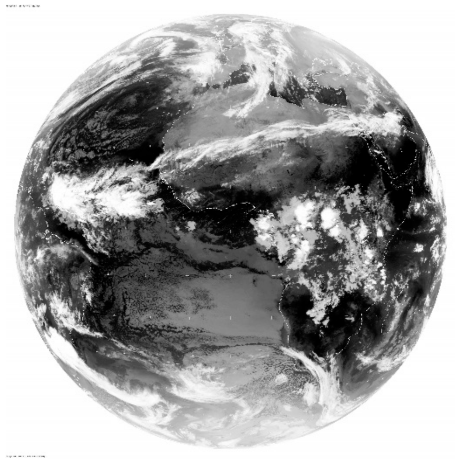
SECOND YEAR: 2604
PLANETARY REMOTE SENSING 5

Scattering inside Earth's atmosphere

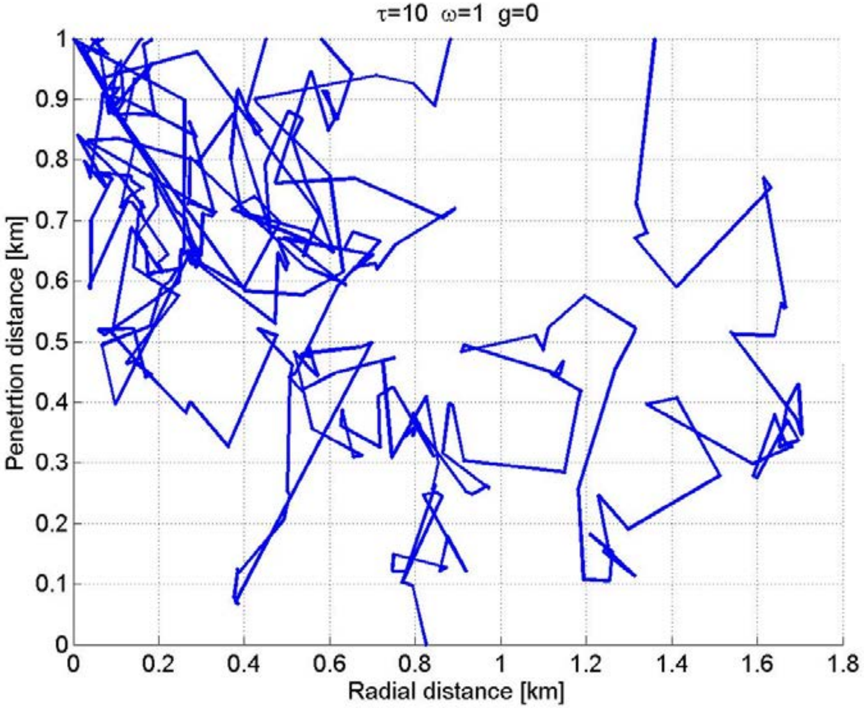
Dr. A. Battaglia
EOS-SRC, Dept. of Physics and Astronomy,
University of Leicester, U.K.

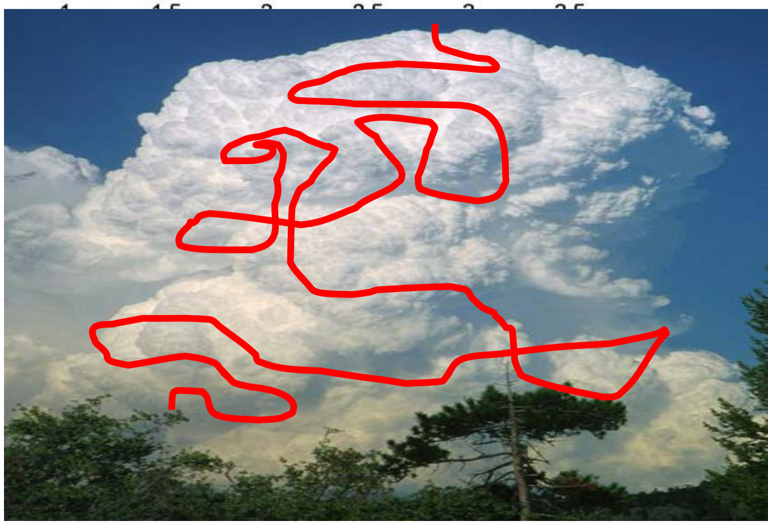
http://www2.le.ac.uk/departments/physics/research/earth-observation-science



The photon journey

10 trajectories

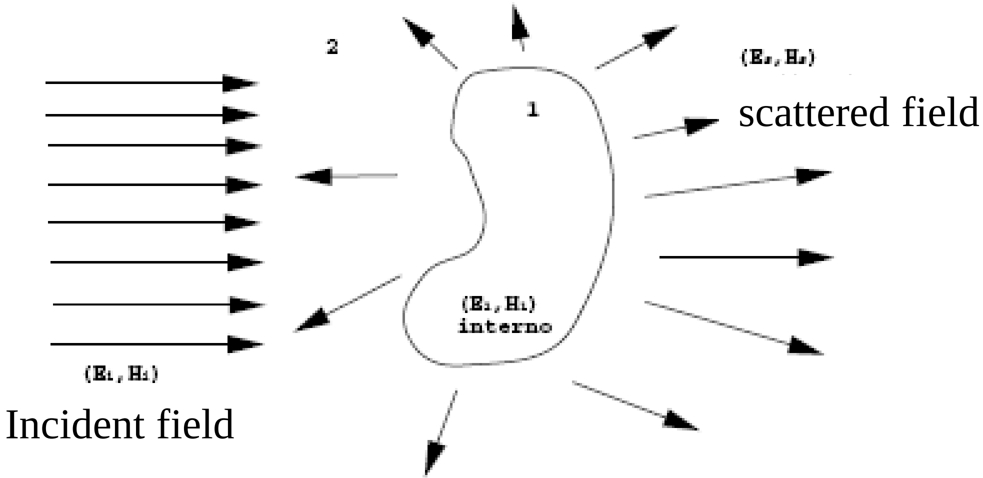




Atmospheric phenomena

Can you explain these phenomena?

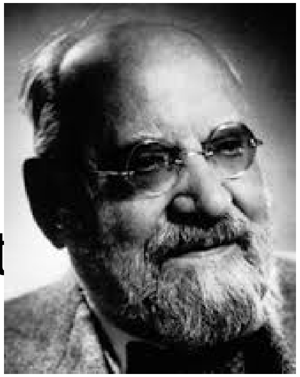
Interaction of Particle with E/M Radiation



$$\nabla \cdot \mathbf{D} = 0$$
$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$$
$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}$$

Basically the problem is ``reduced'' to solving Maxwell's equations with boundary conditions

1869-1957



Solution generally depends on

- Size, shape and composition of the particle ($m=m_r+im_i$)
- Wavelength, polarization and direction of the incident radiation (relative to the particle orientation)

For spheres the problem can be solved analytically ➔ Mie theory (1908, ``Contributions to the optics of turbid media, particularly of colloidal metal solutions'' in "Annalen der Physik)

Mie Scaling property

All scattering and absorption properties scale with the ratio of size of scattering particle (r) to wavelength (λ) of light:

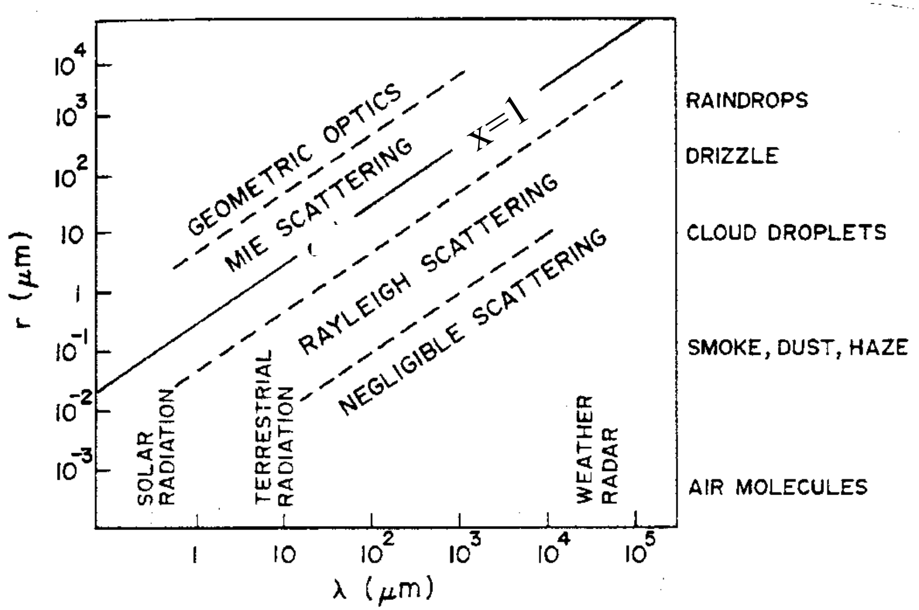
Size parameter $x = 2\pi r / \lambda$

different regimes of atmospheric scattering need to be distinguished.

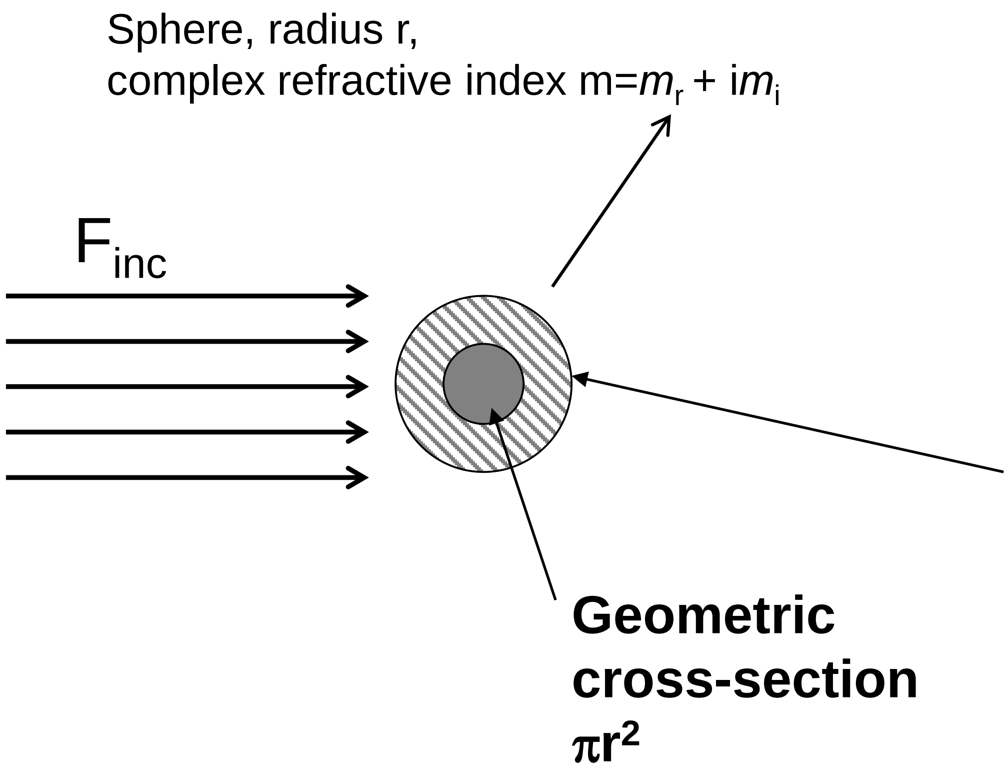
x << 1 **Rayleigh scattering (single Dipole)**

0.1 < *x* < 50 ***``Mie" Scattering***

x> 50 ***Geometric optics***



Interaction of Particle with E/M Radiation: cross sections



Particle scattering/absorption is defined in terms of cross-sectional areas σ & efficiency factors Q

σ_e = effective area projected by the particle that determines extinction

Similarly σ_s, σ_a

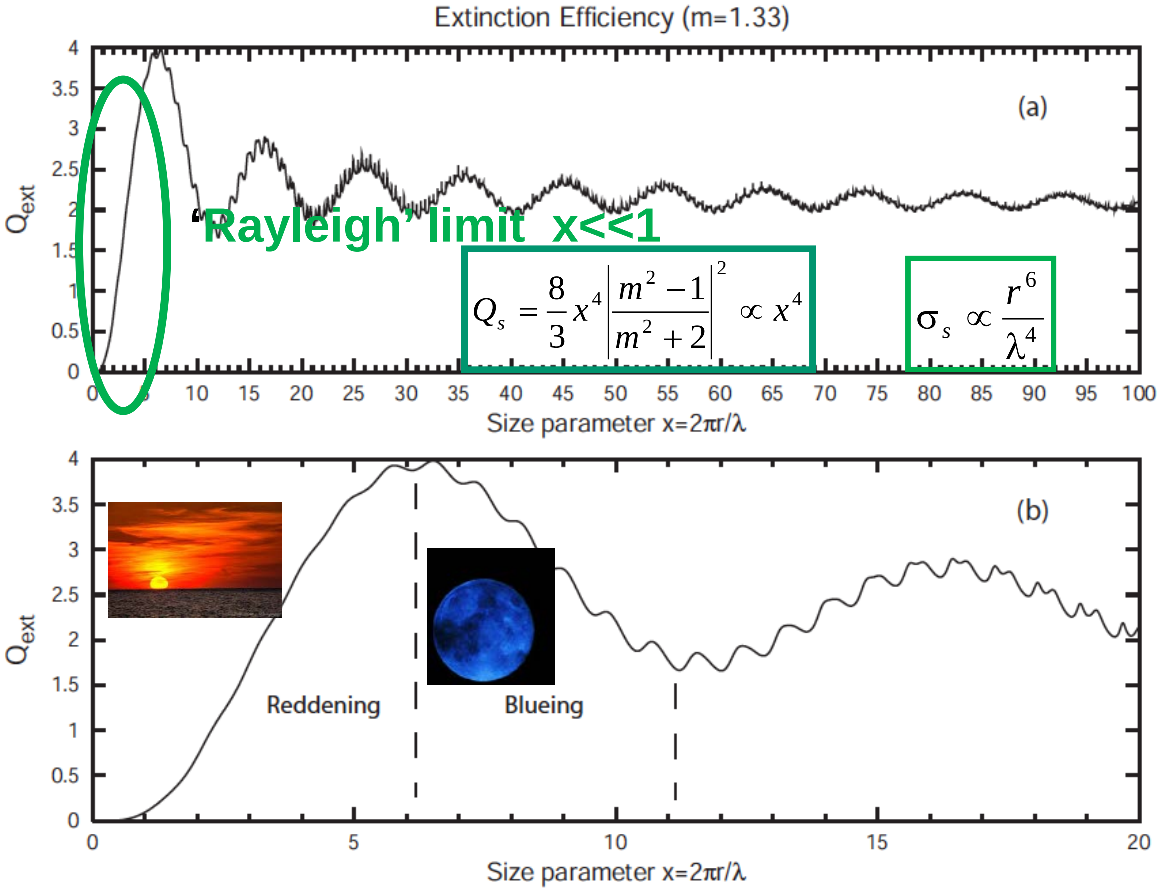
The particles is absorbing and/or scattering some of the incident radiation ➔absorbed/scattered power

$$\sigma_{e,s,a} = \frac{P_{e,s,a}}{F_{inc}}$$

The efficiency factor then follows $$Q_{e,s,a} = \frac{\sigma_{e,s,a}}{\pi r^2} = f(x,m)$$

N.B.: Q can be larger than 1!!!

Extinction efficiency (non absorbing sphere)



Cloud droplets

Non-absorbing water sphere in the VIS (m=1.33+i0)

a) Q_e rises to 4 at x=6 (r~λ), then dampening oscillations around 2

b) $Q_e \rightarrow 2$ as $2\pi r/\lambda \rightarrow \infty$ *extinction paradox*

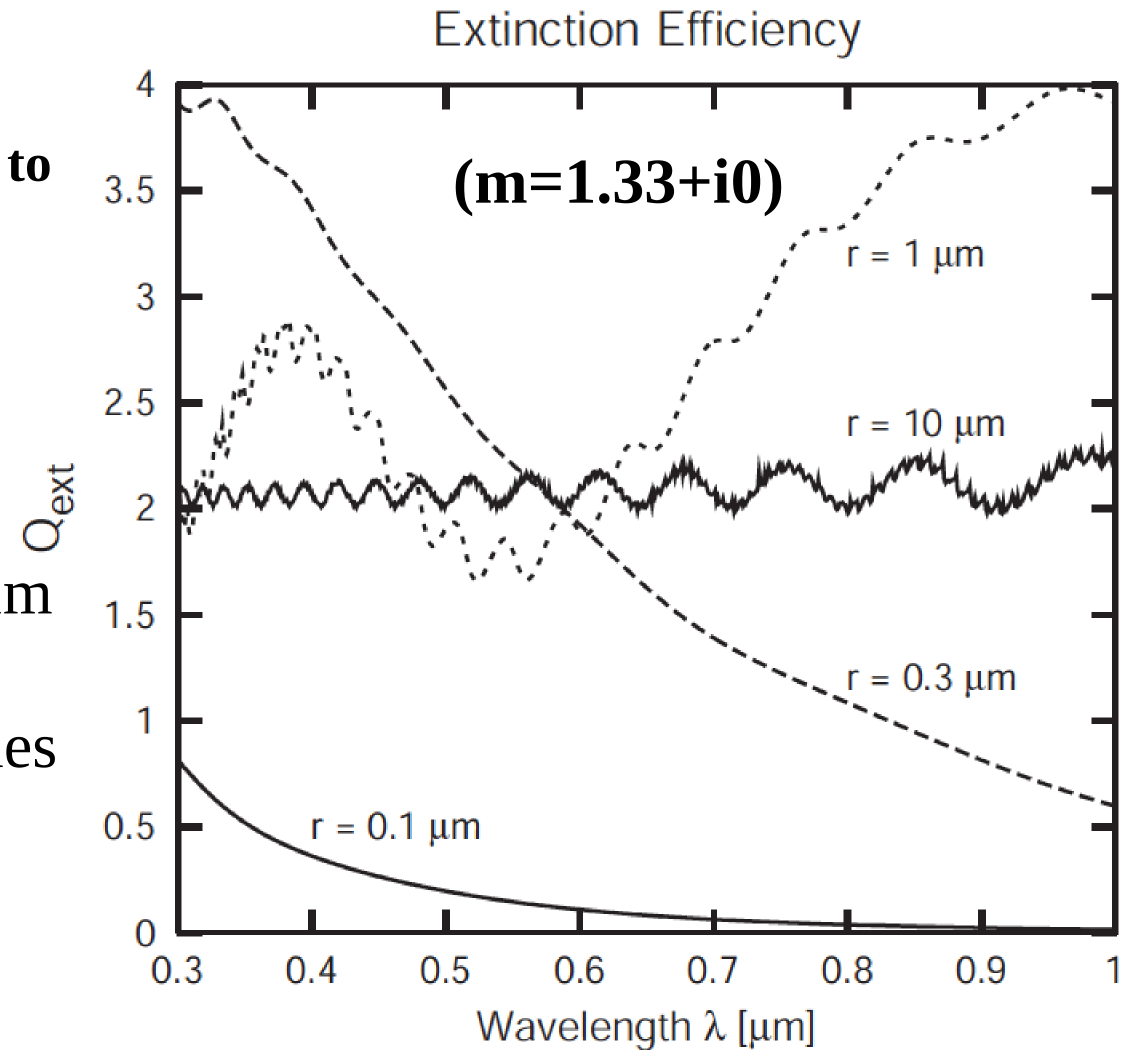
Implications of oscillations for color of scattered light. Assume r=const and λ rising (to the left!)

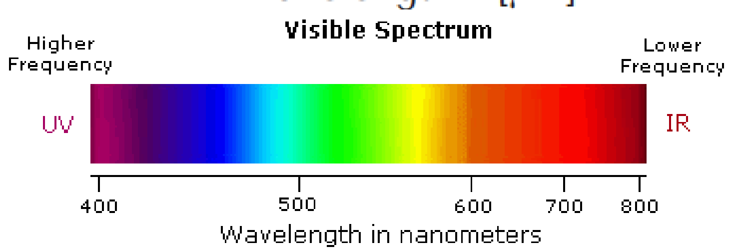
- *x<6*: red less extinguished than blue➔reddening
- *6<x<12*: blue less extinguished than red➔blueing (needs particular narrow size distribution of aerosols).

Extinction efficiency vs wavelength

Looking into light source: maximum signal correpsonds to minimu extinction

a) For aerosols and small haze (r<0.3μm) we still see reddening.
b) For particles around 1 μm we could see greening!
c) For typical cloud particles (r~10μm) we see no colour- selective properties and Q_e~2: white clouds





Extinction and Scattering by Absorbing Spheres

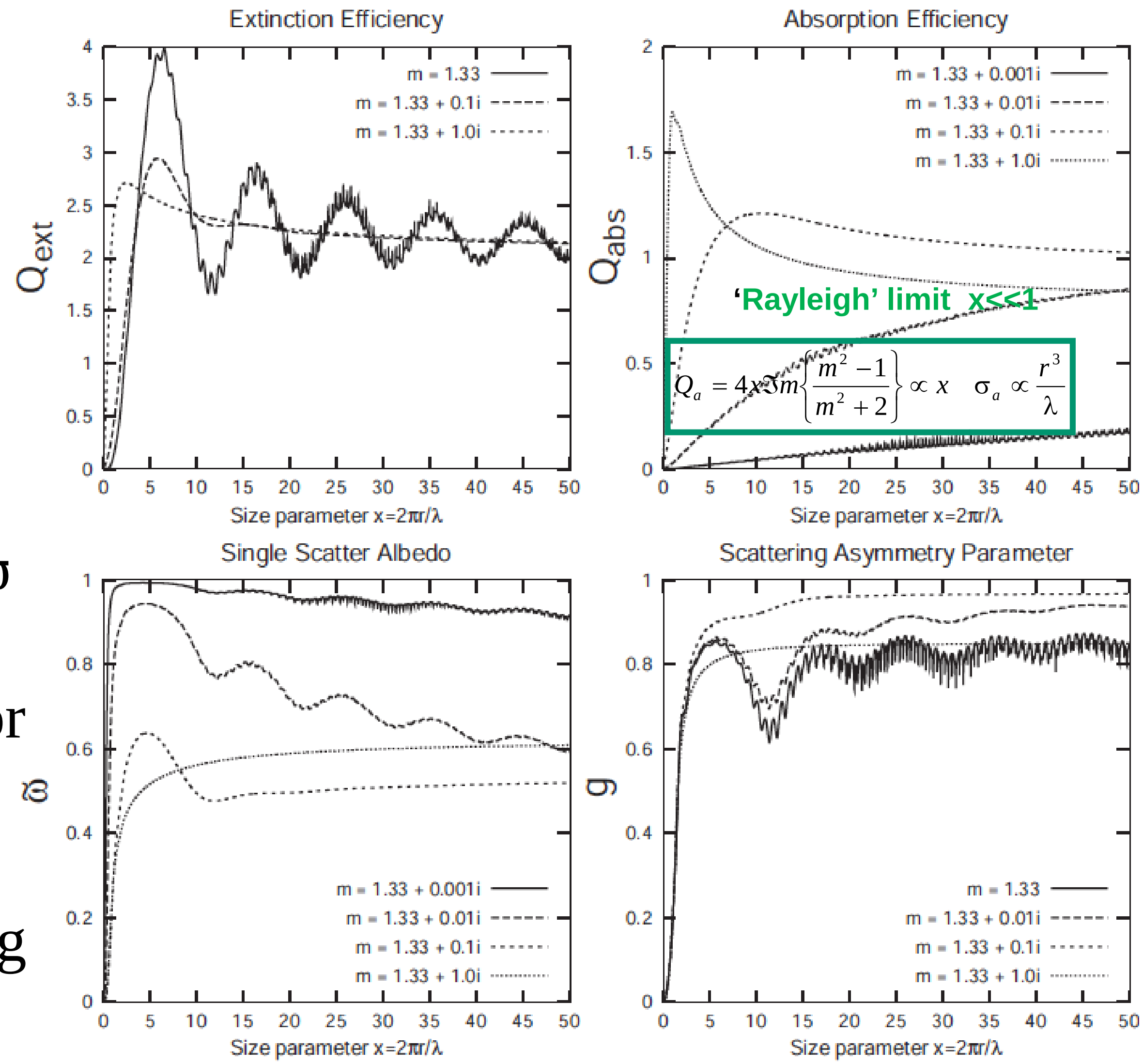
Absorbing water sphere ($m=1.33+im_i$)

a) Absorption largely reduces both the large and fine wiggles.

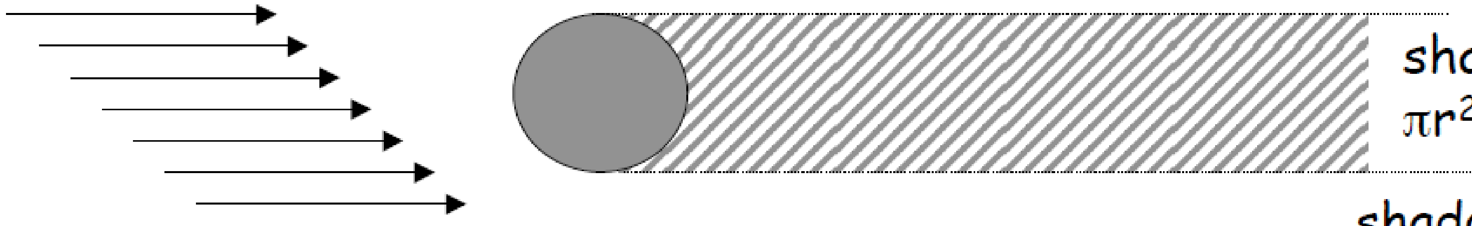
b) Single scatter albedo strongly reduces with increasing m_i (it's 1 for $m_i=0$).

c) For x>10 behaviour of ϖ with increasing m_i is unpredictable (can rise or fall).

d) Forward scattering increases, with g starting at zero in the Rayleigh range.

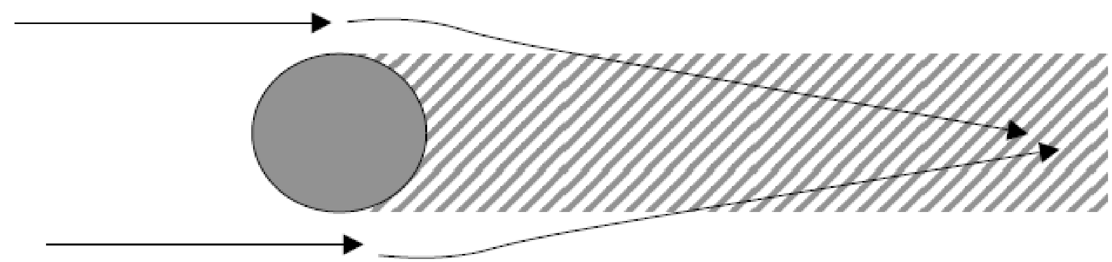


Extinction Paradox



shadow area
πr^2

$$Q_{ext} = \frac{\text{shadow area}}{\pi r^2} = 1\,??$$



combines the effects of absorption and any reflections (scattering) of the sphere.

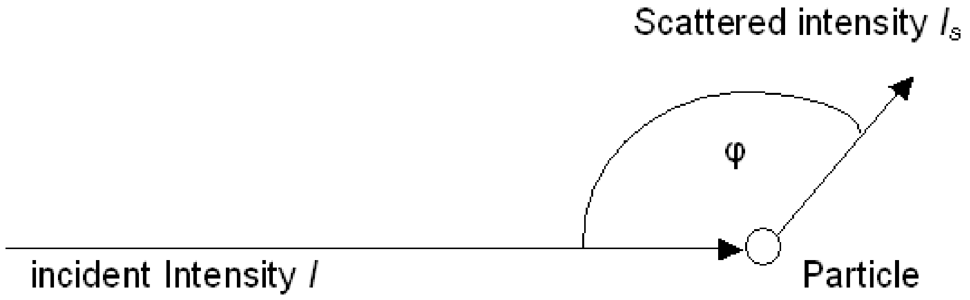
$$Q_{ext} = \frac{\text{shadow area} + \text{area filled by diffraction}}{\pi r^2}$$

$$= \frac{\pi r^2 + \pi r^2}{\pi r^2} = 2$$

SCATTERING PROCESSES

❑ Basic Considerations:

- **Scattering removes fraction of radiation from one direction and directs it into other directions**
- **Photons passing through medium may encounter only one particle (single scattering) or many particles (multiple scattering)**
- **Scattering in the atmosphere occurs from molecules (Rayleigh scattering) and aerosol/cloud particles (Mie scattering)**



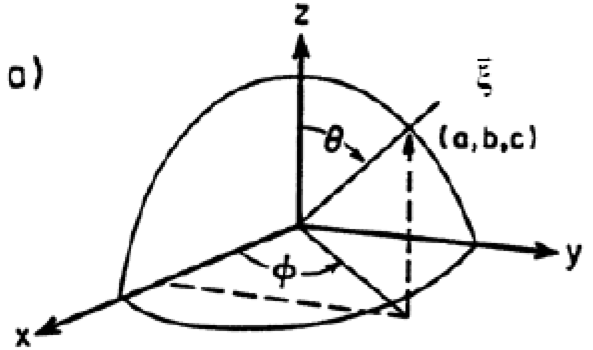
Main Parameters:

- **Scattering cross section σ_s(effective area for scattering)**
- **Scattering phase function p(φ) (direction)**

Scattering Angle and plane of scattering

Unit direction vector :

$$\vec{\xi} = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)$$

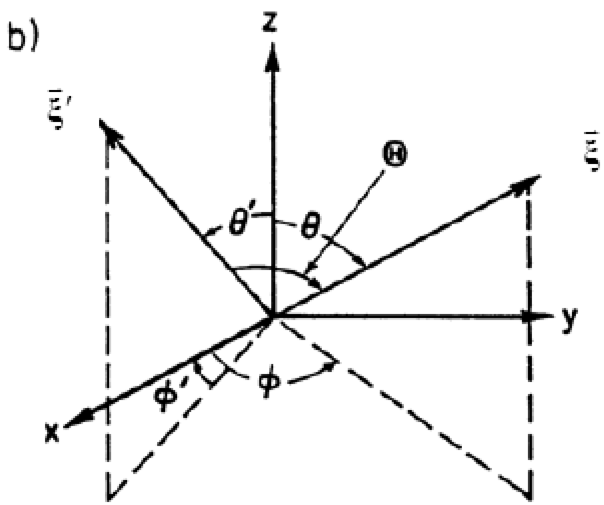


The angle formed between two direction vectors is given by:

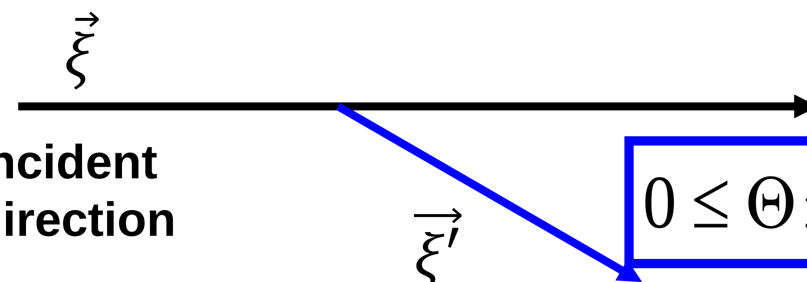
$$\cos\Theta = \xi \bullet \xi'$$

$$= \mu\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\varphi' - \varphi)$$

with μ = cosθ



Viewed in 2-D (in a plane that contains the incident and scattered direction vectors)



Incident direction

$$\boxed{0 \le \Theta \le 180^o}$$

Scattered Direction

Phase Function for spheres

Scattering only depends on the scattering angle

$$Phase\ \ function: \quad P(\cos\Theta) = \frac{4\pi}{\sigma_s}\frac{d\sigma_s(\cos\Theta)}{d\Omega}$$

$$P(\cos\Theta) = P(\theta,\phi,\theta',\phi')$$

Phase function is normalized:

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P(\theta,\phi,\theta',\phi')\sin\theta\, d\theta\, d\phi = 1$$

Isotropic scattering:

$$P(\cos\Theta) = 1$$

A useful parameter to describe the PF shape is the asymmetry parameter

$$g \equiv \langle\cos\Theta\rangle = \int_{4\pi} P\cos\Theta\, d\Omega$$

average cosine of the scattered direction

g = 1 - pure forward scatter

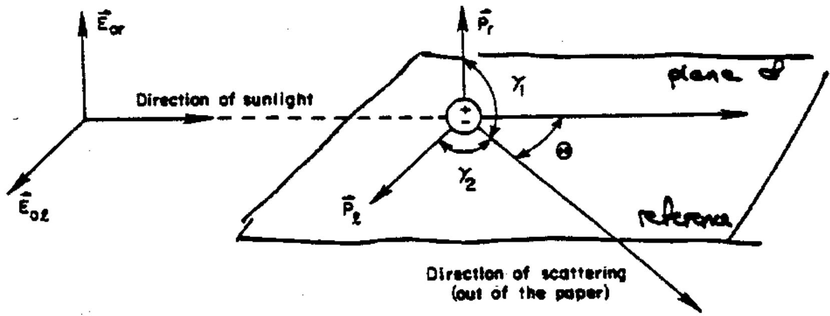
g = 0 - isotropic or symmetric

g = -1 - pure backscatter

Scattering by small particles (e.g. visible on molecules)

Scattering by molecules is referred to as Rayleigh scattering

$$\frac{2\pi r}{\lambda} = x << 1$$



- The whole particle is experiencing the same electromagnetic field when a wave is passing.
- **E/M wave** The em field slightly moves electrons and protons in opposite directions **induces oscillating dipole moment p in molecule** locked to the phase of the forcing field of the em wave.
- **Oscillating dipole** steadily accelerates charges **radiates E/M wave - Hertz Dipole** (*Grant&Phillips, Ch. 13*)**:**
- **Scattered wave is proportional to the Polarizability α**

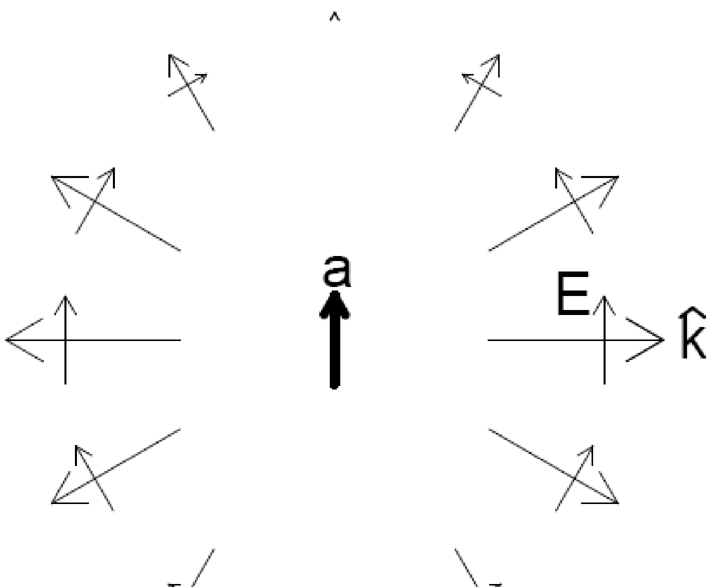
$$\mathbf{p} = \alpha \mathbf{E}_0 \exp(i\omega t)$$

$$\alpha \equiv \frac{\varepsilon - 1}{\varepsilon + 2} = \frac{m^2 - 1}{m^2 + 2}$$

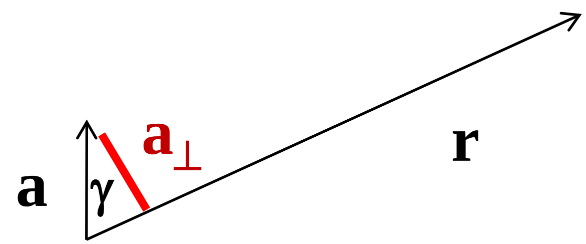
ε= dielectric constant
m=refractive index

Bremsstrahlung

Bremsstrahlung, i.e. "braking radiation", is electromagnetic radiation produced by the acceleration of a charged particle
All classical [electromagnetic radiation is ultimately generated by accelerating electrical charges](#).



$$\mathbf{E}_{rad}(\mathbf{r},\ t) = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{a}_\perp(t - r/c)}{rc^2}$$



$$\mathbf{E}_{rad.\ dip.}(\mathbf{r},\ t) = \frac{1}{4\pi\epsilon_0}\left[\frac{\omega^2}{rc^2}\hat{\mathbf{r}}\times\mathbf{p}\times\hat{\mathbf{r}}\right]\ e^{[j\omega(t-r/c)]}$$

electric field of an electric dipole oscillating in the vertical direction
No E-field along the oscillating direction
Maximum E-field along directions normal to oscillation

$$\mathbf{p} = \mathbf{p}_0\ e^{j\omega t}$$

In the r-p plane orthogonal to r proportional to sin(γ)
γ angle between r and p

The Scattered Wave

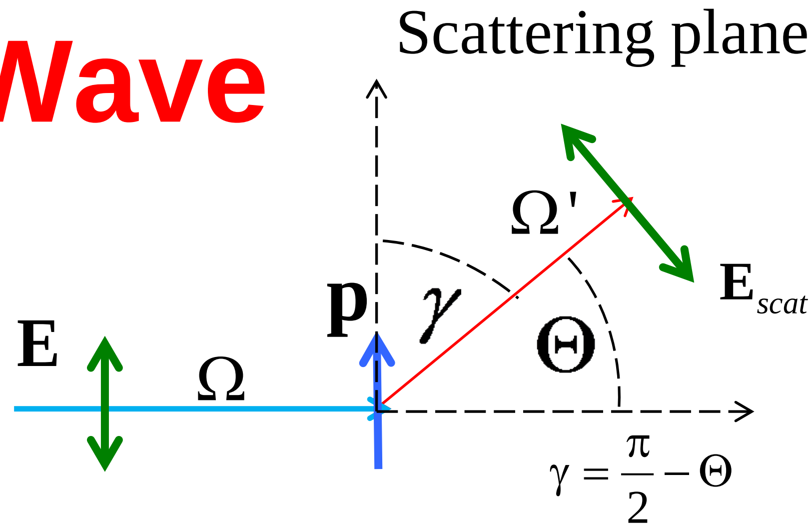
parallel to scat. plane

1. $\mathbf{E}_0 \perp \hat{\Omega}$

2. $\mathbf{p} \parallel \mathbf{E}_0 \underset{\text{when } \alpha \text{ is scalar}}{\rightarrow} \mathbf{p} \perp \hat{\Omega}$

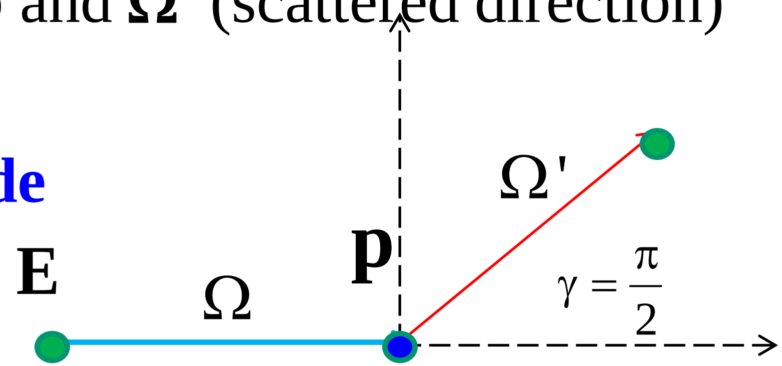
|| mode

E lies on the scattering plane



3. $\mathbf{E}_{scat}$ in a plane spanned by **p** and **Ω'** (scattered direction)

$\boxed{\perp}$ **mode**



4. $\left|\mathbf{E}_{scat}(\hat{\Omega}')\right| \propto \left|\mathbf{p} \times \hat{\Omega}'\right| \propto \begin{cases} \sin\gamma & \parallel \\ 1 & \perp \end{cases}$

5. $\left|\mathbf{E}_{scat}\hat{\Omega}')\right| \propto \left|\frac{d^2\mathbf{p}}{dt^2}\right| = \left|\alpha \mathbf{E}_0 \frac{d^2}{dt^2}\exp(i\omega t)\right| \propto \omega^2$

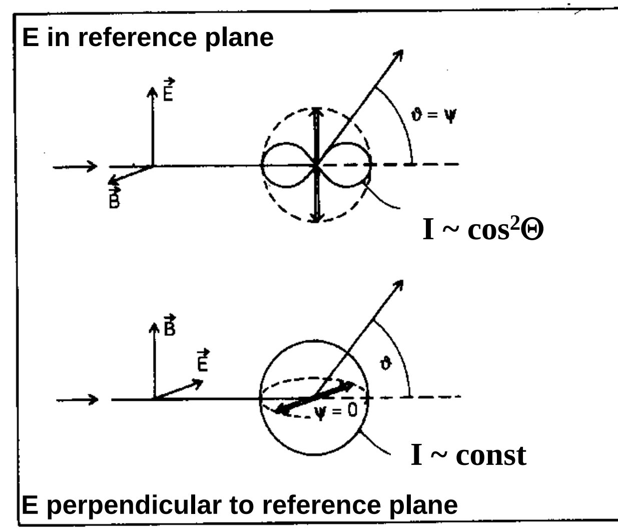
$$I_{scat} \propto \mathbf{E}^2_{scat}(\hat{\Omega}') \propto \begin{cases} \omega^4 \sin^2\gamma & \parallel \\ \omega^4 & \perp \end{cases}$$

Scattered intensity: **σ_s [Rayleigh]~ λ^{-4}**

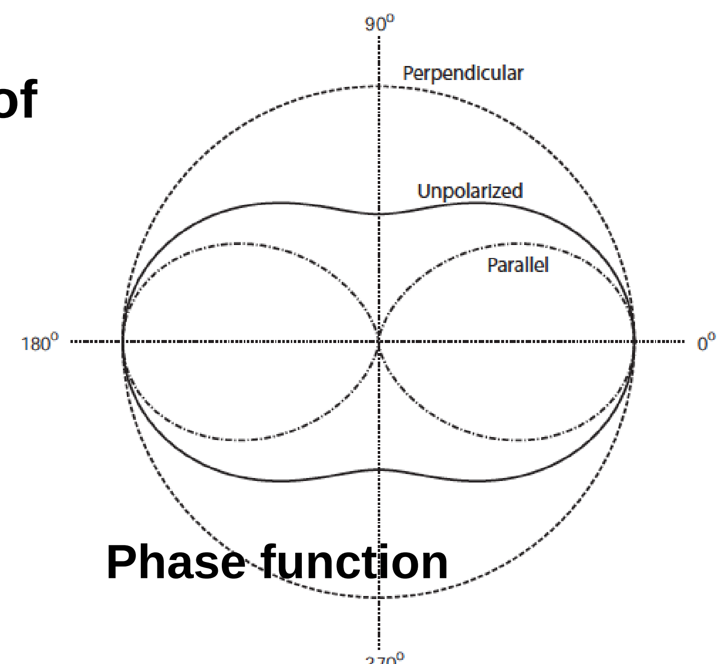
- **Very strong wavelength dependence**
- **Rayleigh scattering is very efficient for short (UV) wavelength**

Rayleigh Scattering and polarization

- Intensity of scattered light will depend on orientation of E vector
  - $I \sim \cos^2\Theta$ for E_l and $I \sim$ const. for E_r
  - Unpolarized light: superposition of 2 dipoles: $I \sim 1+\cos^2\Theta$
  - Close to isotropic for unpolarized light



- (Rayleigh) scattering polarizes light
  - Dipole does not radiate in direction of oscillation
  - maximum of polarisation for 90° scattering angle



$$P_{unpolarized}(\Theta)=3/4(1+\cos^2\Theta)$$

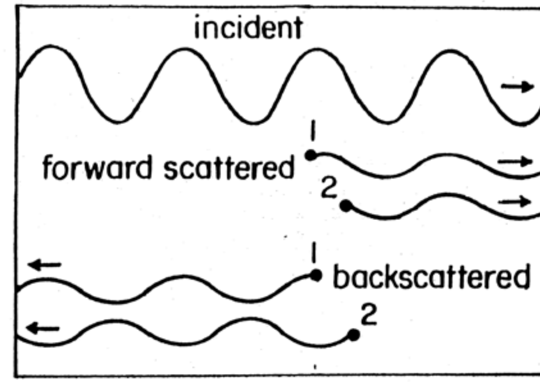
Phase function

Scattering on Particles (Mie Scattering)

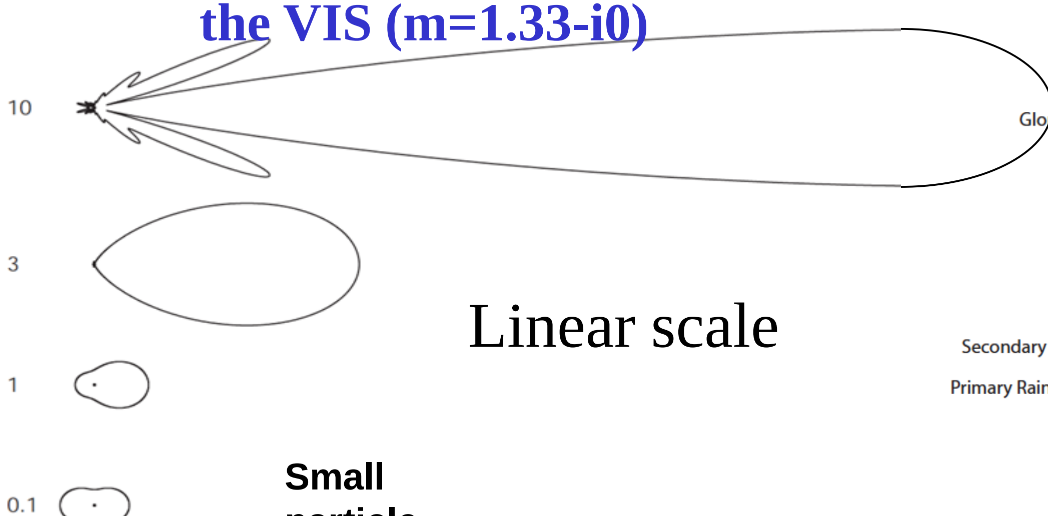
Explained by coherent scattering from many individual particles (dipoles) that form scattering particle

- **Forward moving waves tend to be in phase and thus constructively add**

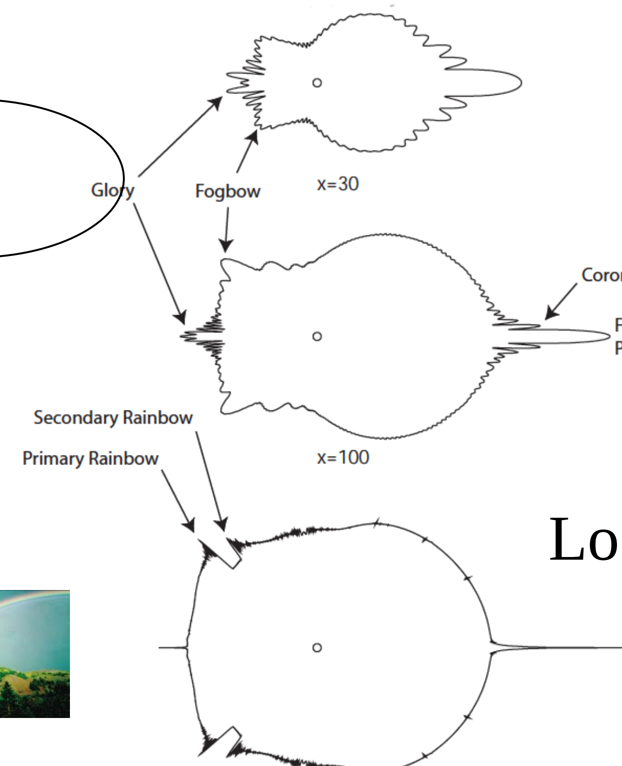
Combination of 2 Dipoles



Non-absorbing water sphere in the VIS (m=1.33-i0)



Linear scale



Logarithmic scale

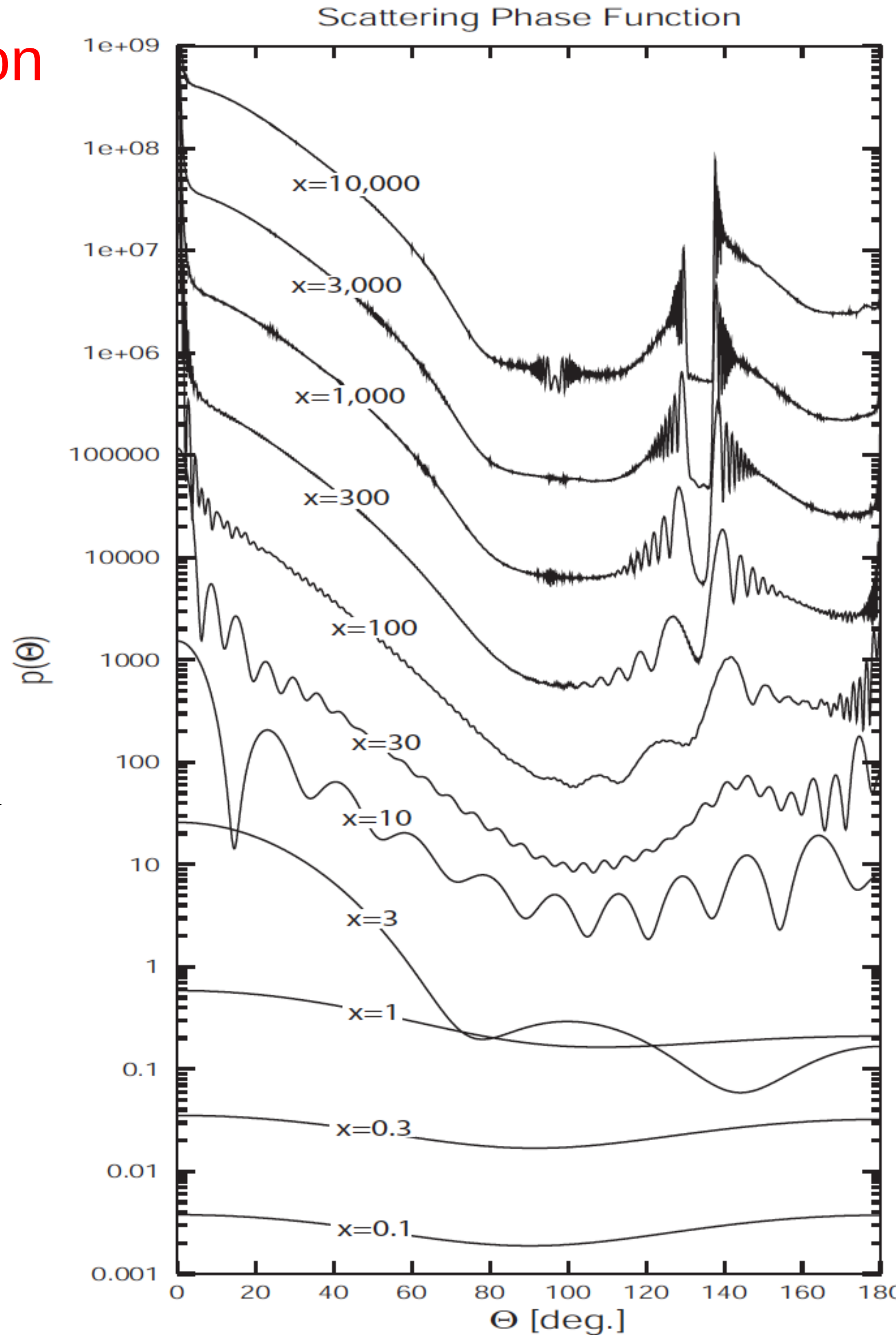
Small particle

Size parameter

Sphere Scattering Phase Function

Non-absorbing water sphere in the VIS (m=1.33-i0), scale is arbitrary!

- For x=0.1 symmetric Rayleigh phase function
- Increasing x leads to increasing forward scattering (depends on m!)
- Number of large wiggles are ~equal to x.
- At x=3 forward scattering is already 100 times larger than sidewards scattering.
- Forward scattering becomes increasingly peaked.
- At x=100 backscattering at 140° starts to develop (rainbow), and at x=1000 also the secondary rainbow develops.
- At x>2000 usually geometric optics is valid but still some dependence on x.



Exticntion, absorption, scattering coefficients

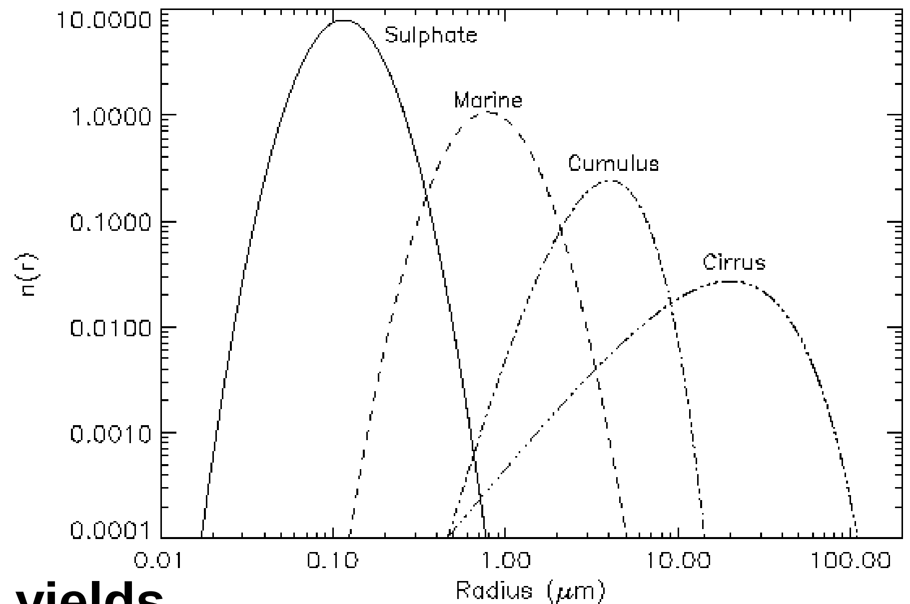
Extinctions, absorptions and scatterings by ensemble of particles are obtained by simply adding (integrating over the size distribution)

Extinctions coefficient:

L^{-4} L

L^{-1} $$k_{e,a,s} = \int_0^\infty n(r)\pi r^2 Q_{e,a,s}(r,\lambda)dr$$

L^2



n(r)= dN/dr : particle size distribution

πr^2: geometric area of particle

Q: efficiency factor

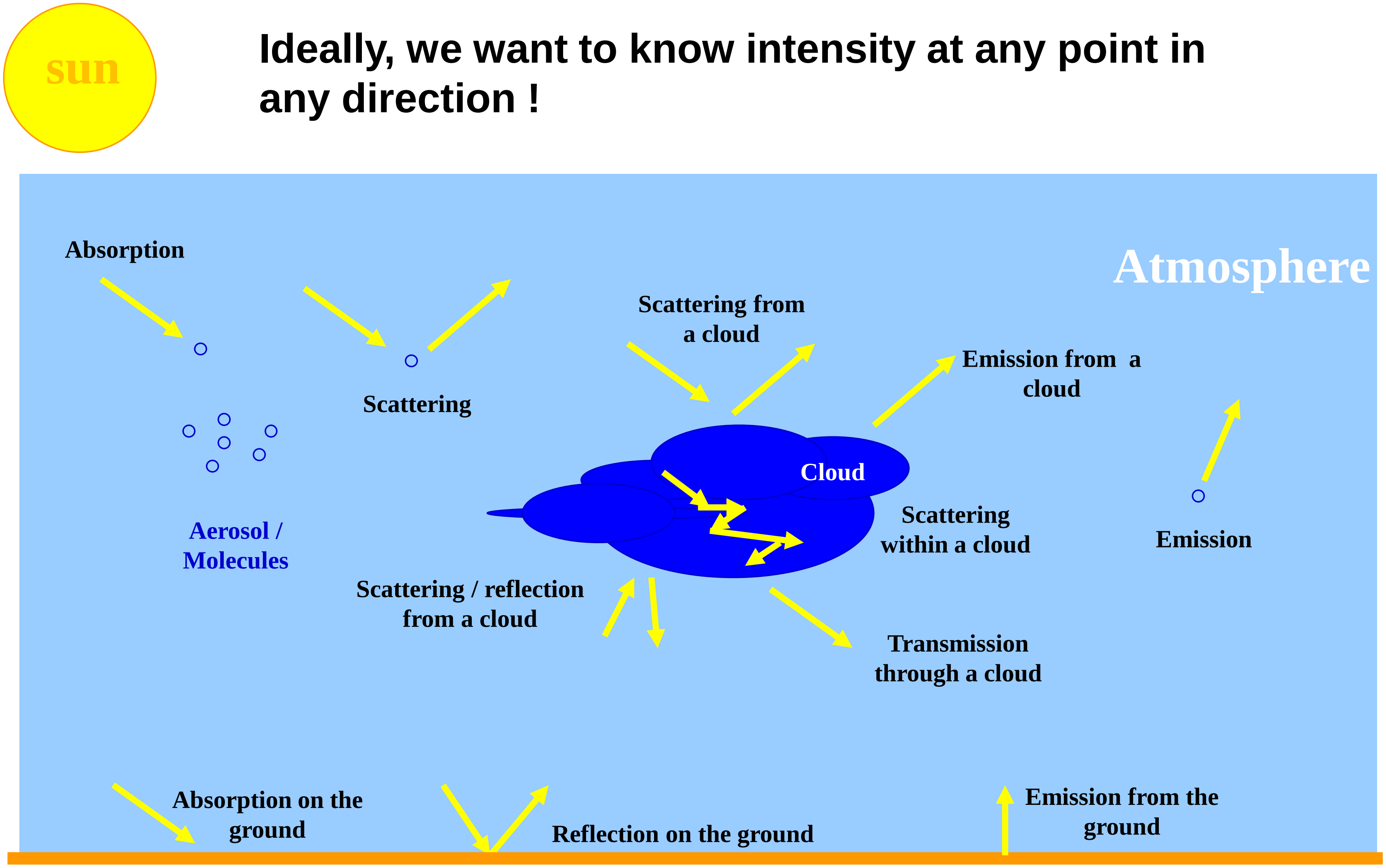
Integration over particle distribution yields smooth behavior for extinctions coefficients

$$\varpi = \frac{k_s}{k_e}\ , \ P(\cos\Theta) = \frac{1}{k_s}\int_0^\infty n(r)Q_s(r)\pi r^2 P(\cos\Theta, r)dr$$

$$g = \frac{1}{k_s}\int_0^\infty n(r)Q_s(r)\pi r^2 g(r)dr$$

Trying to understand the complete Picture

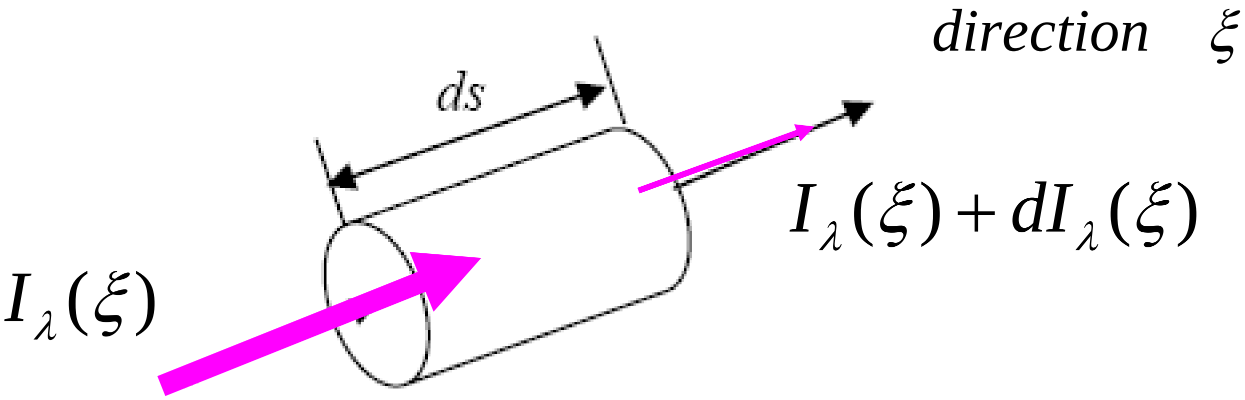
Ideally, we want to know intensity at any point in any direction !



So what is the problem ?

AND IN ANALYTIC FORM

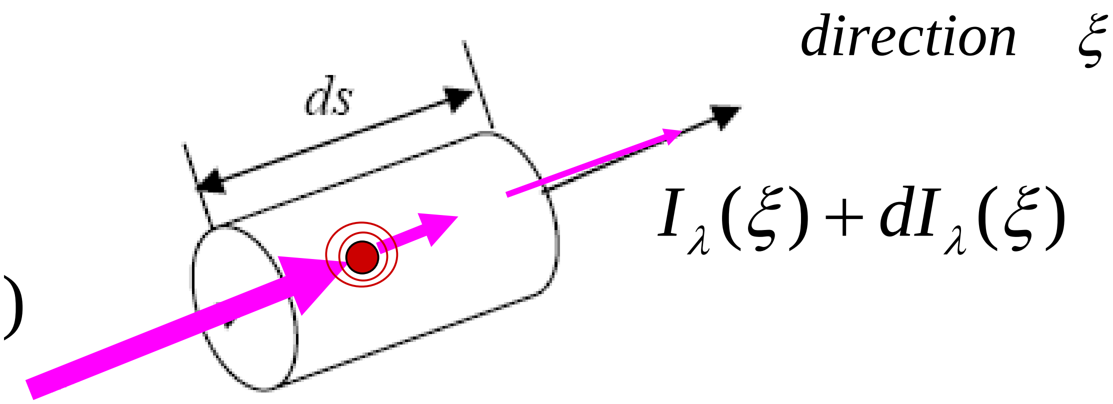
$$\frac{dI_\lambda(\xi)}{ds} =$$



AND IN ANALYTIC FORM

$$\frac{dI_\lambda(\xi)}{ds} = - \qquad - k_a(\lambda)\, I_\lambda(\xi)$$

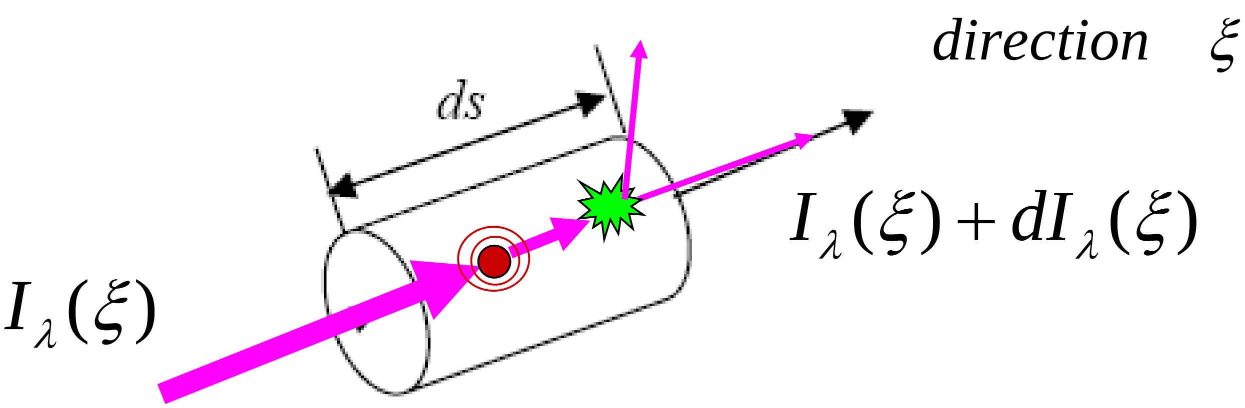
loss by absorption



AND IN ANALYTIC FORM

$$\frac{dI_\lambda(\xi)}{ds} = -k_s(\lambda)\,I_\lambda(\xi) - k_a(\lambda)\,I_\lambda(\xi)$$

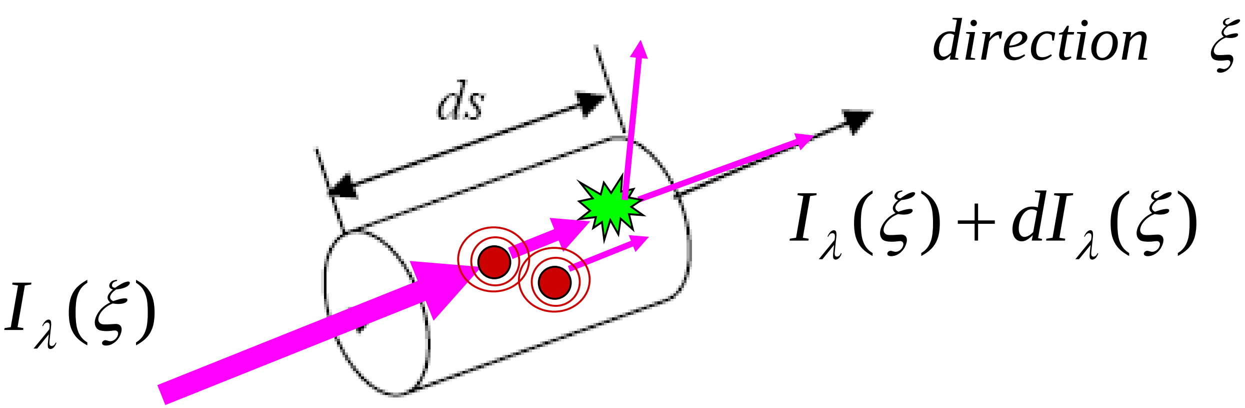
loss by scattering **loss by absorption**



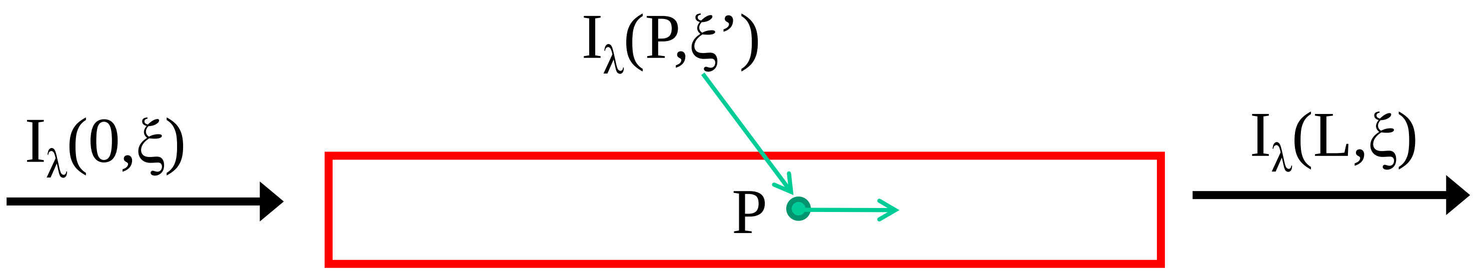
AND IN ANALYTIC FORM

$$\frac{dI_\lambda(\xi)}{ds} = -k_s(\lambda)\, I_\lambda(\xi) - k_a(\lambda)\, I_\lambda(\xi) + k_a(\lambda)\, B_\lambda(T)$$

loss by scattering **loss by absorption** **gain by emission**



ONE MORE SCATTERING EFFECT



- **Light can also be scattered from outside into the viewing direction**
- **This term is difficult to calculate:**

$$\frac{dI_{\lambda,sca}(\xi)}{ds} = \frac{k_s}{4\pi}\int_{\Omega} P(\Theta) I_{\lambda}(\xi') d\Omega$$

Scattering coefficient

Integral over all directions

Scattering Phase function: Probability for scattering into the viewing direction

Intensity from all directions

- **Leads to second signal (additional source) similar to emission term**

AND IN ANALYTIC FORM

$$\frac{dI_\lambda(\xi)}{ds} = -k_s(\lambda)\,I_\lambda(\xi) - k_a(\lambda)\,I_\lambda(\xi) + k_a(\lambda)\,B_\lambda(T) + \boxed{\frac{k_s}{4\pi}\int_\Omega P(\Theta)I_\lambda(\xi')\,d\Omega}$$

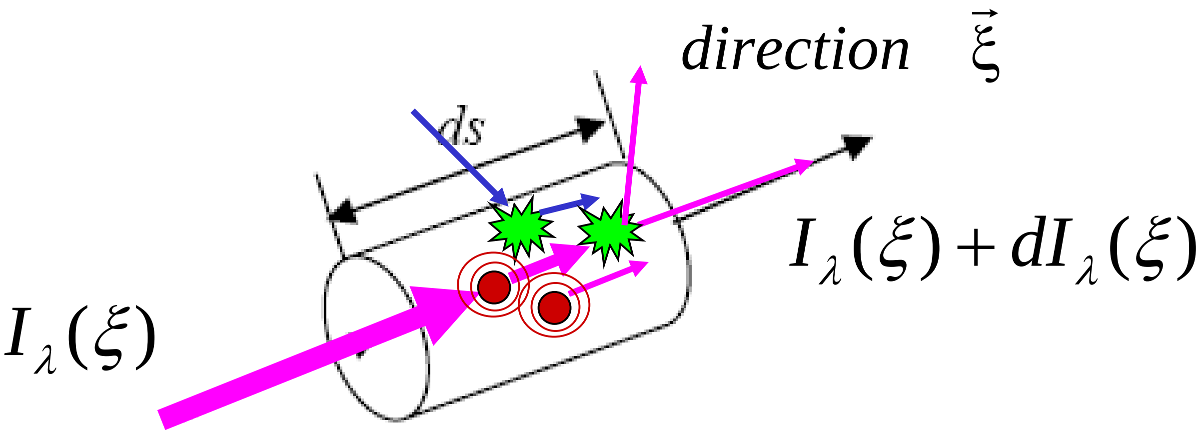
loss by scattering

loss by absorption

gain by emission

$S_{MS}(\lambda)$

gain by multiple scattering



- Most general description of change in intensity after interaction with absorbing, scattering and emitting medium
- Equation is merely a statement of energy conservation
- In addition: boundary conditions for top (incoming flux) and bottom (reflection/emission) of atmosphere

AND IN ANALYTIC FORM

$$\frac{dI_\lambda(\xi)}{ds} = -k_s(\lambda)\,I_\lambda(\xi) - k_a(\lambda)\,I_\lambda(\xi) + k_a(\lambda)\,B_\lambda(T) + \boxed{S_{MS}(\lambda)}$$

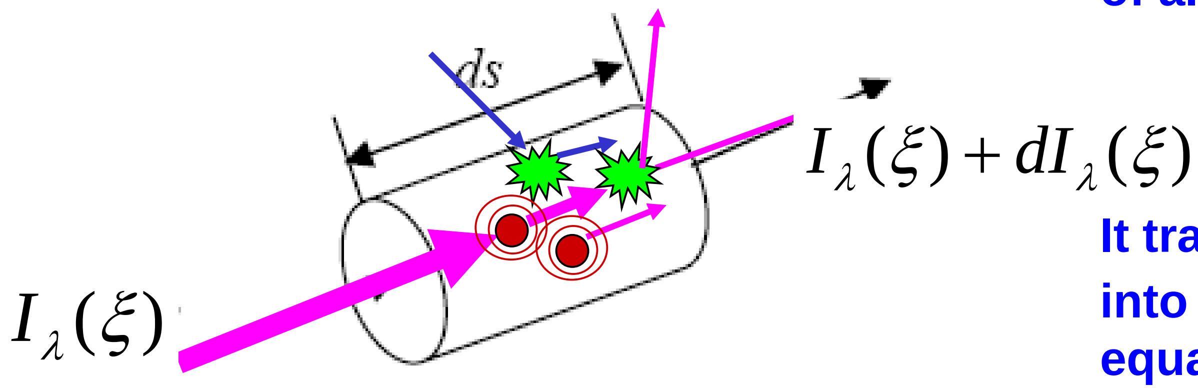
loss by scattering

loss by absorption

gain by emission

gain by multiple scattering

This is the source of all the difficulties



It transforms a differential into an integro-differential equation.

The radiation field at a certain point in a given direction may depend on the radiation field in any other point/direction

Note:

For purely absorbing case this lead to Lambert Beer law

For thermal IR (no scattering) to the Schwartzschild equation

What do you need to know?

- **Scattering, absorption cross sections**
- **Relation between scattering/absorption coefficients and corresponding cross sections**
- **Phase function**
- **Characteristics of Rayleigh and Mie Scattering**
- **Radiative transfer equation**