

Count outcomes

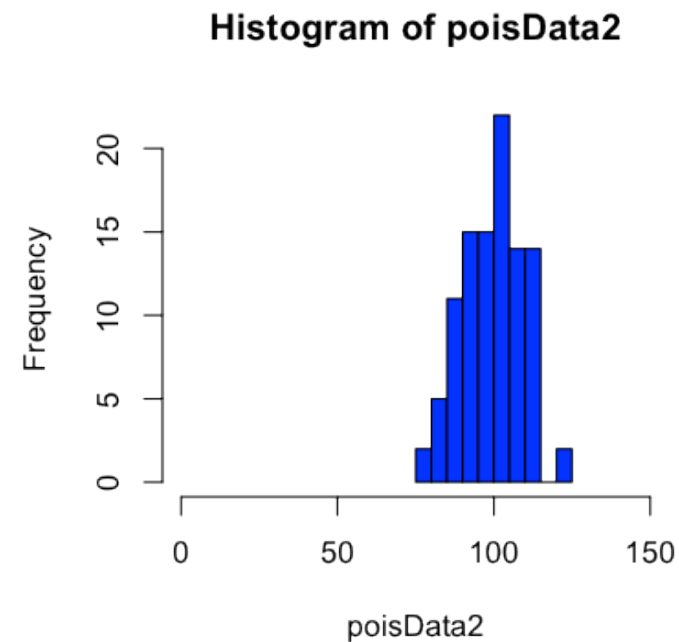
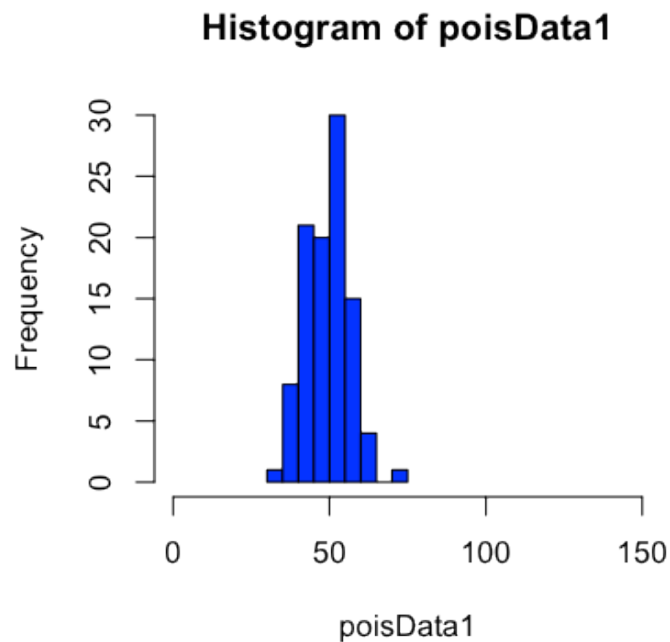
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Key ideas

- Many data take the form of counts
 - Calls to a call center
 - Number of flu cases in an area
 - Number of cars that cross a bridge
- Data may also be in the form of rates
 - Percent of children passing a test
 - Percent of hits to a website from a country
- Linear regression with transformation is an option

Poisson distribution

```
set.seed(3433); par(mfrow=c(1,2))  
poisData2 <- rpois(100,lambda=100); poisData1 <- rpois(100,lambda=50)  
hist(poisData1,col="blue",xlim=c(0,150)); hist(poisData2,col="blue",xlim=c(0,150))
```



Poisson distribution

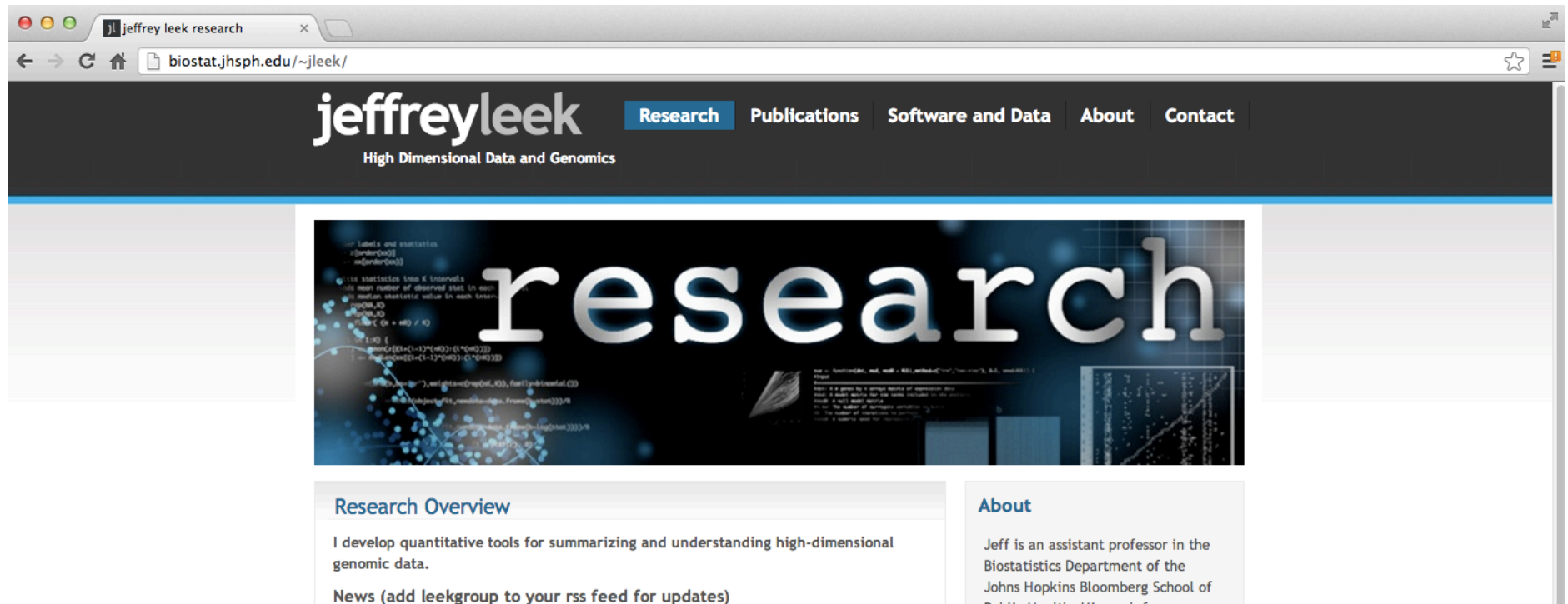
```
c(mean(poisData1),var(poisData1))
```

```
[1] 49.85 49.38
```

```
c(mean(poisData2),var(poisData2))
```

```
[1] 100.12 95.26
```

Example: Leek Group Website Traffic



<http://biostat.jhsph.edu/~jleek/>

Website data

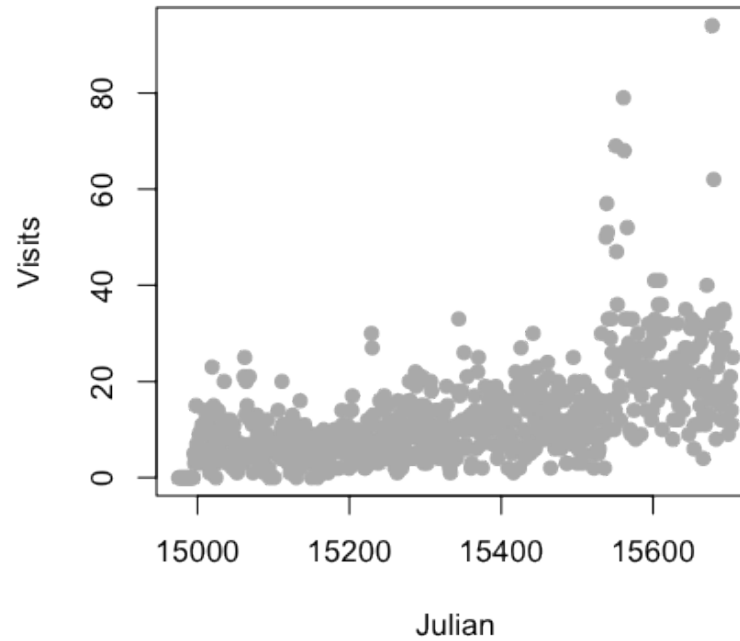
```
download.file("https://dl.dropbox.com/u/7710864/data/gaData.rda",destfile="./data/gaData.rda",metho  
load("./data/gaData.rda")  
gaData$julian <- julian(gaData$date)  
head(gaData)
```

	date	visits	simplystats	julian
1	2011-01-01	0	0	14975
2	2011-01-02	0	0	14976
3	2011-01-03	0	0	14977
4	2011-01-04	0	0	14978
5	2011-01-05	0	0	14979
6	2011-01-06	0	0	14980

<http://skardhamar.github.com/rga/>

Plot data

```
plot(gaData$julian,gaData$visits,pch=19,col="darkgrey",xlab="Julian",ylab="Visits")
```



Linear regression

$$NH_i = b_0 + b_1 JD_i + e_i$$

NH_i - number of hits to the website

JD_i - day of the year (Julian day)

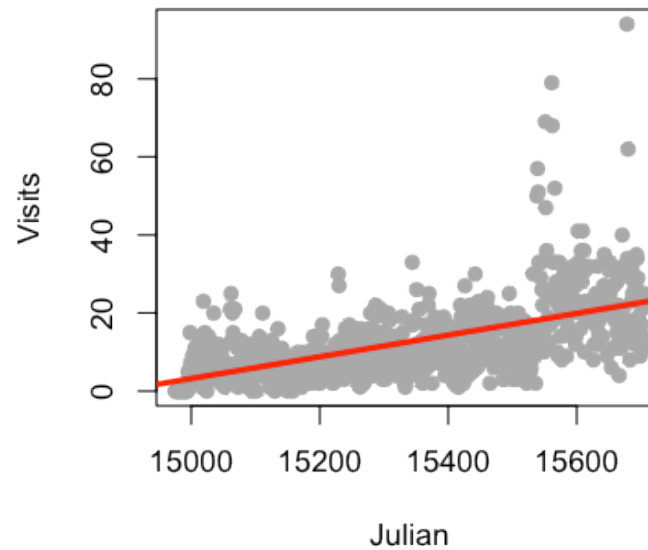
b_0 - number of hits on Julian day 0 (1970-01-01)

b_1 - increase in number of hits per unit day

e_i - variation due to everything we didn't measure

Linear regression line

```
plot(gaData$julian,gaData$visits,pch=19,col="darkgrey",xlab="Julian",ylab="Visits")  
lm1 <- lm(gaData$visits ~ gaData$julian)  
abline(lm1,col="red",lwd=3)
```



Linear vs. Poisson regression

Linear

$$NH_i = b_0 + b_1 JD_i + e_i$$

or

$$E[NH_i | JD_i, b_0, b_1] = b_0 + b_1 JD_i$$

Poisson/log-linear

$$\log(E[NH_i | JD_i, b_0, b_1]) = b_0 + b_1 JD_i$$

or

$$E[NH_i | JD_i, b_0, b_1] = \exp(b_0 + b_1 JD_i)$$

Multiplicative differences

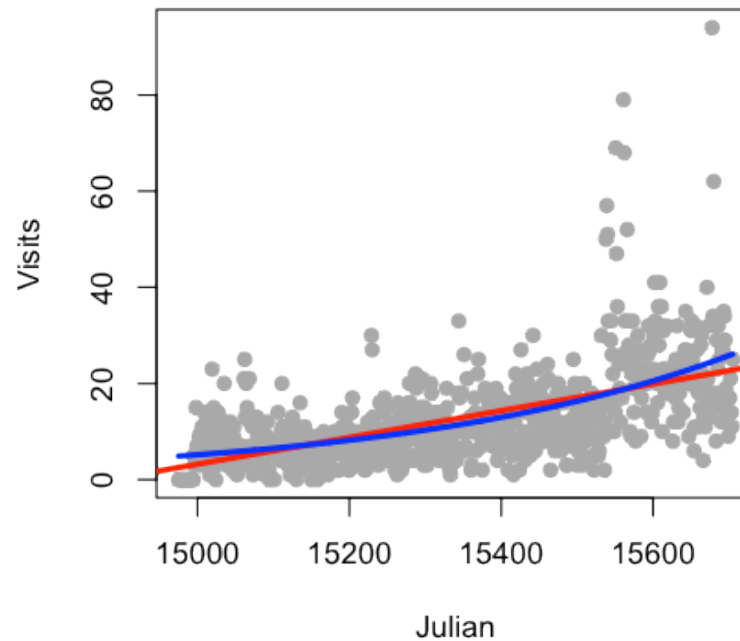
$$E[NH_i | JD_i, b_0, b_1] = \exp(b_0 + b_1 JD_i)$$

$$E[NH_i | JD_i, b_0, b_1] = \exp(b_0) \exp(b_1 JD_i)$$

If JD_i is increased by one unit, $E[NH_i | JD_i, b_0, b_1]$ is multiplied by $\exp(b_1)$

Poisson regression in R

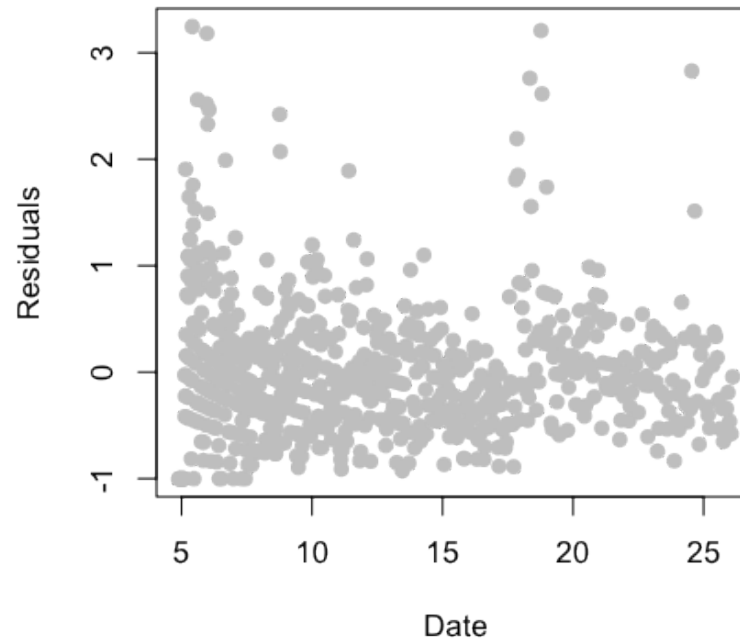
```
plot(gaData$julian,gaData$visits,pch=19,col="darkgrey",xlab="Julian",ylab="Visits")  
glm1 <- glm(gaData$visits ~ gaData$julian,family="poisson")  
abline(lm1,col="red",lwd=3); lines(gaData$julian,glm1$fitted,col="blue",lwd=3)
```



12/19

Mean-variance relationship?

```
plot(glm1$fitted,glm1$residuals,pch=19,col="grey",ylab="Residuals",xlab="Date")
```



Model agnostic standard errors

```
library(sandwich)
confint.agnostic <- function (object, parm, level = 0.95, ...)
{
  cf <- coef(object); pnames <- names(cf)
  if (missing(parm))
    parm <- pnames
  else if (is.numeric(parm))
    parm <- pnames[parm]
  a <- (1 - level)/2; a <- c(a, 1 - a)
  pct <- stats::format.perc(a, 3)
  fac <- qnorm(a)
  ci <- array(NA, dim = c(length(parm), 2L), dimnames = list(parm,
                                                                pct))

  ses <- sqrt(diag(sandwich::vcovHC(object)))[parm]
  ci[] <- cf[parm] + ses %o% fac
  ci
}
```

<http://stackoverflow.com/questions/3817182/vcovhc-and-confidence-interval>

Estimating confidence intervals

```
confint(glm1)
```

```
              2.5 %      97.5 %  
(Intercept) -34.34658 -31.159716  
gaData$julian  0.00219  0.002396
```

```
confint.agnostic(glm1)
```

Rates

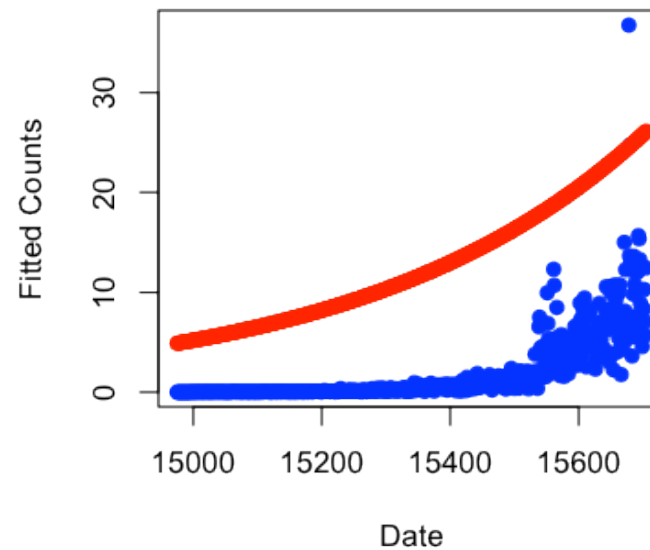
$$E[NHSS_i | JD_i, b_0, b_1] / NH_i = \exp(b_0 + b_1 JD_i)$$

$$\log(E[NHSS_i | JD_i, b_0, b_1]) - \log(NH_i) = b_0 + b_1 JD_i$$

$$\log(E[NHSS_i | JD_i, b_0, b_1]) = \log(NH_i) + b_0 + b_1 JD_i$$

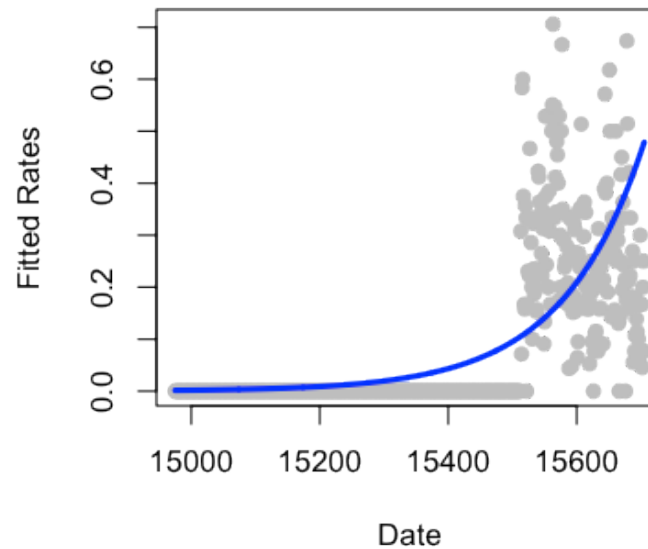
Fitting rates in R

```
glm2 <- glm(gaData$simplystats ~ julian(gaData$date), offset=log(visits+1),  
            family="poisson", data=gaData)  
plot(julian(gaData$date), glm2$fitted, col="blue", pch=19, xlab="Date", ylab="Fitted Counts")  
points(julian(gaData$date), glm1$fitted, col="red", pch=19)
```



Fitting rates in R

```
glm2 <- glm(gaData$simplystats ~ julian(gaData$date), offset=log(visits+1),  
            family="poisson", data=gaData)  
plot(julian(gaData$date), gaData$simplystats/(gaData$visits+1), col="grey", xlab="Date",  
      ylab="Fitted Rates", pch=19)  
lines(julian(gaData$date), glm2$fitted/(gaData$visits+1), col="blue", lwd=3)
```



More information

- [Log-linear models and multiway tables](#)
- [Wikipedia on Poisson regression](#), [Wikipedia on overdispersion](#)
- [Regression models for count data in R](#)
- [pscl package](#) - the function *zeroinfl* fits zero inflated models.