 10^{-7}\,nm$

$1W = 10^3\,mW$

Planck's function: Wien law

By differentiating the Planck function B_λ and equating the result to zero we find the wavelength corresponding to the maximum:

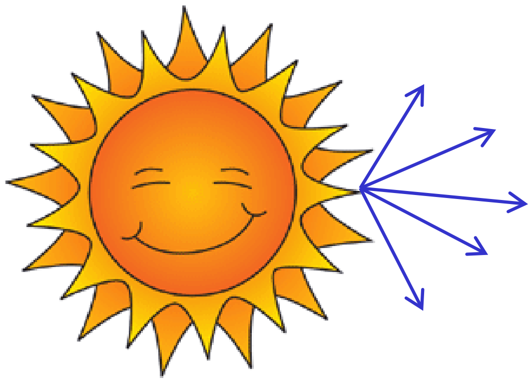
$$\lambda_{max}\,[\mu m]\; T[K] = 2{,}897.8$$



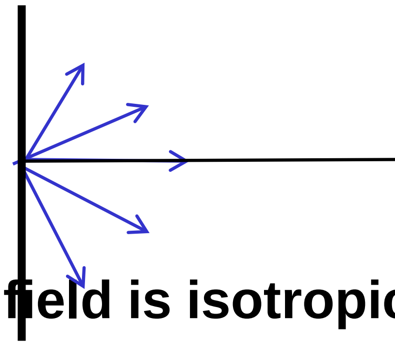
T(sun) ≈6000 K (peaks at approx. 0.5 μm or 500 nm)

T (Earth surface mean)≈ 288 K (peaks at approx. 10 μm)

Spectral flux at the sun surface



$B_\lambda(T=6000K)$



The Radiance field is isotropic

$I_\lambda(\theta,\phi) = B_\lambda(T=5600K) = \text{const} \quad \text{for } 0<\theta<\pi/2$

$$\boxed{F_\lambda = \int_{4\pi} d\Omega \cos\theta \, I_\lambda \qquad \left[W\, m^{-2}\, Hz^{-1}\right]}$$

Integral over one hemisphere

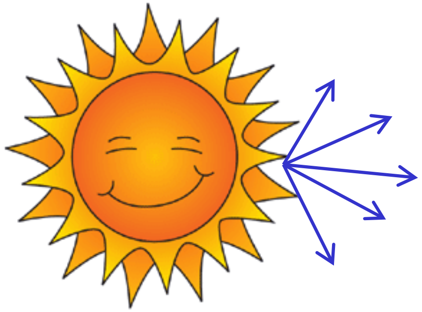
$$F_\lambda^\uparrow = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_\lambda(\theta,\phi)\cos\theta \sin\theta d\theta$$

$$F_\lambda^\uparrow = \int_0^{2\pi} d\phi \int_0^{1} I_\lambda(\theta,\phi)\sin\theta \; d\sin\theta$$

$$F_\lambda^\uparrow = B_\lambda 2\pi \left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2}$$

$$F_\lambda^\uparrow = \pi B_\lambda$$

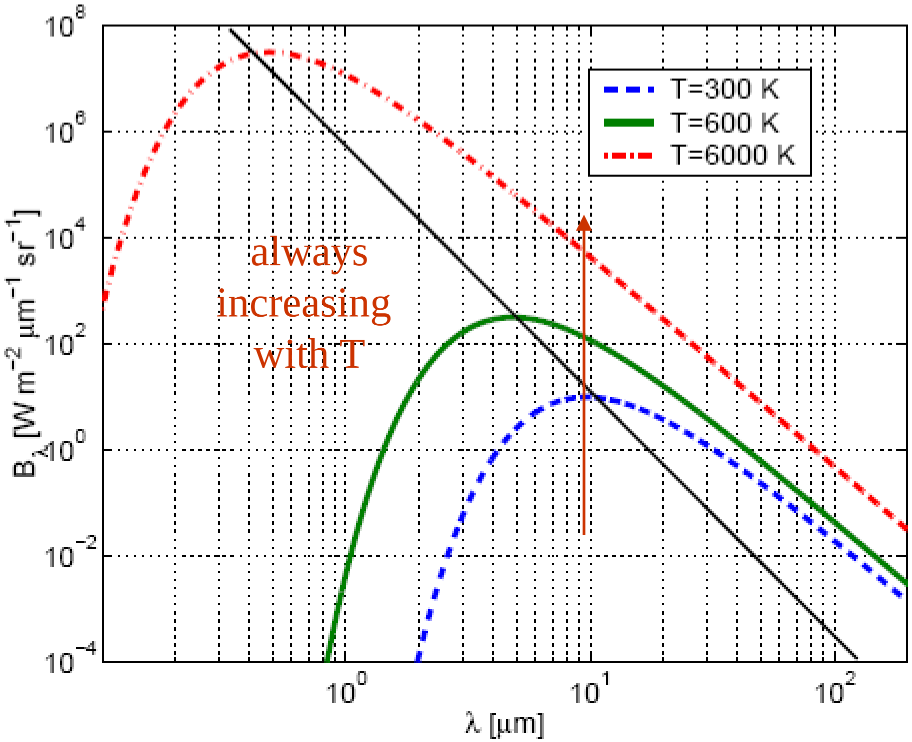
Total flux at sun surface: Planck law



$B_\lambda(T=6000K)$

$$F^{BB} = \pi \int_0^\infty B_\lambda \, d\lambda = \pi \int_0^\infty \frac{2hc^2}{\lambda^5 \left\{ e^{hc/k\lambda T} - 1 \right\}} d\lambda$$

$$x = \frac{hc}{k\lambda T}$$



$$F^{black-body} = \frac{2\pi(k_B T)^4}{h^3 c^2}) \left[\int_0^\infty \frac{x^3 \, dx}{e^x - 1} \right] = \left(\frac{2\pi^5 k_B^4}{15 h^3 c^2} \right) T^4 \equiv \sigma_B T^4$$

$\sigma = 5.67 \times 10^{-8} \, W m^{-2} K^{-4}$

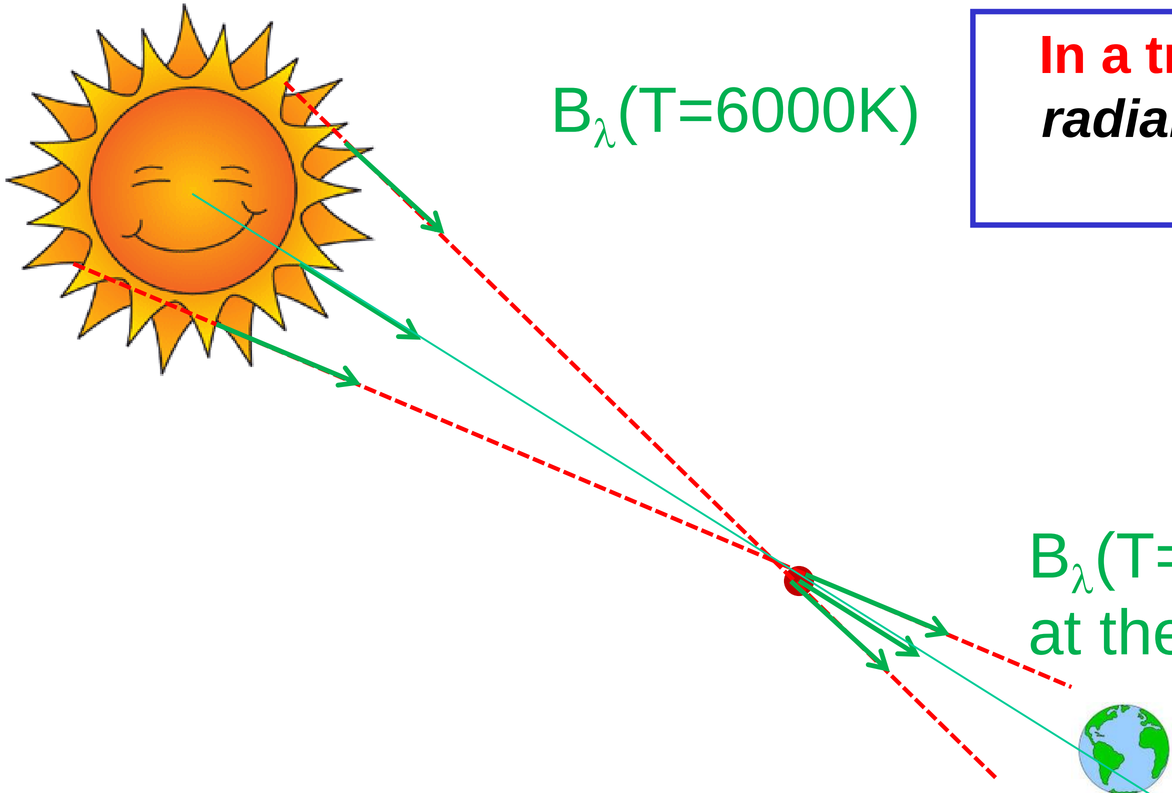
By integrating the blackbody flux over all frequencies we get

$F_{BB} = \sigma_B T^4$ Stefan-Boltzmann Law

T=300 ➔ F=460 W/m^2

T=6000 ➔ F=460 *160000W/m^2

Now travelling to Earth's TOA



In a transparent medium, the ***radiance is constant along a ray***.

$$I\,(P,\hat{\Omega}) = I\,(P',\hat{\Omega})$$

B_λ(T=6000K) i.e. same as at the Sun surface

At TOA The field is very collimated. Inside the atmosphere the radiation field is more diffuse.

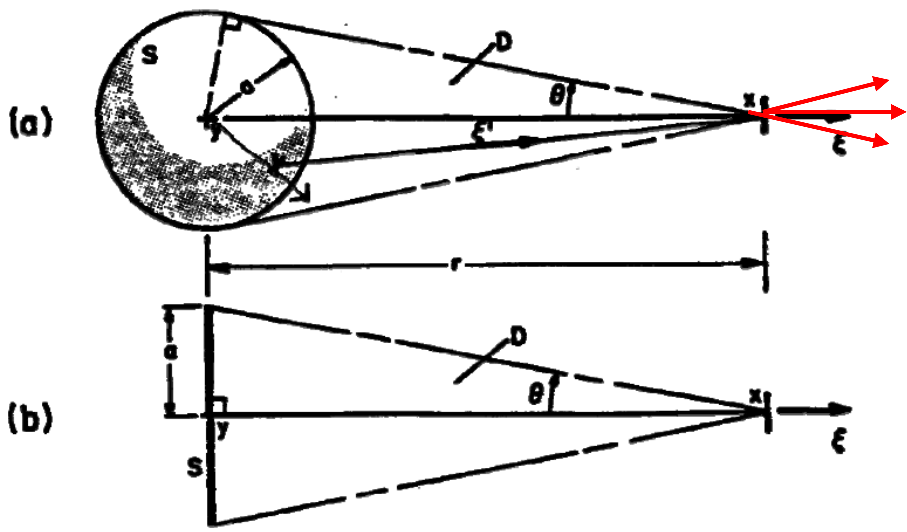
How much radiation is impinging at TOA? ➔Sun 'constant'

How is it spectrally distributed? Sun spectrum

Flux from Sun at distance D

Assume isotropic emission with intensity I_0 from extend source. Compute the flux incident on a surface (with normal in direction of source) at distance D

$$
\begin{aligned}
F(D) &= \int I_0 \cos\theta \, d\Omega \\
&= I_0 \int_{subtended} d\Omega \cos\theta \\
&= I_0 \int_0^{2\pi} d\varphi \int_0^{\theta_0} \cos\theta \sin\theta \, d\theta \\
&= I_0 \pi \sin^2\theta_0
\end{aligned}
$$



$$F(D) = \underbrace{I_0 \pi}_{F(a)} a^2 / D^2 = I_0 \Omega_0(D)$$

Thus ***F(D) ~ 1/D²***

Which is the expected $1/r^2$ dependence: fluxes are preserved

Example: the solar constant=flux from sun at TOA

Sun is approximately blackbody with T=5790K

$$F(D) = I_{Sun}\pi\, R_{sun}^2 / D_{Earth-SUN}^2 = I_{Sun}\,\Omega_{Sun}(D)$$

Intensity from sun:

(Stefan-Boltzman law for BB)

$$I_{Sun} = \frac{\sigma\, T_{sun}^4}{\pi} = \frac{5.67\times 10^{-8}\times 5790^4}{\pi} = 2\times 10^7\,\frac{W}{m^2 sr}$$

Flux observed on Earth:

$F_{Earth} = I_{sun}\,\Omega_{sun} = I_{sun}$ * 0.68*10^{-4} sr ~ 1368 W/m^2

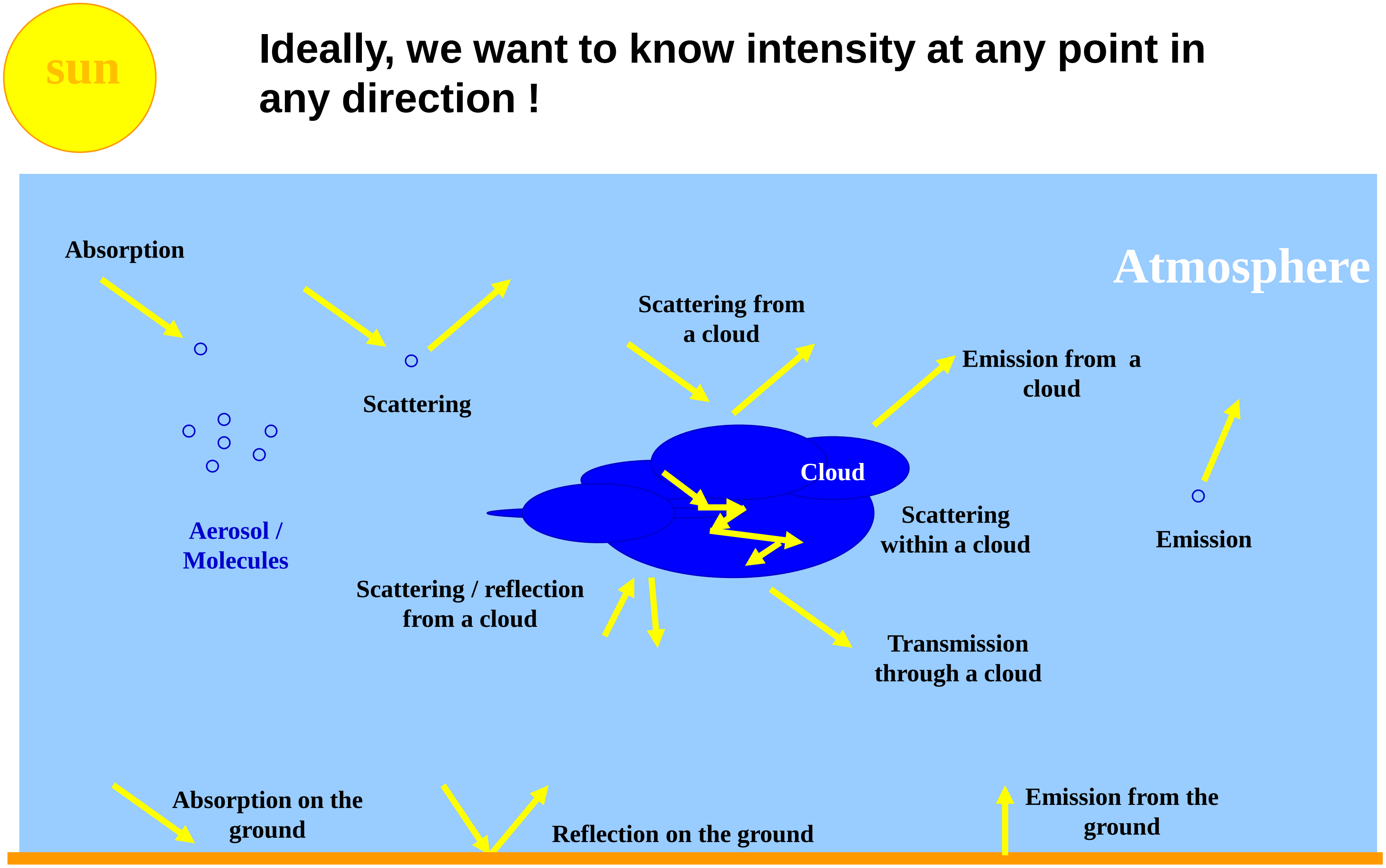
Finally ending the 10-minute boring travel in the vacuum

Home thinking

Can you explain these phenomena?

Trying to understand the complete Picture

Ideally, we want to know intensity at any point in any direction !



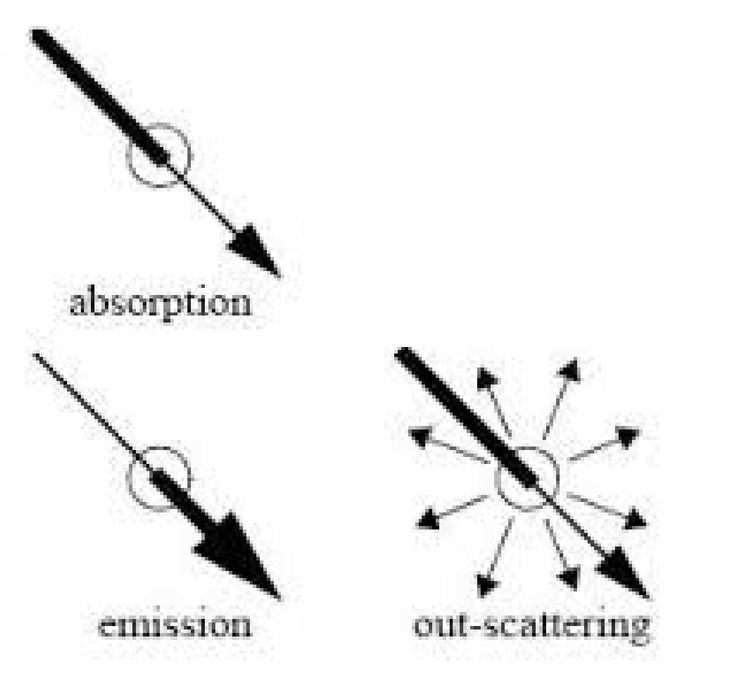
So what is the problem ?

Interaction of Light and Matter

Light (EM radiation) can interact with matter in the following ways:

- **Emission: add (generates) photons**
- **Absorption: removes photons**
- **Scattering: changes direction of photons (and sometimes energy) e.g. when impinging onto a surface, a cloud or an aerosol layer**

In such conditions (the medium is non transparent) radiance is not constant along the ray



Emission and Absorption of E/M Radiation

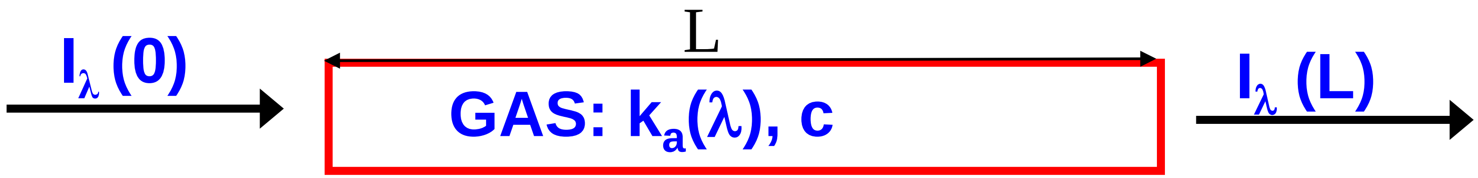
- **Emission:**
  - **Radiation emitted from a body due to its temperature**
  - **Emission of a medium is governed by the Planck function of the medium and the emissivity ε**
  - **Emission signal = $\varepsilon(\lambda) \times B(\lambda,T)$**
  - **Black body: $\varepsilon(\lambda) = 1$**

- **Absorption:**
  - **Radiation absorbed by a body**
  - **Absorptivity: $a(\lambda) = \varepsilon(\lambda)$ [Kirchoff's law in local thermal equilibrium]**

- **Emission and absorption are fundamentally driven by quantum mechanics of molecules (or matter).**

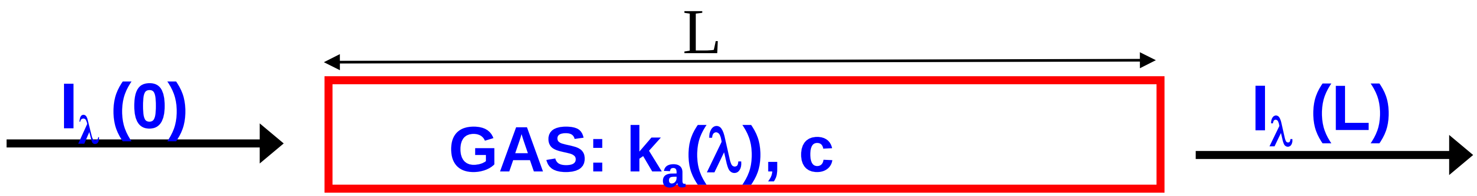
BEER-LAMBERT LAW: absorption

The **Beer-Lambert (-Bouget) Law**: if a signal of intensity I_λ travelling in direction Ω penetrates a distance, dL, in a homogenous medium with **absorption coefficient, k_a** (per unit distance):



BEER-LAMBERT LAW

The Beer-Lambert Law: if a signal of intensity I_λ penetrates a distance, dL, in a homogenous medium with absorption coefficient, k_a (dimensionally L^{-1}):



$$dI_\lambda = - k_a(\lambda)\, I_\lambda\, dL$$

Observed experimentally

Radiance is not constant along the ray!

Then if k_a = constant along the path L

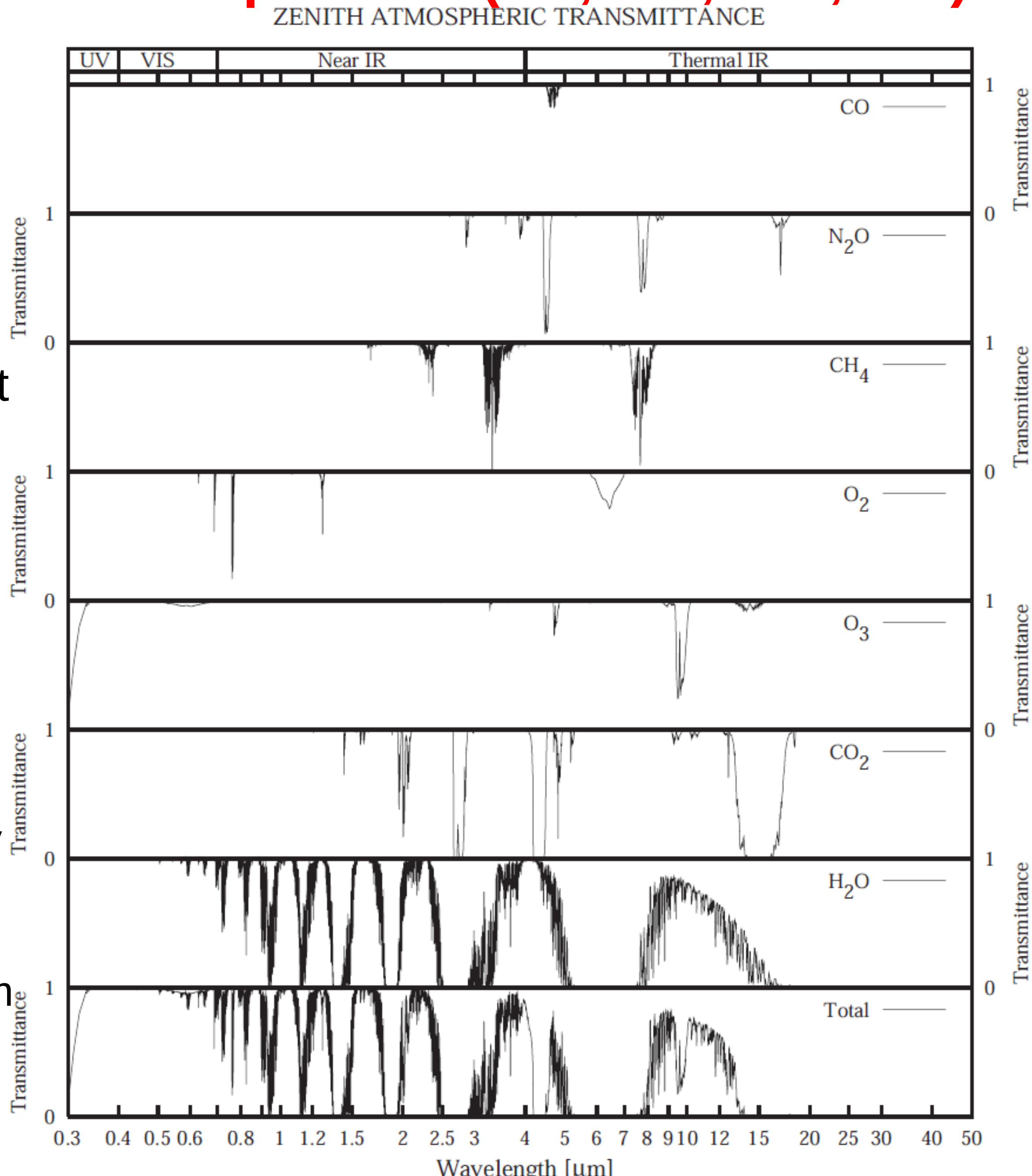
$$I_\lambda (L) = I_\lambda(0) \exp\{- k_a(\lambda)\, L\}$$

transmissivity: $\mathcal{T}(\lambda) = I_\lambda (L) / I_\lambda (0) = \exp\{- k_a(\lambda)\, L\} = \exp\{-\tau\}$

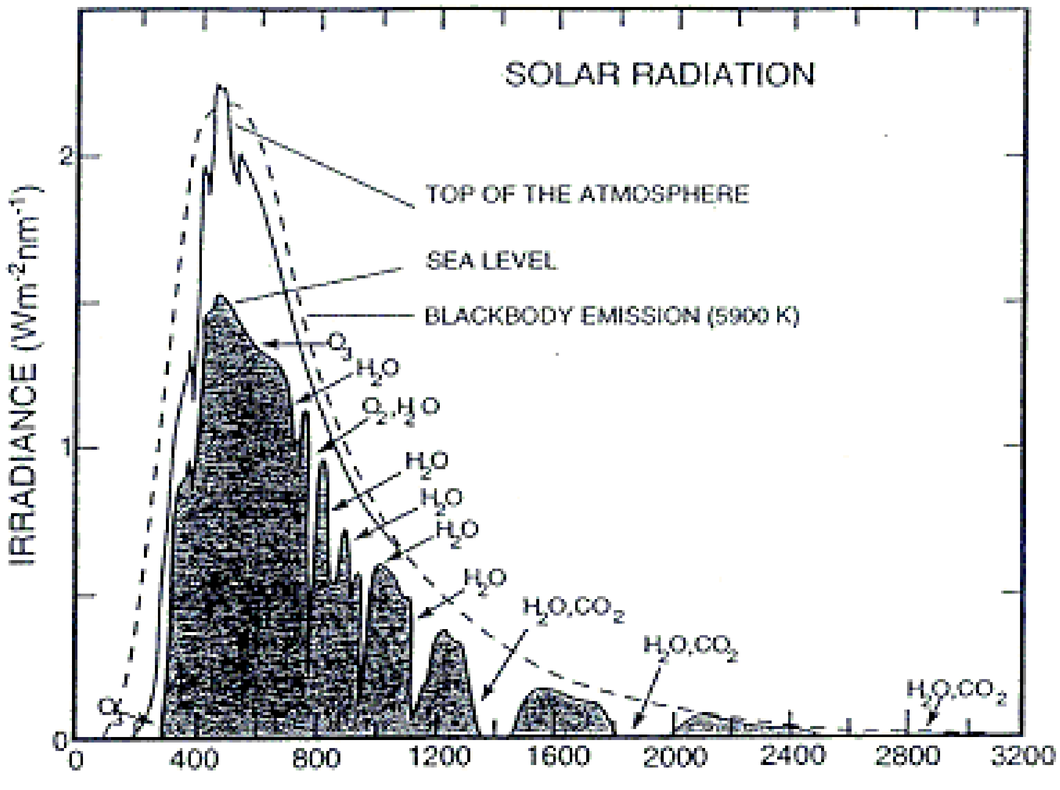
with τ = optical depth

Atmospheric Transmittance Spectrum (UV, VIS, NIR, TIR)

- <0.3µm opaque due to O_2 and O_3 at UV-C
- VIS most transparent except minor abs. by O_2 and O_3
- NIR many lines by H_2O, CO_2, CH_4, N_2O
- Thermal IR many lines/bands by H_2O, CO_2, O_3
- IR window between 8 and 12 µm
- Far IR (50µm-1mm) completely opaque with H_2O main absorber; only useful for sensing upper atmosphere from space



SOLAR SPECTRUM AND ATMOSPHERIC GASES



Main gas: O_3
Other gases: O_2, H_2O, CO_2 [NO_2, OClO, BrO]

0.4 μm **3.2 μm**

Figure 8.2 Spectrum of SW radiation at the top of the atmosphere (solid) and at the earth's surface (shaded), compared against the emission spectrum of a blackbody at 6000 K (dashed). Individual absorbing species indicated. Adapted from Coulson (1975).

Optical thickness: contributors

$I_\lambda(0)$ → L → $I_\lambda(L)$

GAS: $k_a(\lambda)$, c, T

$$I_\lambda(L) = I_\lambda(0)\ \exp\{-k_a(\lambda)\,L\} = I_\lambda(0)\ \exp[-\sigma(\lambda)\,c\,L]$$

with c = the density of molecules per unit volume.

Three factors affects the optical thickness:

- **Spectroscopy: absorption cross section $\sigma(\lambda)$** [cm^2/molecule]
- **Composition/density: $c = \chi\, c_{air}$** [molecules/cm^3]
- **Photon path-length: geometrical distance = L** [km]

mixing ratio of the gas

General form with k_a changing in space: $I_\lambda(L) = I_\lambda(0)\,e^{-\int_0^L \sigma(\lambda,p,T)\,c(p,T)\,dL}$

Transmission of gases

Example

Calculate the transmission of CO_2 lines over a range of 10 km line for different absorption cross sections of a) 10^{-19} cm^2; b) 10^{-21} cm^2; c) 10^{-23} cm^2. Assume CO_2 has the concentrations given by 2.7 * 10^{15} molecules cm^{-3} .

$\mathcal{T}_g(\nu) = \exp[-k_n(\nu) N\ l\ ]$

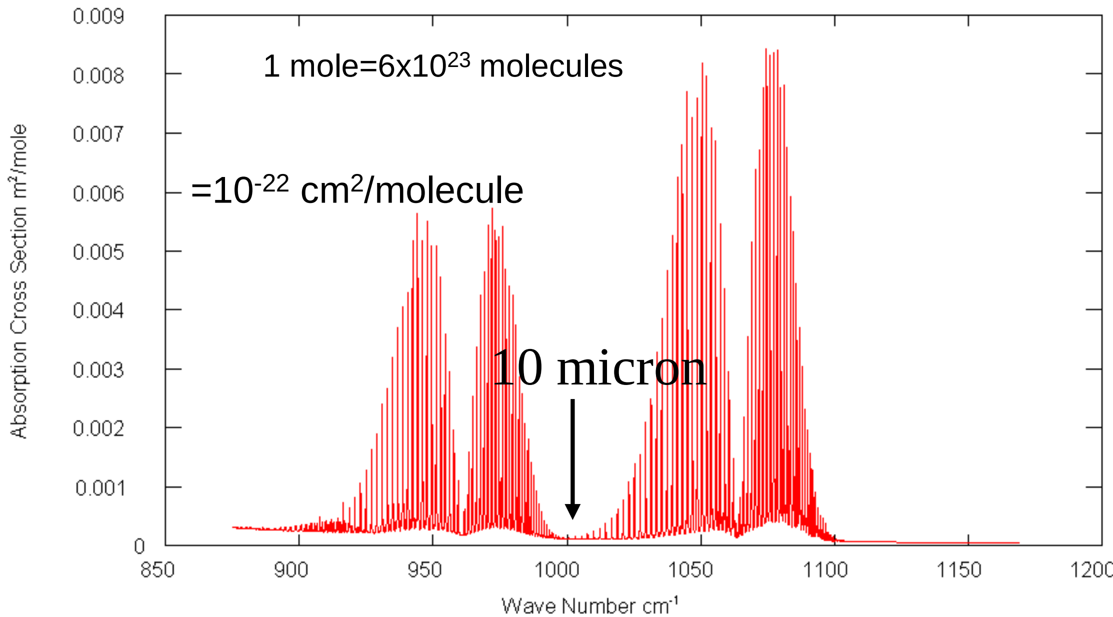
a) $= \exp[-10^{-19}\ \text{cm}^2/\text{mol} * 2.7 * 10^{15}\ \text{mol cm}^{-3} * 10 * 10^5\ \text{cm}]$

$= \exp[-2.7 * 10^{1}] \approx 0.0$

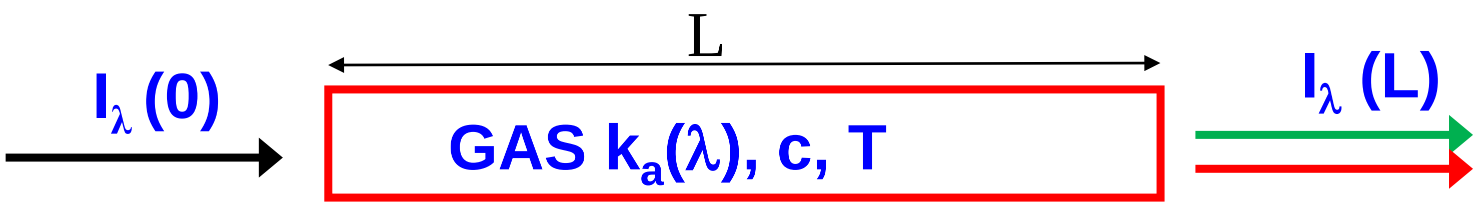
b) $= \exp[-2.7 * 10^{-1}] \approx 0.76$

c) $= \exp[-2.7 * 10^{-3}] \approx 0.997$

a) is opaque
b) has mid-range transmission
c) Is nearly transparent.



Accounting for emission



There are 2 signals emerging:

1. **Radiance transmitted by the gas in the cell:** $\mathcal{T}(\lambda)\, I_o(\lambda)$
2. **Radiance emitted by the gas in the cell:** $\varepsilon(\lambda) \times B(\lambda,T)$

Emissivity is related to absorptance $a(\lambda)$ of the layer (fraction of radiance being absorbed)

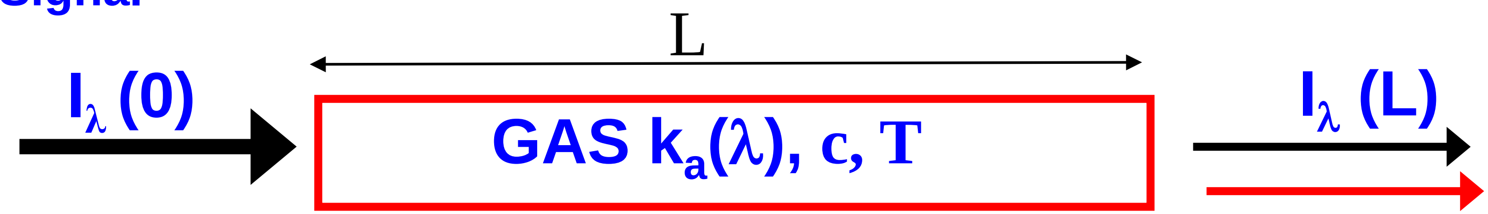
We neglect scattering (valid for gases in the IR)

$\mathcal{T}(\lambda) + a(\lambda) = 1 \Rightarrow a(\lambda) = 1 - \mathcal{T}(\lambda)$ Radiation is either transmitted or absorbed

In thermal equilibrium (Kirchhoff's law) $\varepsilon(\lambda) = a(\lambda) = 1 - \mathcal{T}(\lambda)$

IR TRANSMISSION FOR ISOTHERMAL LAYER

Total Signal



Hence, Total Signal, I(λ) is given by

==> $I_\lambda(L) = \mathcal{T}(\lambda)\, I_\lambda(0) + (1 - \mathcal{T}(\lambda)) \times B(\lambda, T)$

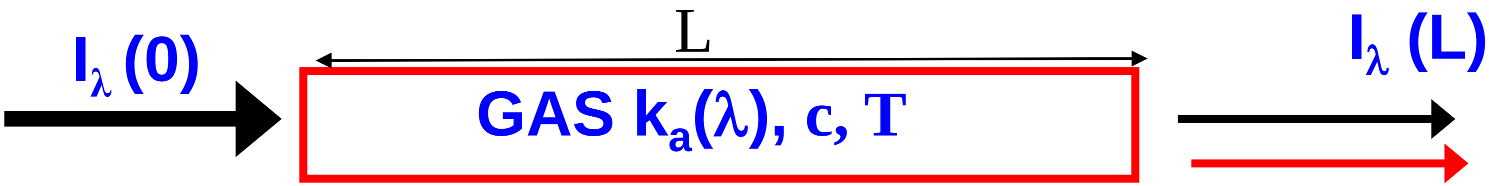
Limiting cases:

1) $\mathcal{T}(\lambda) \approx 1 \rightarrow I_\lambda(L) \approx \mathcal{T}(\lambda)\, I_\lambda(0)$ **[Known as a WINDOW]**

2) $\mathcal{T}(\lambda) = 0 \rightarrow I_\lambda(L) = B(\lambda, T)$ **[Known as 100% absorption or saturation]**

N.B. : If: a) term 1 >> term 2 or b) if T_{gas} = 0 K (!)
then Eqn. 1) would be true and conventional use of $\mathcal{T}(\lambda)$ alone is fine, e.g. uv-visible on Earth or hot source relative to cold gas.

Hot nebula



$$I_\lambda(L) = \mathcal{T}(\lambda)\, I_\lambda(0) + (1 - \mathcal{T}(\lambda)) \times B(\lambda, T)$$

Imagine first the case in which $I_\lambda(0)=0$, i.e. solely emission from the volume of gas (with constant source function).

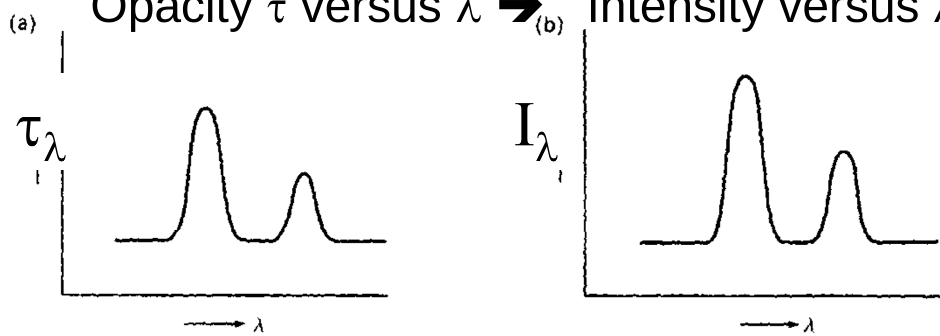
$$I_\lambda = B_\lambda (1 - e^{-\tau_\lambda})$$

We have two limiting cases:

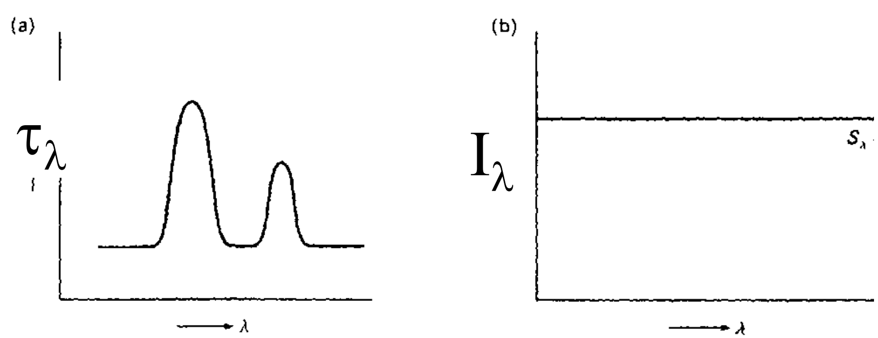
Opacity τ versus λ → Intensity versus λ

- Optically thin case (τ_λ<<1)

$$e^{-\tau_\lambda} \approx 1 - \tau_\lambda \Rightarrow I_\lambda = \tau_\lambda B_\lambda$$



- Optically thick case (τ_λ>>1)

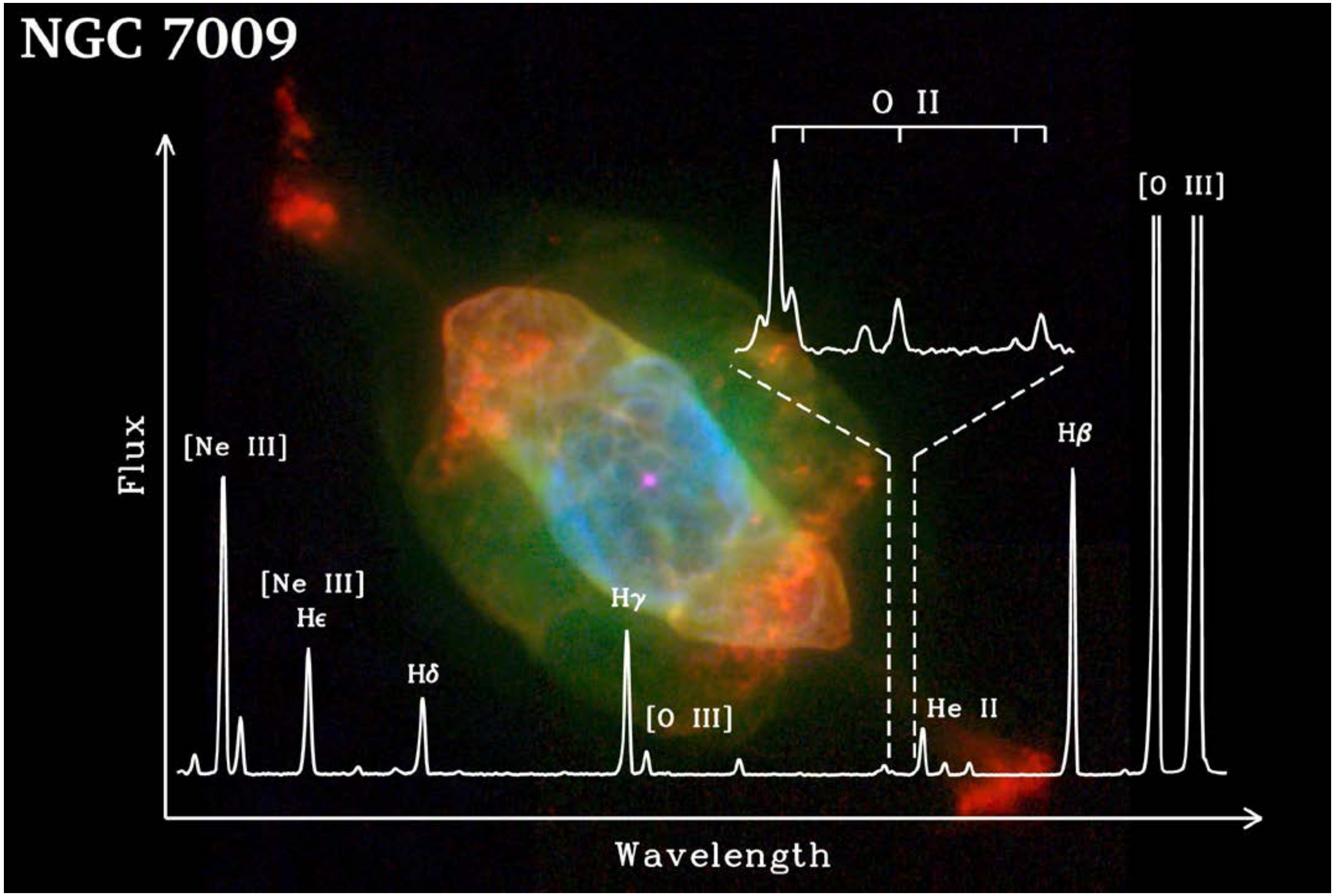


$$e^{-\tau_\lambda} \approx 0 \Rightarrow I_\lambda = B_\lambda$$

It behaves like a black body

Hot low density nebular gas

Example of emission lines for optically thin nebulae



The optical thickness is strongly dependent on wavelength ➔ peaks corresponding to absorption bands of different elements in the nebula

Absorption versus emission

$$I_\lambda(L)=I_\lambda(0)e^{-\tau_\lambda} + B_\lambda(T)(1-e^{-\tau_\lambda})$$

Imagine now $I_\lambda(0)\neq 0$, again with two extreme cases:

- Optically thin case (τ_λ<<1) $\quad I_\lambda(L)=I_\lambda(0)\,(1-\tau_\lambda)+\tau_\lambda B_\lambda(T)=I_\lambda(0)+\tau_\lambda[B_\lambda(T)-I_\lambda(0)]$

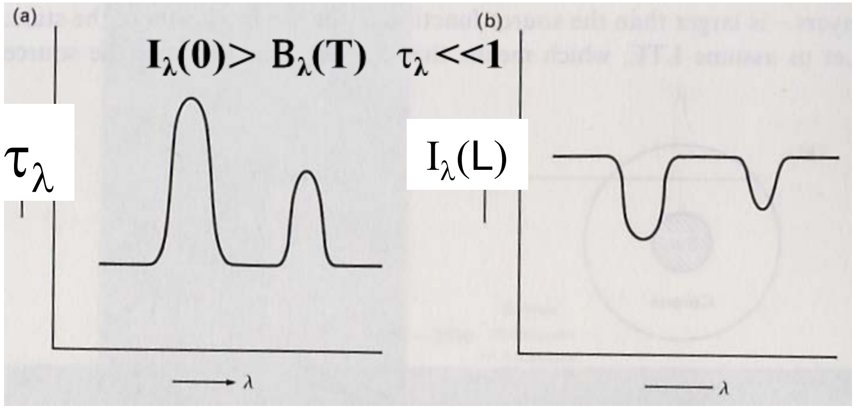
(a) If $I_{\lambda 0}>B_\lambda$, so there is something subtracted from the original intensity which is proportional to the optical depth – we see absorption lines on the continuum intensity I_λ.
EXAMPLE: <u>stellar photospheres</u>

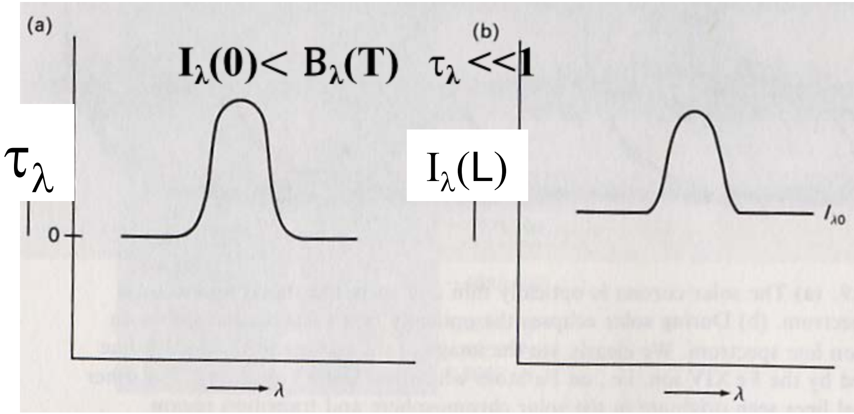
(b) If $I_{\lambda 0}<B_\lambda$, we will see emission lines on top of the background intensity.

Example: <u>Solar UV spectrum</u>

- Optically thick case (τ_λ>>1):

Planck function as before. $\quad I_\lambda(L)=B_\lambda(T)$





Opacity τ versus λ ➔ Intensity versus λ

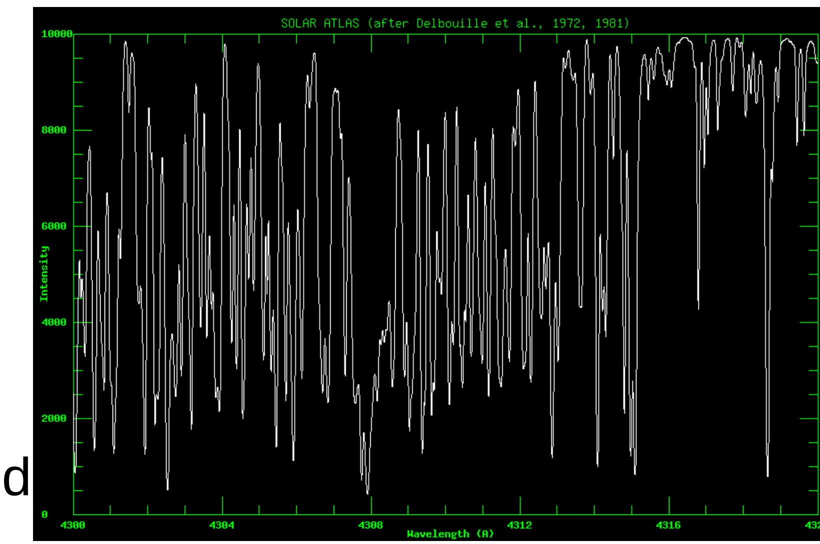
Absorption lines? Outward decreasing temperature

In a star absorption lines are produced if

$I_{\lambda 0} > B_{\lambda}$ i.e. the intensity from deep layers is larger than the source function from top layers.

In LTE, the source function is $B_{\lambda}(T)$, so the Planck function for the deeper layers is larger than the shallower layers. Consequently the deeper layers have a higher temperature than the top layers (since the Planck function increases at all wavelengths with T).

(Instances occur where LTE is not valid, and the source function declines outward in parallel with an increasing temperature).



Solar Spectrum (4300-4320 *angstrom=0.43-0.432 micron*) ➔ absorption lines

Absorption versus emission spectra

Emission line spectra

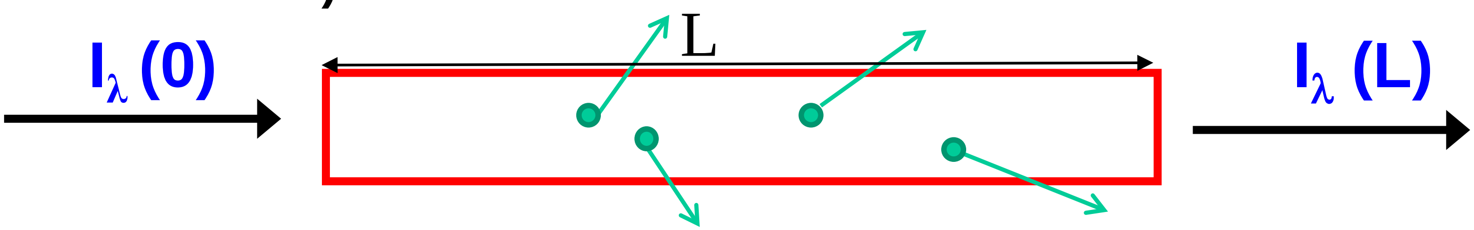
- Optically thin volume of gas with no background illumination (emission nebula)
- Optically thick gas in which the source function increases outwards (UV solar spectrum)

Absorption line spectra

- Optically thick gas in which source function declines outward, generally T decreases outwards (Stellar photospheres)
- Optically thin cold gas penetrated by background radiation (Interstellar matter between us and the star)

BEER-LAMBERT LAW: EXTINCTION LAW

Homogenous non-absorbing medium with scattering coefficient, k_s (per unit distance)



$$dI_\lambda = - k_s(\lambda) I_\lambda \, dL$$

and $I_\lambda(L) = I_\lambda(0) \exp\{- k_s(\lambda) L\}$ **(compare absorbing case)**

Scattering and absorbing media: $I_\lambda(L) = I_\lambda(0) \exp\{- k_e(\lambda) L\}$

with extinction coefficient: $k_e = k_s + k_a$

Extinction: total loss of light due to absorption and scattering of light out of path

The scattering optical thickness of our atmosphere

Compare the atmospheric transmissivity for Rayleigh scattering in UV with larger wavelength (600nm, 900 nm)

λ, nm	σ, cm^2
300	6.00×10^{-26}
400	1.90×10^{-26}
600	3.80×10^{-27}
1000	4.90×10^{-28}
10,000	4.85×10^{-32}

Transmission

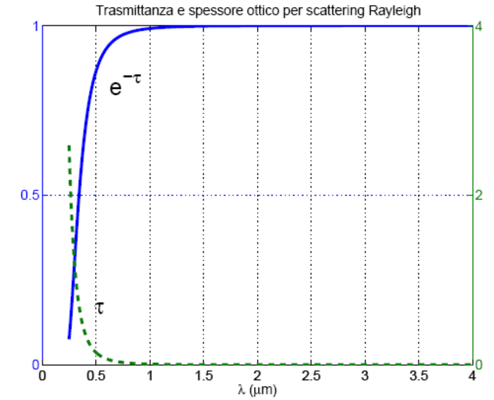
$$\mathcal{T}(\lambda) = I_\lambda(L) / I_\lambda(0)$$
$$= \exp(-\tau) = \exp\{- k_s(\lambda)\, L\}$$
$$= \exp\{- \sigma\, c\, L\} = \exp\{- \sigma\, VCD\}$$

with Vertical Column Density of air (VCD) = 2.3×10^{25} molec/cm^2

Transmissivity: $\mathcal{T}(\lambda=300nm) = 0.25$ $\tau = 1.4$

$\mathcal{T}(\lambda=600nm) = 0.92$ $\tau = 0.08$

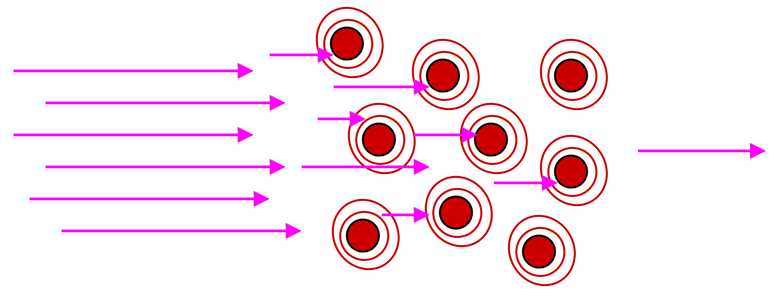
$\mathcal{T}(\lambda=1000nm) = 0.99$ $\tau = 0.01$



Only 25% of direct light reaches surfaces in UV at 300 nm but 99% at 1000nm.

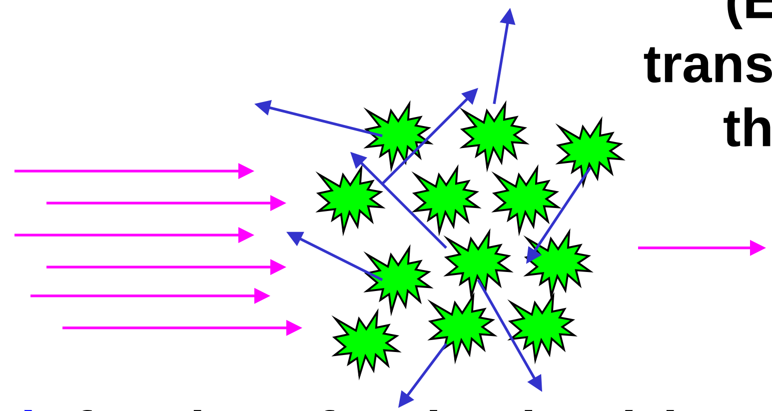
Extinction: Scattering plus Absorption

Absorption removes radiant energy from an E/M field, converting it to other forms of energy.



(Effect on transmittance is the same)

Scattering does not remove energy from an E/M field, but can redirect it, thereby making it a "source" of radiation for another direction.



Single scatter albedo (SSA): fraction of extinction (absorption + scattering) due to scattering processes:

$$\omega_o = \sigma_s / \sigma_e \qquad \sigma_e = \sigma_a + \sigma_s$$

$\omega_o = 1$ **all scattering (conservative)**

$\omega_o = 0$ **all absorption (non - conservative)**

How can we compute scattering and absorption cross sections of single particles?
How can we relate them to scattering and absorption coefficients?

What do you need to know?

- ❑ **Converting from Radiance to Fluxes**
- ❑ **Black body radiation and related laws**
- ❑ **Emission and Absorption**
- ❑ **Isothermal layer model**

$I_o(\lambda)$ → **GAS** $k_a(\lambda)$, T → $I_\lambda(L)$

$$\boxed{I_\lambda(L) = \mathcal{T}(\lambda)\, I_\lambda(0) + (1 - \mathcal{T}(\lambda)) \times B(\lambda, T)}$$

- ❑ **Lambert Beer Law (optical depth, transmissivity)**

Good textbooks (available in the university library) :

G.W. Petty – A first introduction in atmospheric radiation

G.L. Stephens – Remote sensing of the lower atmosphere – an introduction

Problem I

Based on what you know about their appearance, attempt to characterize the apparent spectral dependence of k_e and $\tilde{\omega}$ in the visible spectral range for the following substances:

- red wine
- milk
- chocolate milk
- a cloud
- a plume of exhaust from a diesel truck
- the cloud-free atmosphere (based on the color of the setting sun).

Red wine: wine is not turbid (milky), therefore there's not much scattering going on in comparison to extinction. The SSA is likely close to zero, but extinction is relatively strong, depends on λ, so that red light is transmitted far more readily than shorter λs.

Chocolate milk: Like milk, chocolate milk is rather opaque (extinction coefficient is large), and the opacity is mostly due to scattering. Unlike pure milk, chocolate milk is not white but brownish gray. This suggests that the SSA is somewhat less than one (there is some absorption) and that absorption is slightly stronger for shorter (bluer) wavelengths.

Problem II

Assume the measured extinction through a 10 cm water-ink suspension to be 70%. How large is $\boldsymbol{k_e}$?

Problem III

A cloud layer has a vertical profile of k_e that is quadratic in altitude z between cloud base z_{base} and cloud top z_{top}, with maximum k_e at the midpont of the cloud. At the base and top of the cloud, k_e=0.

a) Find the quadratic equation that describes $k_e(z)$ within the cloud layer.

b) Find the expression for the optical path τ measured vertically through the cloud layer.

c) Typical values for the above parameters for solar radiation incident on a thin cloud layer migt be $z_{base} = 1.0\ km$, $z_{top} = 1.2\ km$, and $k_e = 0.015\ m^{-1}$. Compute the total optical depth for this case.

d) Based on your answer to (c), compute the vertical transmission through the cloud.

Problem IV

A ground-based radiometer operating at $\lambda = 0.45\mu m$ is used to measure the solar intensity $I_\lambda(0)$. For a solar zenith angle of $\theta = 30°, I_\lambda(0) = 1.74 \times 10^7\ Wm^{-2}\mathrm{\mu}m^{-1}sr^{-1}$, and for $\theta = 60°, I_\lambda(0) = 1.14 \times 10^7\ Wm^{-2}\mathrm{\mu}m^{-1}sr^{-1}$ is measured. From this information, determine the top-of-the-atmosphere solar intensity S_λ and the atmospheric optical thickness τ_λ.