Black-Litterman implementation

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[VERY DRAFT]

While mean-variance optimization (1952) is still a big reference in the academia, it didn't find much traction among practitioners. Unrealistic and extreme solutions (in terms of portfolio weights) and issues trying to join that framework with the reality of the investment desks, analysts' contributions and macro strategists meant it is really not so widely used.

While some of its results still need to be considered, Black-Litterman (1990) suggested a different approach to portfolio optimization which seems better suited for the real-life asset manager.

The starting points of their proposal are:

- The reference is the portfolio representing markets' equilibrium (as derived via the CAPM)[1].
- To this starting point specific views about the market can be added by the investor.
- The portfolio weights resulting from the following optimization reflect those views, returning portfolio weights which are a function of the starting point, the views and the confidence about them.

The dataset

For this paper a set of 5y weekly returns (1566 observations), covering the period from 06-Jun-14 to 31-May-19, for the MSCI ACWI regional indices are used.

The capitalization weights are printed alongside:

```
##              Region          Ticker Weight
## 1 AsiaPacificExJap NDUECAPF Index  3.93%
## 2               EM  NDUEEGF Index 11.99%
## 3       EuropeExUK NDDUE15X Index 14.25%
## 4            Japan   NDDUJN Index   7.1%
## 5    NorthAmerica   NDDUNA Index 58.17%
## 6               UK   NDDUUK Index  4.56%
```

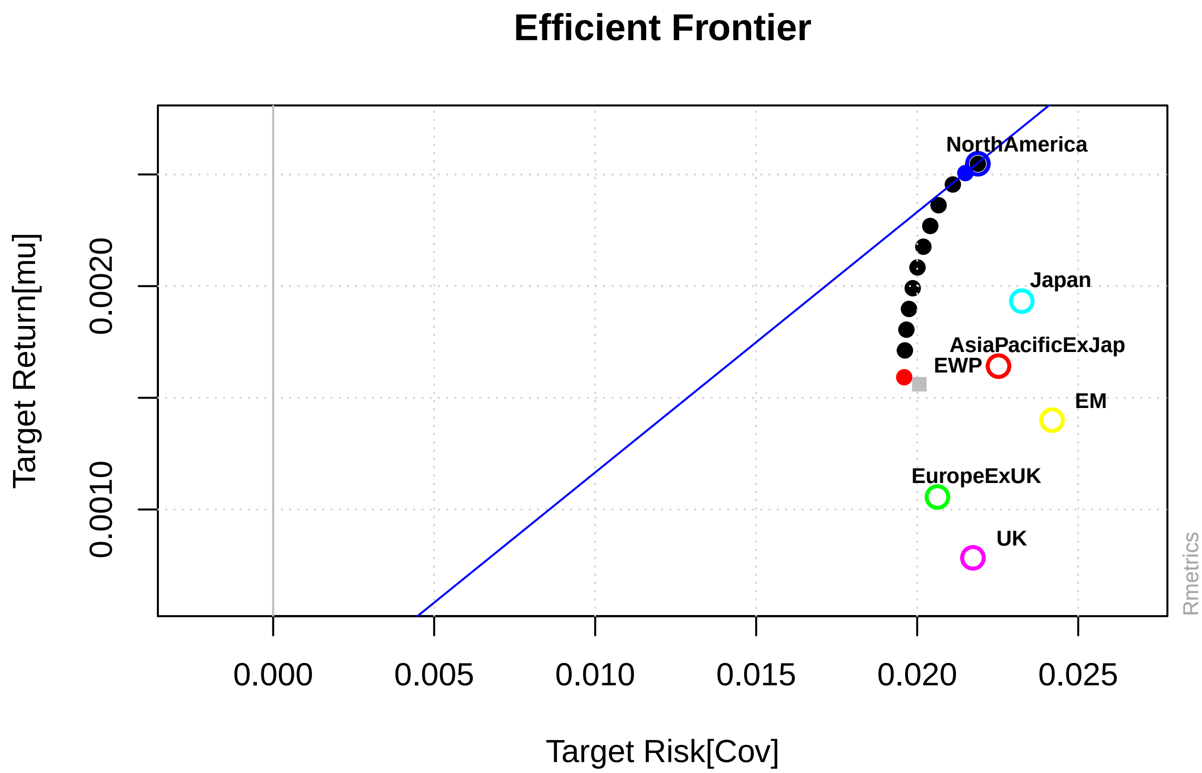
A short overview of the Markowitz framework

The essence is to find a set of portfolio weights balancing the expected return against the contribution to portfolio risk for each individual asset. When the return from an additional unit of an asset is lower than the contribution to risk, you just stop buying that asset. Unfortunately this approach (whenever the constaints are not too overwhelming) typically returns an unreasonably extreme portfolio.

The easy way to show this is by showing some key portfolio obtained using the mean-variance framework.

This is the starting point given the dataset using the standard representation of the risky portfolios in the context of the risk/reward space:

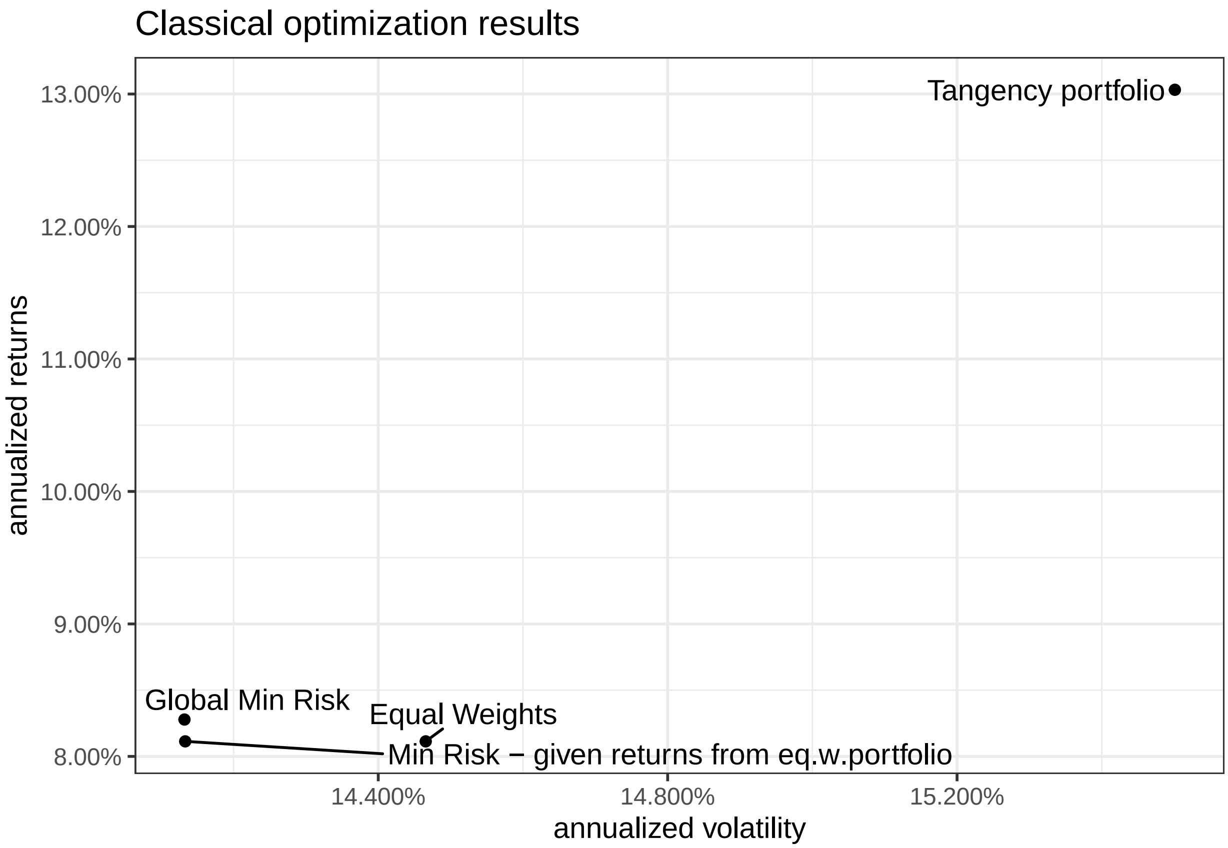
[1] The mean-variance optimization lack this natural starting point and this is one of the reason why the resulting weights from the optimization process, once the full set of returns have been supplied, typically fail to make sense.



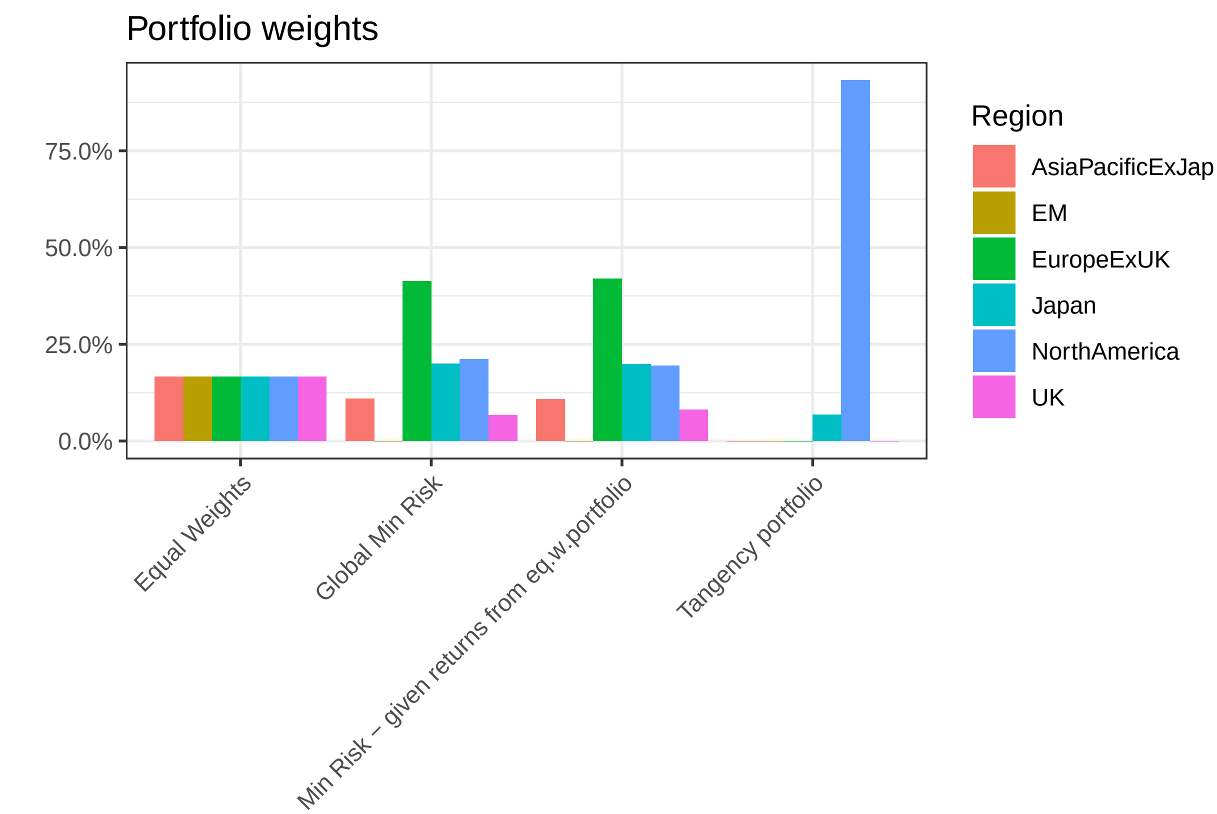
These are the weights for:

1. Equal weight portfolio (just an example of a feasible portfolio)
2. Minimum risk portfolio given returns equal to equal-weight portfolio
3. Global minimum risk portfolio
4. Tangency portfolio (maximum Sharpe) unconstrained

with their expected volatility and returns.



Classical optimization results
annualized returns
13.00%
12.00%
11.00%
10.00%
9.00%
8.00%
Tangency portfolio
Global Min Risk
Equal Weights
Min Risk − given returns from eq.w.portfolio
14.400%
14.800%
15.200%
annualized volatility



Without adding constraints the optimal portfolio from the Markowitz optimization is typically rather extreeme.

The Black-Litterman model

Equilibrum returns

The starting point for the BL model is the equilibrium weights represented by the capitalization weight of the relevant market. In case of a benchmark based portfolio, this translates into a neutral portfolio where the weights are the same weights as the benchmark. What is the reason to deviate from the benchmark? It is when the investor views are different from those represented by the equilibrium (ie, by the benchmark weights).

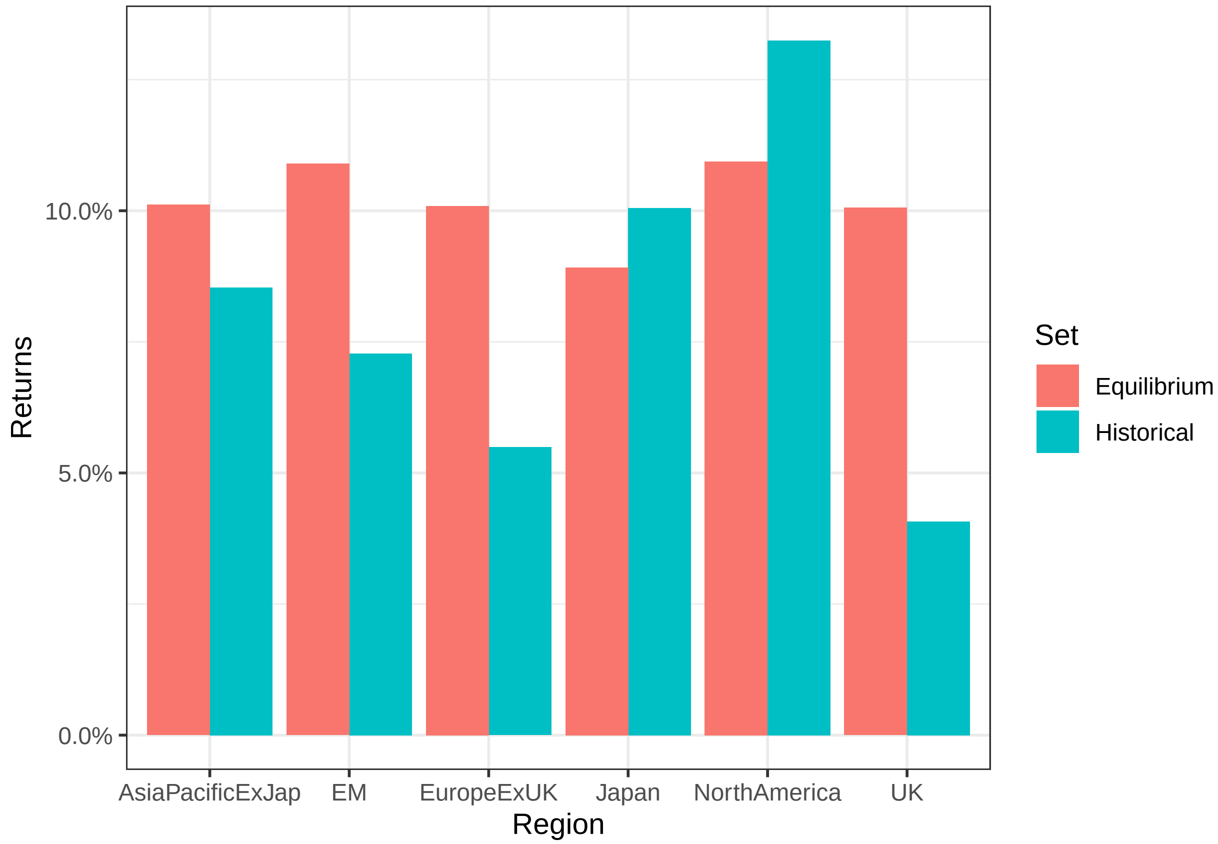
The immediate next step is to derive the exptected returns from the equilibrium weights given the covariance matrix[2]. Knowing the weights, the covariance matrix the equilibrium returns are the result of the reverse optimization process represented by the following equation:

$$\mu^* = \lambda \Sigma w_{mkt}$$

where λ is a parameter representing risk-aversion (higher λ means more willingness to take risk, lower λ a higher risk-aversion), 'Σ is the covariance matrix[3] and w_{mkt} are the market weights.

Using our dataset and implementing an estimate of the covariance matrix mildly sophisticated[4] the expected returns of the different regions is represented in chart xxxxx.

[2] Estimating the covariance matrix is one of the crucial aspects of the model.
[3] Hopefully something better than the sample covariance matrix!
[4] *Minimum volume ellipsoid* form the mass package.



Investors views

The second essential component of the BL model is the addition of the investor view(s). They can be represented in absolute terms (eg, EM will return 12% over the following period), in relative terms (eg, EM will return 2% over NorthAmerica) and as a combo of different components (eg, Japan and Asia Pacific ex Japan will return -2% vs. Europe Ex-UK).

In extreme synthesis, the BL model portfolio will be the weighted average of the market equilibrium portfolio and portfolios representing the views.

In case of constraints the resulting portfolio will be delivered by the optimization process on the ex-post expected returns (equilibrium + views).

What is crucial is the fact the investor needs to be specific about the results of her views; we can't just say "component 1 will outperform component 2"; we need to be clear about a) the economic result and b) how confident we are with that view.

A working example

User case - single view

Let's assume the strategists have a view where EM is expected to underperform Japan by 5% on an annual basis. Once this is translated in the same time unit of the dataset (WEEKLY). The view is identified by the following statement:

```
## 1 : -1*EM+1*Japan=0.0221880078490092  + eps. Confidence: 3509.098
```

Attached to the view elements and the expected outcome, the model works out (based on the estimate of the covariance matrix - based on the historical dataset) the confidence associated to that view defined as the reciprocal of the variance of the returns associated to the view itself.

$$confidence = \frac{1}{pickMatrix * covarMatrix * pickMatrix'}$$

where $pickMatrix$ is the matrix where the elements of the view are picked from the asset list.

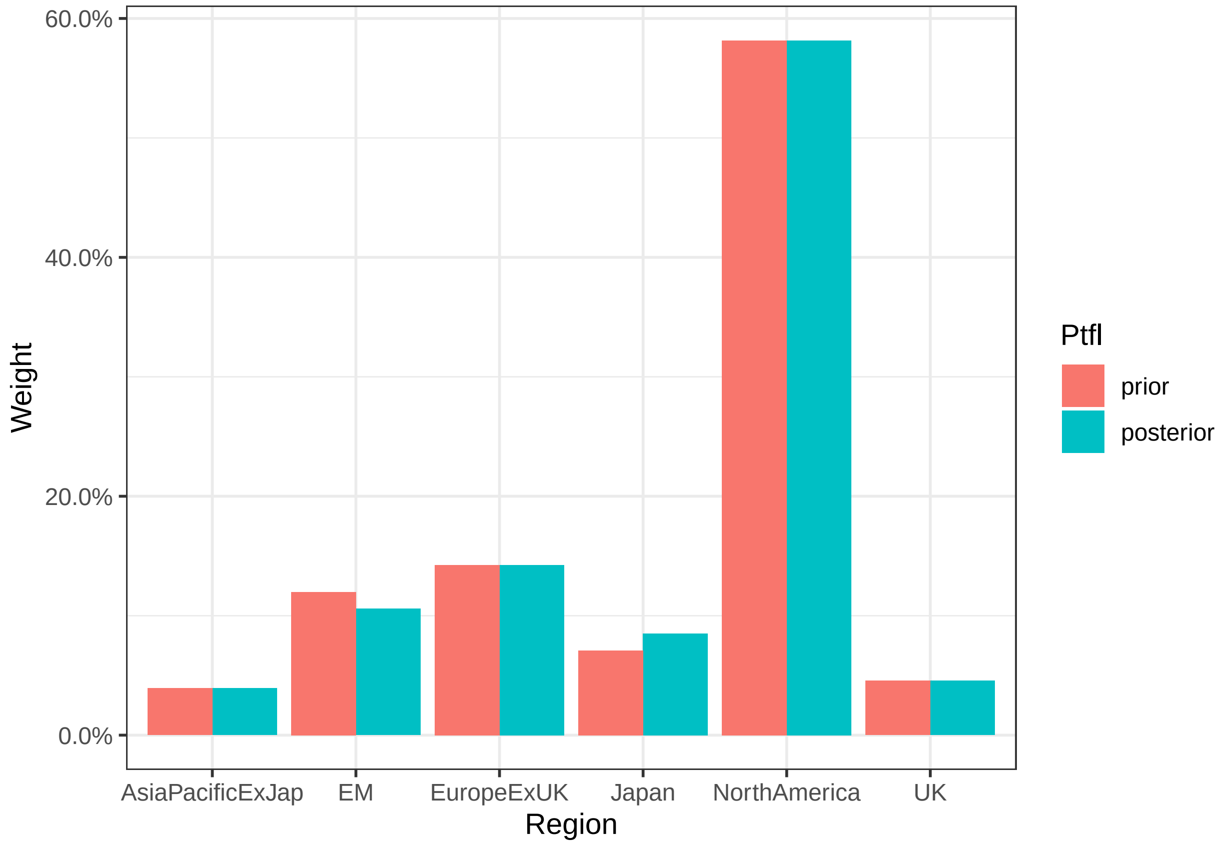
Based on the view the model delivers the posterior expected returns

```
## Prior means:
## AsiaPacificExJap               EM       EuropeExUK            Japan
##      0.001945138      0.002095249      0.001940579      0.001715163
##     NorthAmerica               UK
##      0.002103611      0.001933956
## Posterior means:
## AsiaPacificExJap               EM       EuropeExUK            Japan
##      0.001931069      0.002076568      0.001936110      0.001724398
##     NorthAmerica               UK
##      0.002099598      0.001928019
## Posterior covariance:
##                  AsiaPacificExJap           EM   EuropeExUK        Japan
## AsiaPacificExJap     0.0003729660 0.0003921405 0.0002586786 0.0002567699
## EM                   0.0003921405 0.0004439684 0.0002810103 0.0002642311
## EuropeExUK           0.0002586786 0.0002810103 0.0003283525 0.0002380103
## Japan                0.0002567699 0.0002642311 0.0002380103 0.0003530907
## NorthAmerica         0.0002341384 0.0002492256 0.0002464441 0.0002106082
## UK                   0.0002616559 0.0002832628 0.0002932330 0.0002261338
##                  NorthAmerica           UK
## AsiaPacificExJap 0.0002341384 0.0002616559
## EM               0.0002492256 0.0002832628
## EuropeExUK       0.0002464441 0.0002932330
## Japan            0.0002106082 0.0002261338
## NorthAmerica     0.0003196341 0.0002499046
## UK               0.0002499046 0.0003490816
```

Given the new market expectations we can then apply the optimizer and get the new weights:

```
##        Ptfl      Mean       Vol AsiaPacificExJap        EM EuropeExUK
## 1     prior 0.1059675 0.1199007       0.03926789 0.1199397  0.1425226
## 2 posterior 0.1054287 0.1198243       0.03926788 0.1057887  0.1425226
##        Japan NorthAmerica         UK
## 1 0.07099147    0.5816815 0.04559672
## 2 0.08514251    0.5816816 0.04559672
```

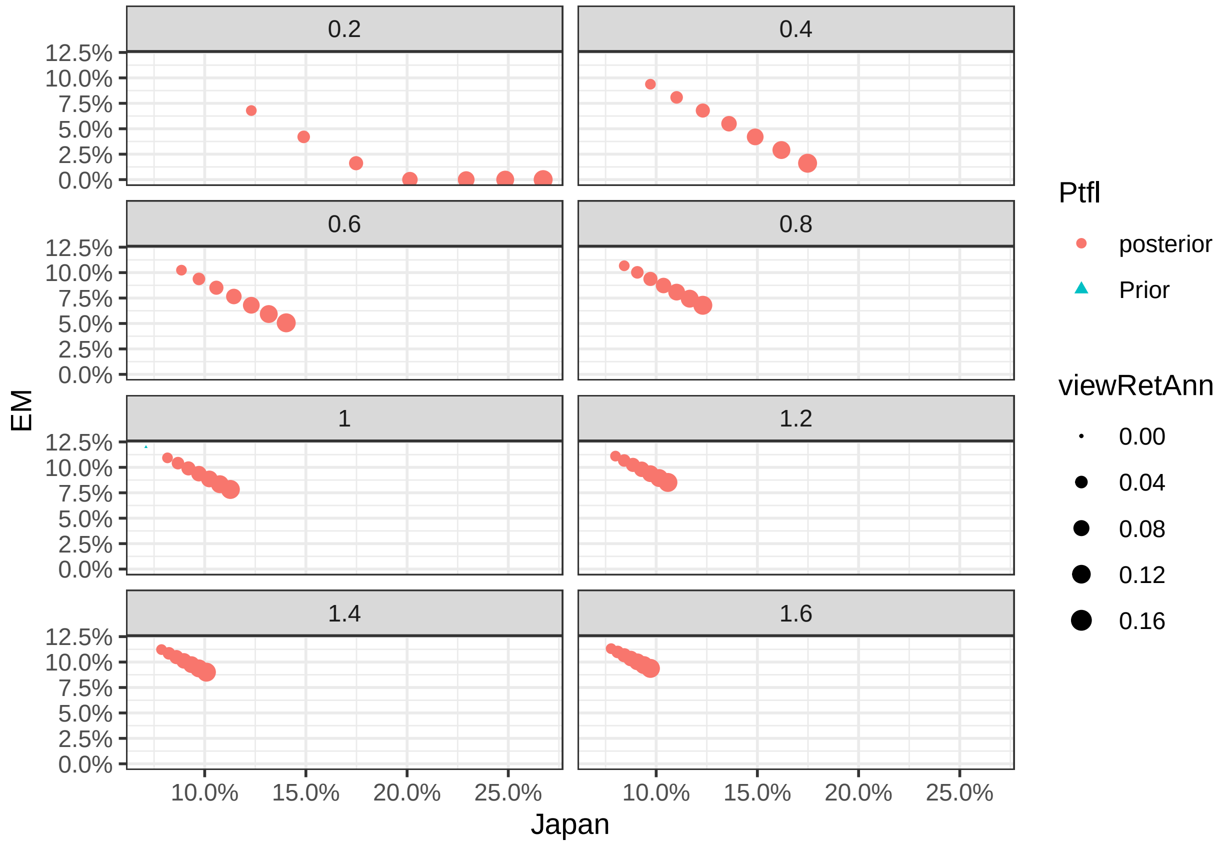
In graphical terms:



Results shifting the confidence and or the expected returns

One of the key aspect of the model is the fact it can be explained as a linear combination of different portfolios. The linearity of the underlying model can be easily illustrated by the result of the trade-off between risk-aversion (the λ) and the expected returns of the view.

The following chart provides for different values of risk-aversion the change in weights between Japan and EM due to different expected returns of the view (as the size of the point)



A standalone app

[screenshots of the Shiny package:

- selection of view elements
- confidence multiplier slide
- resulting posterior returns
- resulting weights (change vs. equilibrium)

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