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MARKET LIQUIDITY RISK: A SCENARIO BASED APPROACH

This paper explains the StatPro approach for measuring Liquidity Risk. The traditional problem of Liquidity Risk is that the data needed for calibrating these models is only available for liquid instruments, trading on a regular basis and for which books of bid/ask and volumes are available. For this reason the current approaches to measuring Liquidity Risk fail providing any indication for the most opaque and illiquid instruments, or where the measurement of Liquidity Risk is mostly needed.
StatPro has introduced a new approach based on liquidity scenarios, which is universal, because it covers potentially any financial asset, from equities, to bonds, to OTC derivatives under a homogeneous and consistent approach.
The Liquidity Risk measure is divided into six different components. The most important component re-builds, with a quantitative approach based on observed market data, the fair value bid and asks of all the financial instruments that can be priced via an arbitrage-free pricing function, providing a solid and consistent benchmark of Liquidity Risk.

1. Market Liquidity Risk

What is Liquidity Risk? We can provide at least two definitions for it.

Funding Liquidity Risk. This definition refers to the Asset Liability Management (ALM) of an institution – normally a bank – identifying the gaps in the funding of the institution's assets. E.g. in a bank there is usually a funding gap as the liabilities contain short-term deposits in large part against assets that invest in longer term horizons. Funding gaps generate a funding risk, the risk of rolling the short term funding at growing costs or even the risk of not being able to roll/over the shorter term liabilities.

Market Liquidity Risk. This is the risk of losing a certain amount of money when liquidating one or more positions in a portfolio. In financial terms, the loss is generated by the difference between the price at which the financial asset is marked and the price at which it can be sold.

This paper focuses on Market Liquidity Risk.

When all financial assets that lie in our portfolios have quotes on market bid and ask prices with their respective volumes (including book volumes), computing Liquidity Risk is straightforward.
Having the book of bids and asks at disposal, we can measure with objectivity and precision how much we would be losing by liquidating our positions, in one or more units of time.
The current approaches for measuring Liquidity Risk are centered around bid/ask and volumes.
The most popular measure is still the number of days needed for liquidating a position, obtained by dividing the position size by the daily trading volume.

However, there are a number of problems related to this approach:

a. *The Liquidity Risk Paradox*. Information on bid/asks, book and volumes is only available for liquid instruments, i.e. financial assets that trade on a daily basis in fairly transparent markets or trading venues. This information is not available for the most opaque and illiquid instruments. In other words, the financial information required to calibrate the “traditional” models of measurement of liquidity risk is not available for the instruments where a measure of this risk is mostly needed.

b. *Proliferation of Trading Venues*. The recent past has seen a proliferation of trading venues for many financial instruments including equities. Trades on stocks were traditionally concentrated in one main market; today their trading is split around multiple venues, spreading the information of volumes, books, bid and asks. This scattering of information opens new questions on the traditional approaches. Do we have the information on consolidated volumes across the different trading places? Have we access to all the venues where one stock is traded once we decide to liquidate the position? Should we take into account the consolidated volumes or only the volumes and bid/asks of the trading venues we have access to?

c. *OTC Volumes vs Trading Venues Volumes*. The last 10 years have been also accompanied by an increase of electronic trading venues for fixed income products. Bonds are now often traded electronically and information on traded volumes is available in open platforms. The issue is that the OTC market retains most of the liquidity in the fixed income space and the information on bid and asks on a trading venue may even be misleading: e.g. assume we have a bond issue that trades on average 100,000 $ in a regulated market but that in the OTC market you can find easily quotes for multiples of these volumes; if your position on the bond counts in millions the information conveyed by the regulated market becomes irrelevant.

These issues constitute a structural impediment to a coherent and universal approach to the measurement of Liquidity Risk. The traditional approaches will constantly fail to deliver any information on illiquid bonds, OTC derivatives, certificates, leaving us uncovered where the problem of liquidity mostly hits.
Is there an alternative? Can we design a model framework that is at the same time:

- <u>universal</u>, covering all financial instruments;
- <u>coherent</u>, applying a consistent methodology to all instruments;
- <u>objective</u>, using a quantitative method based on observable market data to build in full or in part the Liquidity Risk measure.

These questions have inspired our research on the topic and have, in a way, driven it. The remainder of the paper explains our approach to measuring Liquidity Risk and how it complies with the requirements described above.

2. A Scenario Based Approach

The constraints described above have compelled us to follow a different route for modelling Liquidity Risk. We have created a scenario-based approach building in our system a number of pre-defined scenarios for measuring Liquidity Risk under *Normal*, *Stressed*, *Highly Stressed* market situations.
We will explain how these scenarios are created and how the risk is modelled in paragraph 4.

A scenario, given a financial asset, returns the percentage liquidity loss for the instrument in the two cases of being long and short of the instrument. The loss is determined by the distance between the mid price at which the asset is valued and the scenario bid (long case) or the scenario ask (short case).
E.g. for a certain bond and for the *Normal* scenario, we assume a loss of 0.50% when long and a loss of 0.70% when short.

3. Explanation of Liquidity Measures

Before explaining the different components of the Liquidity Risk measure, we explain in more detail what measures are computed by our system at asset level and how:

- they are compounded for determining the Liquidity Risk at portfolio level;
- they can be used to determine the Bid and Ask of an instrument.

Our system computes initially two measures of Liquidity Risk for each asset and under every scenario *S* (where *S* can be the Normal, Stressed, Highly Stressed and any other User-Defined scenario):

H^L = loss (haircut) for a unit owned in a "long" asset;

H^S = loss (haircut) for a unit owned in a "short" asset;

These measures are computed on a daily basis.

Portfolio Liquidity Loss
Given a portfolio of n assets, let *i*, where i = 1, 2, …, n, be the index. The portfolio liquidity loss (PLL) can be defined as:

$$PLL = \Sigma (E_i * H_i) \qquad (1)$$

Where,

E_i = the i-th Asset Exposure as defined in Appendix 1.
H_i = the total liquidity loss at asset level. It is $H^L{}_i$ when the i-th asset is long and $H^S{}_i$ when short. In both cases, the multiplication of the Exposure by the haircut will determine a loss. Liquidity Risk always plays against the holder of the position.

Computing Bid and Asks
The same liquidity haircuts can be used to compute the bid and ask of each asset under every scenario S. The formulas will be different depending on the StatPro Product Type to which each asset belongs.

For products with absolute variation type and namely no.21, Generic OTC:

$$Bid = UNPV – H^L \qquad (2)$$
$$Ask = UNPV + H^S \qquad (3).$$

Where,

UNPV = Unitary NPV of the OTC derivative.

For all the other products:

$$Bid = P * (1 - H^L) \qquad (4)$$
$$Ask = P * (1 + H^S) \qquad (5).$$

Where,

P = Current reference (mid) price of the financial instrument.

4. The Components of Liquidity Risk

In this paragraph we illustrate how the liquidity haircut H is generated. The value H is made of six components that we list here below:

- Specific (H_1)
- Fair Value Bid/Ask (H_2)
- Pricer (H_3)

- Nominal (H_4)
- Market Cap (H_5)
- % Owned (H_6).

In the following paragraphs we explain how each of the components is computed. We will start with the component no.2, the Fair Value Bid and Ask and leave the component 1 to the end.

4.1. H_2 – Fair Value Bid and Ask

This haircut component is among the 6 probably the most important one and the one with the highest quantitative content. We dedicate the next three paragraphs to it.

4.1.1. Making Markets in Illiquid Instruments

As we explained in par. 1, the widespread approach to modeling Liquidity Risk fail to give an answer for all those instruments that are not traded in a regulated market and for which there is no information on bid/asks and volumes. The two major asset classes that suffer from these limitations are bonds and OTC derivatives. For many of these instruments, it is often not possible to find a transparent market price and it is practically impossible finding information on live bid/asks and on traded volumes.

Nevertheless these instruments are traded, often on a regular basis in the OTC markets. These markets remain firmly in the hands of the investment banks, who dominate the way OTC prices are made.
The real question then becomes how a Market Maker produces a price of such instruments. How does he compute a fair price and how does he define a bid and ask price when these prices are not observed in the market or not backed by sufficient volumes?

If you ignore equities for a moment, most financial instrument values are linked to the value of simpler instruments. A pricing function links the value of a bond or of a specific OTC contract (option, certificate, and so on) to a number of other financial instruments that are traded independently. In StatPro, we call these instruments *risk factors*, as their value determines the outcome of the pricing function: risk factors are the inputs of a pricing function.
For example, the price of a fixed-coupon bond can be replicated using a Libor curve built off the IRS curve, the CDS curve of the issuer and the basis between CDS and bond credit spread.
The evolution of financial technology and the derivative market let us today compute a Fair-Value price for any product that has an arbitrage-free pricing function.

When a market-maker is called to quote bids and asks for illiquid products, he will take the pricing function of the instrument and will use it to create two prices.
The bid price will be created by inserting the bid or asks of the underlying derivatives into the pricing function, depending on the exposure generated from the purchase of the instrument.
As an example, let's consider an illiquid convertible bond. If the market maker is hit on the bid, he will acquire a long position on the bond and will have to do a number of trades to hedge the risks generated by the position; he will:

- pay the ask of the relevant IRS to hedge interest rate risk;
- sell at the bid an option on the underlying equity to hedge volatility risk;
- trade the underlying equity to hedge the net delta exposure;
- buy protection at the ask in the CDS of the underlying issuer.

The market maker will insert the bids and asks of these derivative instruments into the pricing function, based on the hedge actions that he will need to perform.

For the same reason, when computing the ask price, he will invert the bid and asks inputted in the pricing function.

The selection of the bids and asks of the derivatives that populate the pricing function is a careful process that is ultimately driven by the exposure to the derivative generated by the purchase/sale of the instrument being quoted. Coming back to our previous example, if the market maker was asked to make a market for a reverse convertible instead of a convertible bond, the hedge on implied volatility would be entirely reversed, as a long position on the former would generate a short volatility position. Consequently, the bid of the reverse convertible will be created by inserting the ask of the implied volatility into the pricing function, instead of the implied volatility bid used for the convertible bond bid.

4.1.2. Replicating the Computation of Prices in Illiquid Instruments

The second component of our Liquidity Risk measure replicates the mechanism described above. We fill each scenario with the bids and asks of all the risk factors used in our system for pricing financial instruments, from IRS to CDS Tranche correlation, from Inflations Swaps to Implied Volatilities.
The *Normal* scenario contains the "current" bid and asks and is updated periodically. The *Stressed* and *Highly Stressed* scenarios contain the bid and asks observed during moments of financial chaos. These pieces of data are available and observable for the majority of the *risk factors* available in our solution.

Our Liquidity Risk Engine uses the bid and asks of the underlying derivatives and the pricing functions to reconstruct the Fair Value Bid and Ask of each financial instrument, understanding the exposure of each instrument to each risk factor and selecting accordingly the bid and asks of the underlying derivatives.
In other words, the engine replicates the algorithms used by market-makers in creating prices and links the "cash" markets to the derivative markets, providing a unified and *consistent* framework for both worlds.

The approach is *universal*, because it allows defining in one scenario the bid and asks of a limited and controllable number of derivative instruments for generating Liquidity Risk for a potentially unlimited number of instruments and asset classes.

Finally, the approach is also *objective*, as the bid and asks for most of the derivatives are observed in the market, the information is available and is updated on a regular basis.

4.1.3. Computation of Fair Value Bid/Ask Liquidity Haircut

This paragraph explains more formally how we compute the component H_2.

Let's define with P_0 the Fair Value Price of a financial instrument.

$$P_0 = f\,(RF_1, RF_2, \ldots, RF_n) \qquad (6).$$

Where,
RF_i is the i-th risk factor MID price, contributing for the given financial instrument.

We define with Δ^-_i and Δ^+_i the shocks applied to the i-th Risk Factor for obtaining its bid (RF^-_i) and its ask (RF^+_i). E.g. for IRS 3y EUR Δ^-_i = 1 bps (0.0001), Δ^+_i = 1 bps (0.0001).
These shocks populate our Liquidity Scenarios.

The Liquidity Risk Engine will use the following algorithm:

i. gathers the haircuts for each risk factor;
ii. creates a positive and a negative price scenario for that risk factor (RF^-_i, RF^+_i);
iii. computes the instrument price *P* for all shocks and stores them;
iv. composes all the negative price variations, neglecting terms of higher order, and stores the result for the long side of liquidity risk; composes all positive variations and stores the result for the short side of liquidity risk. (Below more details on this);
v. the scenario results will be stored and exported to the risk API.

During step *iii* the algorithm re-prices the instrument using the risk factor bid RF^-_i and the risk factor ask, RF^+_i, keeping the other risk factors at mid price. Normally, one of the prices will be higher than P_0 and we call this price P^+_i, the other will be lower than P_0 and we call it P^-_i.

Composition of scenarios from different risk factors
The composition of liquidity scenarios coming from different risk factors (step *iv* of the algorithm) will be done linearly neglecting terms of order higher than the first. For example, suppose a product depends on risk factor 1 and 2. Let P_0, be the reference price, P^+_i the computed price scenario that gives a price increase for risk factor i, and P^-_i the price scenario that results in a price decrease for risk factor i.
For a **relative/futures haircut type** we compute the haircut H^S_2 to be applied for the short side,

$$H^S_2 = [(P^+_1/P_0 - 1) + (P^+_2/P_0 - 1)] ,$$

and the haircut H^L to be applied for the long side,

$$H^L_2 = \mathrm{Min}\ ([1 - P^-_1/P_0 + 1 - P^-_2/P_0], 100\%) .$$

For an **absolute haircut type** (e.g. OTC derivative instruments) we compute the haircut H^S to be applied to the short side,

$$H^S_2 = (P^+_1 - P_0) + (P^+_2 - P_0) ,$$

and the haircut H^L to be applied for the long side,

$$H^L_2 = - (P^-_1 - P_0) - (P^-_2 - P_0) .$$

Notes on scenario composition
In the composition of liquidity scenario coming from risk factors depending from more than one market quote, such as interest rates, all the underlying market quotes should be moved together.

4.2. H_3 – Pricer Haircut

The Fair Value Bid/Ask component takes into consideration many risk dimensions, including maturity, currency, multiple exposures to many risk factors in more complex products and so on so forth.
However, Liquidity Risk can also be driven by the asset class, the type of instrument. Obvious examples are ABSs, asset-backed securities. Although their risk factors are similar to those of a plain bond, during the credit crisis these instruments had a significant liquidity risk premium.
The third component of our scenario framework allows users to define an additional haircut by pricing function, or micro-asset class. Haircuts can be defined for each of the 250+ pricing functions available in the system, highlighting if the haircut is relative or absolute and distinguishing between long and short side.

Pricing function	**Haircut**	**Shock type**	**Side**
mortgagebackedsecurity	1.5%	relative	both
zerocouponbond	0.08%	relative	long
zerocouponbond	0.09%	relative	short
irsswap	0.15%	absolute	both

4.3. H_4 – Nominal Haircut

In the fixed income world, one essential measure of liquidity for a bond is the size of the notional oustanding. An issue of 50m€ will be certainly less liquid than an issue of 5bn€.
The Liquidity Scenario lets the user define a matrix of haircuts for each “notional bucket”. The table below provides an example of such input.

Outstanding Notional	Currency	Side	Haircut	Shock type
5,000 M	USD	both	0.20%	relative
1,000 M	USD	both	0.50%	relative
500 M	USD	both	0.75%	relative
100 M	USD	both	1.0%	relative
50 M	USD	both	2.0%	relative
Unknown	USD	both	0.75%	relative

The Liquidity Engine will read the data of nominal issued for each bond and will interpolate logarithmically between the rows of the above table to assign the proper haircut to the relevant bond.

4.4. H_5 – Equity Market Capitalization

This haircut refers entirely to Equities. The liquidity of stocks is intimately linked to market capitalization. A big capitalization offers more liquidity than a small one.
Similarly to the previous component, a grid of Market Capitalizations can be defined, which assigns the proper haircut to each bucket and the Engine will use logarithmic interpolation for intermediate market capitalizations.

Market Cap	Currency	Side	Haircut
2,000 M	USD	both	0.20%
500 M	USD	both	0.50%
100 M	USD	both	0.75%
50 M	USD	both	1.0%
1 M	USD	both	2.0%
unknown	USD	both	0.75%

As the above sample table shows, it is possible to differentiate by Currency and to identify a Long and Short side.

4.5. H_6 – Percentage Owned

Another important measure of Liquidity Risk in the Equity world is the percentage of Market Capitalization in a stock owned by a certain portfolio. If a portfolio owns 0.000001% of the Market Cap of certain stock it will face less liquidity risk than owning 10%.

Unlike other haircuts, this component is not linked to single assets, but is referred to a portfolio of assets.

As in the above cases, our software lets the user define a grid of haircuts, based on different buckets of ownership. Below, we provide an example of such grid. The term p_e is the percentage of ownership and is computed as,

$p_e = Q * P / M_c$.

The term Q identifies the quantity owned in the portfolio and the price P is the stock price at which the position is valued. The term M_c is instead the Market Capitalization of the stock.

p_e	**Currency**	**Side**	**Haircut**
50%	all-currencies	both	50%
25%	all-currencies	both	12.5%
10%	all-currencies	both	5.0%
5%	all-currencies	both	2.0%
1%	all-currencies	both	0.5%
0.1%	all-currencies	both	0.001%
Unknown	all-currencies	both	0.5%

Also in this case logarithmic interpolation will be used for intermediate values of p_e.

4.6. H_1 – Specific

The last liquidity component is an arbitrary asset-specific haircut that can be assigned as add-on to a specific issue if the above five components are not able to model the specific liquidity risk of one asset.

Unlike the five previous haircuts which are always defined as positive numbers (liquidity haircuts always play *against* the holder of a long/short position on the asset, reducing the price if long and increasing it if short) this component can be either negative or positive in the Liquidity Scenario, implying eventually a reduction of liquidity risk.

5. Total Haircut

We denote with H_i for i = 1, …, 6 the haircuts for the liquidity-risk components. Then, the global haircut H_g for the asset can be computed as,

$$H_g = H_1 + H_2 + H_3 + H_4 + H_5 + H_6 ,$$

i.e., summing up the haircuts for each component. This formula holds for both haircut types and both sides.

APPENDIX 1. Computation of Exposure per Product Type

The following table summarizes the rules for computing the term E, Exposure of formula (1).

Product Name	Number	Exposure
Equities	1	Price*Quantity*PriceFactor / FX Rate
Mutual Funds	2	Price*Quantity*PriceFactor / FX Rate
Warrants/Certificates*	3	Price*Quantity*PriceFactor / FX Rate
Indices	4	Price*Quantity*PriceFactor / FX Rate
Bonds	7	Price*Quantity*PriceFactor / FX Rate
Futures	8	Price*Quantity*PriceFactor / FX Rate
Options	9	Price*Quantity*PriceFactor / FX Rate
Liquidity	11	Price*Quantity*PriceFactor / FX Rate
Structured Bonds	12	Price*Quantity*PriceFactor / FX Rate
Generic OTCs	21	Quantity*PriceFactor / (FX Rate)
OTC Options	22	Price*Quantity*PriceFactor / (FX Rate)
Single-Leg Futures	30	Price*Quantity*PriceFactor / (FX Rate)