Multi-Currency Performance Attribution

The two main drivers of global investment performance are local asset returns and currency exchange rate returns. These two sources represent distinct investment decisions, and should be attributed independently, as argued by Singer and Karnosky. This article presents a refined and generalized version of the Singer-Karnosky model for multi-currency attribution. In particular, the authors explicitly account for the cross product, thus fully explaining portfolio returns. Moreover, the methodology in this article attributes the local component with complete generality, naturally accounts for portfolio leverage, and cleanly handles cases where cash return differs from the risk-free rate.

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INTRODUCTION

There are two main drivers of global asset returns. The first is price appreciation of the asset in the local currency of denomination. The second main driver results from converting local asset values back to the base currency, or numeraire, of the investor.

In addition to these two main sources, there is a third, typically minor, contributor to global asset returns. This derives from repatriating local *profits* back to the base currency of the investor. We refer to this as the *Cross Product*. Although the Cross Product is intuitive and clearly defined, it has typically been either neglected or treated in an approximate fashion in previous works on multi-currency attribution.

One of the early works on multi-currency attribution is by Ankrim and Hensel (1994). They focus their currency analysis on the exchange-rate return, decomposing it into a "forward premium" (due to interest rate differentials) and a "currency surprise." They argue that since the forward premium can be "locked in" at the start of the period, the active manager should only be given credit for the currency surprise. One shortcoming with this approach is that it focuses on exchange-rate return rather than true currency return. In practice, the holder of foreign currency earns not only the exchange-rate return but also the local risk-free return. Omitting risk-free contributions from the currency effect leads to material distortions in performance attribution, particularly as the analysis period becomes longer. Another shortcoming with the approach of Ankrim and Hensel is that they do not explicitly account for the Cross Product.

The leading work on the subject of multi-currency attribution is by Singer and Karnosky (1995). They were the first to properly include risk-free rates as a component of the currency return, and the first to develop an attribution framework recognizing the distinction and separability of the market decision and the currency decision. The separability of these two decisions is best seen in a simple example. Consider a U.K. manager investing in U.K. and European stocks. The manager targets a 50/50 market exposure by investing half the portfolio in U.K. stocks and the other half in European stocks. The manager then decides on an 80/20 currency allocation by entering a currency forward contract (with 30 percent notional value) that goes long the pound and short the euro. The currency forward contract, in effect, decouples the market and currency exposures.

While the Singer-Karnosky framework is fundamentally correct, it does have shortcomings in its treatment of the Cross Product. Specifically, in an attempt to make the Cross Product disappear, they use continuously compounded (log) returns. One basic difficulty with log returns is that they do not agree with actual portfolio returns, which are ultimately what practitioners care about. A more serious problem is that log returns do not satisfy the portfolio property. That is, the weighted sum of log returns does not equal the log return of the portfolio. This leads to small discrepancies with the top-level portfolio return.

Another work in multi-currency attribution is due to Paape (2003). She asserts that it is not possible to cleanly separate market and currency effects as proposed by Singer and Karnosky. Instead, she defines allocation and selection effects in terms of numeraire returns and introduces a currency effect that vanishes for the case in which there is no currency overlay. The main shortcoming of Paape's approach is that it abandons the central achievement of Singer-Karnosky: namely, the separation of market and currency effects. For instance, her approach does not distinguish whether a positive allocation effect associated with an over-weight of Japan was due to a rising Japanese market or a rising yen. Moreover, in practice, the currency manager is responsible for the entire active currency position, not just the portion from the currency overlay. That is, an active exposure to yen due to an unhedged over-weight of the Japanese equity market constitutes an active currency bet, although it would not result in a currency effect in Paape's approach.

A more recent work on multi-currency attribution is by Bacon (2007). Several of the attribution effects defined by Bacon are not intuitive. For instance, the weighted average currency effect does not aggregate to the portfolio level currency effect *even for a single period*. To plug this gap, Bacon introduces a "compounding effect." Unfortunately, no financial motivation or transparency is provided for this term.

In this paper, we present a refined and generalized version of the Singer-Karnosky attribution model. Contrary to Paape's assertion, we obtain a clean and precise separation of market and currency effects, while fully retaining the financial intuition of the Singer-Karnosky framework. More specifically, this article makes four main contributions. First, we explicitly account for the Cross Product, thus giving added transparency to a real source of portfolio return. Second, our approach provides complete flexibility in the attribution of the local (*i.e.*, market) component. That is, the local component can be attributed to allocation and selection effects (*e.g.*, countries or industries), to factors, or to any other decision variables that reflect the investment process. Third, we show how to naturally account for net cash positions (*i.e.*, leverage). This allows one to analyze currency overlays that may have nonzero value. Finally, we discuss how to handle the case where the cash return differs from the official risk-free rate.

DECOMPOSING ASSET RETURNS

Our approach to multi-currency attribution starts by decomposing asset returns into three fundamental sources. Each source has a clear financial interpretation. Once decomposed, the components can be aggregated to the portfolio level and attributed separately.

Let l_n be the local return of asset n, denominated in currency k. If currency k appreciates by an amount e_k relative to the numeraire, then the base return r_n of the asset is given by the standard formula,

$$r_n = l_n + e_k + l_n e_k . \tag{1}$$

The exchange-rate return e_k depends on fluctuations in the spot rate over the investment period. Let $S_k(t)$ be the spot price in base currency of one unit of currency k at time t. The exchange-rate return of currency k from time t_0 to t_f is given by

$$e_k = \frac{S_k(t_f)}{S_k(t_0)} - 1 . \tag{2}$$

As a concrete example, consider an investment in Sony stock from the U.S. dollar numeraire perspective. For simplicity, suppose that the price of Sony is ¥100 per share at the start of the period, and the exchange rate is 0.010 dollars per yen (*i.e*., ¥100/$). Each share of Sony is thus worth $1.00 at the start of the period. At the end of the period, suppose the yen has appreciated 10 percent, to 0.011 dollars (*i.e*., ¥90.91/$), and Sony earns a local return of 10 percent, thus closing at ¥110 per share. At the end of the period, each share of Sony is valued at $1.21 (110/90.91), for a 21 percent return in US dollars. Substituting $l_n = 0.10$ and $e_k = 0.10$ into Equation 1 leads to a 21 percent return, consistent with an ending price of $1.21 per share.

In multi-currency attribution, it proves useful to work in terms of *excess* returns, which are defined relative to risk-free rates. The risk-free rate represents the "opportunity cost" of investing. That is, investment decisions should be made by comparing expected returns to the alternative investment in the risk-free asset.

Let ρ_k be the risk-free return of currency k, and let ρ_{base} be the corresponding return for the base currency. Rewriting Equation 1 in terms of *excess returns*, we obtain

$$r_n - \rho_{base} = (l_n - \rho_k) + (\rho_k + e_k + \rho_k e_k - \rho_{base}) + (l_n - \rho_k) e_k . \tag{3}$$

This important result decomposes the *base excess return*, $(r_n - \rho_{base})$, into three fundamental contributions. The first is the *local excess return*, $(l_n - \rho_k)$, which represents the "market" component and answers whether the local asset return outperforms the local risk-free rate. It is worth pointing out that the local excess return is independent of numeraire. Also note that ρ_k depends on the asset n under consideration. For instance, in the case of Sony, ρ_k is the risk-free return of yen, whereas for IBM, ρ_k is the risk-free return of U.S. dollars.

The second component of Equation 3 is the *currency excess return* c_k, defined as

$$c_k = \rho_k + e_k + \rho_k e_k - \rho_{base} . \tag{4}$$

This quantity answers whether holding cash in currency k outperforms holding cash in the base currency.

The third component of Equation 3 is the *Cross Product*, $(l_n - \rho_k) e_k$. This arises from the conversion of local excess *profits* back to the base currency. Note that this portion of returns cannot be currency hedged because the local excess return of the asset is unknown at the start of the period. Significant Cross Products appear only when large local excess returns occur jointly with large exchange-rate fluctuations. Although the Cross Product is typically small, it should be retained to fully account for the portfolio performance.

It is important to note that Equation 3 applies equally to cash as well as risky assets. In theory, the local excess return of cash is zero, and many performance attribution systems employ this as an operational assumption. In reality, however, the local excess return of cash will be nonzero. For instance, cash proceeds from a short sale typically earn less than the risk-free rate, whereas commercial paper usually earns more. Depending on data availability and the degree of precision desired, the local excess return of cash can also be cleanly attributed within our attribution framework, as discussed below.

AGGREGATING TO PORTFOLIO LEVEL

Consider a portfolio with weights w_n^P. The portfolio base excess return R_P is given by the weighted average of the underlying base excess returns,

$$R_P = \sum_n w_n^P (r_n - \rho_{base}) . \tag{5}$$

Applying Equation 3, the portfolio base excess return can be decomposed into three contributions:

$$R_P = L_P + C_P + Q_P , \tag{6}$$

where

$$L_P = \sum_n w_n^P (l_n - \rho_k) , \tag{7}$$

is the local excess (*i.e.*, market) contribution;

$$C_P = \sum_n w_n^P c_k \, , \tag{8}$$

is the currency effect and

$$Q_P = \sum_n w_n^P \left(l_n - \rho_k\right) e_k \, , \tag{9}$$

is the Cross Product. We now show how to attribute each source in greater detail.

LOCAL EXCESS RETURNS ATTRIBUTION

Previous works on multi-currency attribution have decomposed local excess return into country allocation and selection effect. A major point of this paper, however, is that there is complete freedom in attributing local excess returns. This is accomplished by simply feeding local excess returns into the desired attribution model.

To illustrate this point, we attribute local excess return for two distinct investment processes. The first is a sector-based approach that uses economic sectors rather than country sectors. The second investment process we consider is a factor-based approach.

SECTOR-BASED ATTRIBUTION

A sector-based investment process consists of two basic decisions. The first is sector allocation, where the manager seeks to over-weight outperforming sectors and under-weight underperforming ones. The second decision is security selection within sectors, where the objective is to over-weight outperforming assets within the sector and under-weight underperforming ones.

Let W_i^P and W_i^B denote, respectively, the portfolio and benchmark weights for sector *i*. The portfolio sector return (local excess) is given by

$$l_i^P = \frac{1}{W_i^P} \sum_{n \in i} w_n^P \left(l_n - \rho_k\right), \tag{10}$$

where w_n^P is the portfolio weight in asset *n*. The local excess return for the entire benchmark is[1]

$$L_B = \sum_n w_n^B \left(l_n - \rho_k\right). \tag{11}$$

With the sector weights and returns thus defined, the active local excess return is attributed according to the Brinson model[2] (1985),

$$L_A = \sum_i \left(W_i^P - W_i^B\right)\left(l_i^B - L_B\right) + \sum_i W_i^P \left(l_i^P - l_i^B\right). \tag{12}$$

The first term in Equation 12 is the allocation effect, which measures the active weighting decision at the sector level. The validity of this expression hinges on the portfolio and benchmark weights each summing to 1; this always holds when cash positions are properly included. Usually, cash should be treated as a distinct sector, although for some investment processes it may be appropriate to consider cash as assets within sectors. The quantity $(l_i^B - L_B)$ is termed the *relative return* of the sector. Positive allocation effect is achieved by over-weighting sectors with positive relative return, or under-weighting those with negative relative return.

The second term in Equation 12 is the selection effect, which measures the active weighting decision within sectors. We term $(l_i^P - l_i^B)$ to be the *active return* of the sector.

FACTOR-BASED ATTRIBUTION

In a factor-based investment strategy, local excess returns are explained by a parsimonious set of factors. For equities, the factors might represent countries, industries, and styles (*e.g.*, Value, Size, or Momentum). In the fixed income world, the factors might correspond to characteristic movements in the yield curve (*e.g.*, Shift or Twist) or to credit spreads in various quality groupings and/or sectors.

One strength of the factor approach is that it decomposes asset returns into a systematic (factor) component and a diversifiable (idiosyncratic) component. This segmentation provides considerable insight into the sources of return and risk. Another benefit of the factor framework is that it disentangles otherwise competing effects, thereby allowing the portfolio to be analyzed along multiple dimensions simultaneously (*e.g.*, countries, industries, *and* styles). By contrast, the sector approach considers only one segmentation variable at a time (*e.g.*, countries *or* industries *or* styles).

Let X_{nm} be the exposure of asset *n* to factor *m*. The local excess return of the asset is decomposed into contributions from individual factors,

$$l_n - \rho_k = \sum_m X_{nm} f_m + u_n, \quad (13)$$

where f_m is the factor return and u_n is the *idiosyncratic*, or *specific*, return. The factor returns are estimated by cross-sectional regression and can be interpreted as the returns on long/short portfolios that have unit exposure to the factor under consideration and zero exposure to other factors. The Value factor return, for example, is the return of a portfolio that has unit exposure to Value but is neutral to industries, countries, and other styles.

The active exposure to factor *m* is given by

$$X_m^A = \sum_n w_n^A X_{nm}, \quad (14)$$

where w_n^A is active weight (defined as the difference between portfolio and benchmark weight) in asset *n*. The active local excess return can thus be attributed as

$$L_A = \sum_m X_m^A f_m + \sum_n w_n^A u_n. \quad (15)$$

The first term gives the contribution from factors, and the second term provides the stock-specific contribution. Positive return is achieved by having positive exposures to factors with positive payoffs, or by overweighting assets with positive specific returns.

Cash has zero exposure to all risk factors. If the local excess return of cash is nonzero, it will be reflected in nonzero contributions from the idiosyncratic term.

CURRENCY RETURNS ATTRIBUTION

The portfolio currency return is given by

$$C_P = \sum_n w_n^P c_k, \quad (16)$$

where w_n^P is the portfolio weight in asset *n*, and c_k is the currency excess return, as given by Equation 4. The summation in Equation 16 must include all cash and noncash assets. Let W_k^P denote the portfolio weight in currency *k*,

$$W_k^P = \sum_{n \in k} w_n^P. \quad (17)$$

The portfolio currency effect can be written in terms of currency weights as

$$C_P = \sum_k W_k^P c_k. \quad (18)$$

Intuitively, positive weight in a currency that outperforms the base currency contributes positively to the portfolio currency effect.

Net currency weights are due to explicit (cash) as well as implicit (risky assets) positions. It is useful and informative to identify each source separately. Risky assets correspond to anything that is noncash, such as stocks, long-term bonds, or commodities. Cash assets must include actual cash, such as short-term government debt, as well as "synthetic" cash that results from derivative instruments. For instance, a stock index futures contract can be disaggregated into a long position in the underlying index and a short position in cash. Similarly, a currency forward is equivalent to going long one currency and taking an offsetting short position in another currency.

Let $W_{k,c}^P$ be the portfolio cash weight of currency *k*,

$$W_{k,c}^P = \sum_{\substack{n \in k \\ (cash)}} w_n^P. \quad (19)$$

If the portfolio has no cash, then $W_{k,c}^P = 0$ for all currencies. If the portfolio has a currency overlay with net zero value, then $W_{k,c}^P$ may be individually nonzero, but sum to zero over the entire portfolio. If the portfolio is positively levered, then the net portfolio cash weight will be negative.

If we restrict the sum in Equation 17 to only risky assets, we obtain

$$W_{k,r}^P = \sum_{\substack{n \in k \\ (risky)}} w_n^P. \quad (20)$$

Note that the risky weights $W_{k,r}^P$ do not in general sum to one. If the portfolio is positively levered, for instance, the sum of risky weights is greater than one.

The portfolio currency weight is the sum of the cash weight and the risky weight,

$$W_k^P = W_{k,c}^P + W_{k,r}^P. \quad (21)$$

Note that currency weights always sum to one.

For the attribution of active currency returns, one could simply substitute the active currency weights W_k^A (*i.e.*, the difference between portfolio and benchmark weights) into Equation 18. This would give the correct result at the portfolio level but give misleading results at the currency level. For instance, if the benchmark has a currency excess return of two percent, then a currency with an excess return of only one percent represents an *underperforming* currency. The manager deserves positive active currency effect from overweighting currencies that outperform *relative to the benchmark*. Therefore, for attribution of active currency returns, we use the following expression,

$$C_A = \sum_k \left(W_k^P - W_k^B\right)\left(c_k - C_B\right). \tag{22}$$

This is analogous to the definition of allocation effect in Equation 12. The quantity (c_k - C_B) is termed the *currency relative return*.

CROSS-PRODUCT ATTRIBUTION

The Cross Product arises from repatriation of local excess profits back to the base currency and must be retained to fully account for portfolio return. The Cross Product, however, does *not* correspond to an active management decision. Rather, it results from the combined effect of two *separate* decisions (*i.e.*, local investment and currency). Consequently, the aim of cross-product attribution is to provide *transparency* into this source of return but not to elucidate the impact of an active management decision.

In Equation 9, the portfolio Cross Product is written as a sum over assets. It is useful to rewrite it as a sum over currencies,

$$Q_P = \sum_k e_k G_k^P. \tag{23}$$

where G_k^P is the portfolio local excess return contribution from all assets (including cash) denominated in currency k, *i.e.*,

$$G_k^P = \sum_{n \in k} w_n^P \left(l_n - \rho_k\right). \tag{24}$$

The active Cross Product is given by the difference between the portfolio and benchmark Cross Products, and can be written as

$$Q_A = \sum_k e_k \left(G_k^P - G_k^B\right). \tag{25}$$

The local excess return contribution in Equation 24 represents a sum over all assets (including cash) within a currency bucket. If we assume that cash earns the risk-free rate, then the local excess return contributions are due solely to risky assets, and the active Cross Product can be written as

$$Q_A = \sum_k e_k \left(W_{k,r}^P l_k^P - W_{k,r}^B l_k^B\right). \tag{26}$$

where $W_{k,r}^P$ is portfolio weight of risky assets denominated in currency k, and l_k^P is the portfolio local excess return to risky assets denominated in currency k (as in Equation 10).

EXAMPLE

We now consider a concrete example using the MSCI *All Country World Index Investable Market Index* (ACWI IMI). The MSCI ACWI IMI aims to reflect the full breadth of the global investment opportunity set by targeting 99 percent of the float-adjusted capitalization in developed and emerging markets. We select as our portfolio the MSCI ACWI IMI Value Index, but include only assets denominated in euros, U.K. pounds, U.S. dollars, and yen. For the benchmark, we use the MSCI ACWI IMI Growth Index, and similarly exclude all assets not denominated in one of these four currencies. Our analysis period is October 2008, and the base currency is taken as the U.S. dollar. We suppose that the portfolio manager applies an active currency overlay strategy (described below). To illustrate the effect of leverage (or cash drag), we also include a 5 percent net cash position in the portfolio.

In Table 1 we present a summary view of the performance attribution results. The total return of the portfolio was down by 12.68 percent, versus an 18.91 percent drop for the benchmark. The active return was thus 624 basis points (bps). The risk-free rate contributed 14 bps to both the portfolio and benchmark, and represents the risk-free return for the base currency (U.S. dollar). Most of the active return came from the currency effect (398 bps), although the local excess contribution was also significant (215 bps). The Cross Product, meanwhile, contributed a relatively minor 11 bps to active return. We now drill into the individual sources to gain better insight into the drivers of performance.

In Table 2 we attribute the currency effect. The upper panel reports the various currency weights. The portfolio equity (*i.e.*, risky) weight was 95 percent, thus showing that the portfolio was less than fully invested. The portfolio cash weight column indicates the curren-

Table 1: Summary Results of Performance Attribution

Source	Portfolio Return	Benchmark Return	Active Return
Local Excess	-15.30%	-17.44%	2.15%
Currency Effect	2.22%	-1.76%	3.98%
Cross Product	0.26%	0.15%	0.11%
Risk Free	0.14%	0.14%	0.00%
Total	-12.68%	-18.91%	6.24%

Table 2

(a) Currency weights

Currency	Portfolio Stock Weight	Portfolio Cash Weight	Portfolio Currency Weight	Benchmark Currency Weight	Active Currency Weight
Euro	0.18	-0.18	0.00	0.17	-0.17
Pound	0.11	-0.11	0.00	0.11	-0.11
Yen	0.11	0.17	0.28	0.12	0.17
Dollar	0.55	0.17	0.72	0.60	0.12
Total	0.95	0.05	1.00	1.00	0.00

(b) Currency returns and attribution effects

Currency	Exchange Rate Return	Risk Free Return	Currency Excess Return	Portfolio Currency Effect	Benchmark Currency Effect	Currency Relative Return	Active Currency Effect
Euro	-9.72%	0.40%	-9.50%	0.00%	-1.63%	-7.74%	1.33%
Pound	-9.35%	0.47%	-9.07%	0.00%	-1.03%	-7.31%	0.83%
Yen	7.95%	0.05%	7.86%	2.22%	0.91%	9.62%	1.60%
Dollar	0.00%	0.14%	0.00%	0.00%	0.00%	1.76%	0.21%
Total				2.22%	-1.76%		3.98%

cy overlay. The manager decides to hedge exposure to the euro and pound, while increasing exposure to the yen and dollar. The currency overlay can be implemented by currency forward contracts or by buying and selling currencies in the spot market. The total portfolio cash weight is five percent, thus indicating cash drag. The portfolio currency weights are the sum of the equity weights and the cash weights, and always sum to one. The active currency portfolio takes a long position in the yen and dollar, and a short position in the euro and pound. Active currency weights always sum to zero.

In the lower panel of Table 2 we show the currency returns and attribution effects. The yen appreciated 7.95 percent versus the dollar, while the euro and pound each depreciated by over nine percent. The euro and pound had relatively high risk-free rates, while the yen and dollar yielded relatively little. The currency excess return column is computed as in Equation 4. Holding yen, for example, outperformed holding dollars by 7.86 percent in October 2008. The portfolio currency effect (222 bps) is given by the product of the portfolio currency weight and the currency excess return, and is entirely explained by the yen. By contrast, the benchmark held large positions in the euro and pound, which adversely impacted the performance. The currency relative return is the difference between the currency

Table 3: Factor-based Performance Attribution of Local Excess Returns

Group	Portfolio Return Contrib	Benchmark Return Contrib	Active Return Contrib
World	-17.76%	-18.69%	0.93%
Styles	0.22%	0.95%	-0.72%
Industries	0.48%	-0.51%	0.99%
Countries	0.75%	0.72%	0.03%
Specific	1.01%	0.09%	0.92%
Total	-15.30%	-17.44%	2.15%

Table 4: Style Drilldown for Factor-based Attribution

Factor	Portfolio Exposure	Benchmark Exposure	Active Exposure	Factor Return	Portfolio Contrib	Benchmark Contrib	Active Contrib
Momentum	-0.12	0.17	-0.29	0.83%	-0.10%	0.14%	-0.24%
Volatility	-0.08	-0.10	0.02	-3.80%	0.31%	0.38%	-0.07%
Value	0.33	-0.26	0.59	-0.40%	-0.13%	0.11%	-0.24%
Size	0.35	0.22	0.14	0.85%	0.30%	0.19%	0.12%
NL Size	-0.14	-0.02	-0.12	-0.04%	0.01%	0.00%	0.00%
Growth	-0.31	0.28	-0.60	-0.67%	0.21%	-0.19%	0.40%
Liquidity	-0.13	0.14	-0.27	-0.47%	0.06%	-0.07%	0.13%
Leverage	0.22	-0.20	0.43	-1.96%	-0.43%	0.40%	-0.83%
Total					0.22%	0.95%	-0.72%

excess return and the benchmark currency effect. The dollar thus outperformed by 176 bps on a relative basis. The active currency effect is the product of the active currency weight and the currency relative return. The total, 3.98 percent, is the result of over-weighting the low-yielding dollar and yen and under-weighting the high-yielding euro and pound.

In Table 3 we attribute the local excess returns using the Barra Global Equity Model (GEM2), a multifactor risk model. This model is described by Menchero, Morozov, and Shepard (2009). In this example, the World factor is the largest contributor to both portfolio and benchmark return. The World factor essentially represents the cap-weighted world portfolio, and the negative contributions to portfolio and benchmark returns reflect the severe drop in the global equity markets during October 2008. The World factor, however, contributed 93 bps to active return. This can be understood as the positive effect of cash drag in a down month.

In Table 3 we see that industries and stock-specific bets both made significant positive contributions to active return, whereas styles detracted 72 bps. The performance analyst may want to drill into the various groups to better understand the sources of these contributions. For example, in Table 4, we drill down to the individual style factor level. The negative active exposure to the Growth factor contributed 40 bps to active return, whereas the Value factor detracted 24 bps. However, the two most dramatic style factors in October 2008 were Volatility and Leverage, both of which dropped sharply. The Volatility factor roughly represents a portfolio that goes long high-beta stocks and goes short low-beta stocks, and typically performs poorly in down markets. The Leverage factor is roughly represented by a portfolio that goes long highly levered firms and shorts firms with low leverage, and often performs poorly during credit crises. The Volatility factor only

Table 5: Stock-specific Drilldown Showing Top Five and Bottom Five Contributors

Asset Name	Portfolio Weight	Benchmark Weight	Active Weight	Specific Return	Active Contrib
Exxon Mobil	0.041	0.000	0.041	9.7%	0.40%
BP	0.016	0.000	0.016	20.9%	0.33%
JP Morgan Chase	0.017	0.000	0.017	13.8%	0.24%
Total	0.013	0.000	0.013	14.5%	0.19%
Pepsi Co	0.000	0.013	-0.013	-13.2%	0.17%
…	…	…	…	…	…
ING	0.004	0.000	0.004	-32.1%	-0.13%
HSBC	0.019	0.000	0.019	-7.8%	-0.15%
Royal Bank Scotland	0.005	0.000	0.005	-41.4%	-0.22%
Apple	0.000	0.011	-0.011	21.0%	-0.23%
Volkswagen	0.000	0.006	-0.006	101.7%	-0.64%
Total	1.000	1.000	0.000		0.92%

Table 6: Sector-based Performance Attribution of Local Excess Returns Using GICS Sectors

GICS Sector	Port Weight	Bench Weight	Active Weight	Port Return	Bench Return	Active Return	Relative Return	Alloc Effect	Selec Effect	Total Effect
Energy	0.153	0.068	0.085	-7.33%	-26.53%	19.20%	-9.09%	-0.78%	2.94%	2.16%
Materials	0.038	0.067	-0.029	-25.59%	-20.52%	-5.07%	-3.08%	0.09%	-0.19%	-0.11%
Industrials	0.082	0.133	-0.051	-20.14%	-20.59%	0.45%	-3.15%	0.16%	0.04%	0.20%
Cons. Dscr	0.087	0.108	-0.021	-19.96%	-14.06%	-5.90%	3.39%	-0.07%	-0.51%	-0.58%
Cons. Stpls	0.065	0.142	-0.077	-9.46%	-11.15%	1.69%	6.29%	-0.48%	0.11%	-0.38%
Health Care	0.090	0.115	-0.025	-7.89%	-12.77%	4.88%	4.67%	-0.12%	0.44%	0.32%
Financials	0.292	0.081	0.210	-24.18%	-25.69%	1.51%	-8.25%	-1.74%	0.44%	-1.30%
IT	0.020	0.222	-0.202	-27.15%	-17.93%	-9.22%	-0.49%	0.10%	-0.18%	-0.08%
Telecom	0.063	0.022	0.041	-6.83%	-13.69%	6.86%	3.75%	0.15%	0.43%	0.59%
Utilities	0.060	0.044	0.016	-7.58%	-14.16%	6.58%	3.29%	0.05%	0.39%	0.45%
Cash	0.050	0.000	0.050	0.00%	0.00%	0.00%	17.44%	0.87%	0.00%	0.87%
Total	1.000	1.000	0.000	-15.30%	-17.44%	2.14%		-1.76%	3.90%	2.15%

had a minor impact on active return due to the small exposure. Unfortunately, the portfolio had a large positive active exposure to Leverage, which detracted 83 bps from active performance.

In Table 5 we show the stock-specific drilldown. There could potentially be thousands of assets in this view, so we show only the top and bottom five contributors to active return. Interestingly, several large oil companies performed well in October 2008. These firms were in the portfolio, but not the benchmark, which therefore helped active performance. The portfolio also benefited by being under-weight PepsiCo, which underperformed for the month. Three of the largest negative contributors were underperforming financial stocks. The largest detractor, however, was Volkswagen, which was under-weight but performed spectacularly. Note that if we allow for cash to earn a return different than

Table 7: Cross Product Performance Attribution

Currency	Port Equity Weight	Bench Equity Weight	Port Equity Return	Bench Equity Return	Port Local Excess	Bench Local Excess	Exchange Rate Return	Port Cross Product	Bench Cross Product	Active Cross Product
Euro	0.184	0.172	-18.18%	-13.79%	-3.34%	-2.37%	-9.72%	0.33%	0.23%	0.09%
Pound	0.109	0.114	-9.90%	-12.30%	-1.08%	-1.40%	-9.35%	0.10%	0.13%	-0.03%
Yen	0.112	0.116	-18.86%	-23.47%	-2.10%	-2.72%	7.95%	-0.17%	-0.22%	0.05%
Dollar	0.546	0.598	-16.10%	-18.30%	-8.79%	-10.94%	0.00%	0.00%	0.00%	0.00%
Total	0.950	1.000	-16.12%	-17.44%	-15.30%	-17.44%		0.26%	0.15%	0.11%

the risk-free rate, then cash assets would also appear in this view with nonzero specific return.

In Table 6 we present a sector-based attribution of the active local excess return by sectors, classified according to the Global Industry Classification Standard (GICS®). The allocation decisions hurt active performance by 176 bps. This was due mostly to a large overweight position in the Financials sector, which severely underperformed. In the sector-based analysis, cash drag manifests itself as an allocation effect. That is, cash represented the top-performing sector in October 2008, and the five percent overweight contributed 87 bps to allocation effect. On the selection side, most of the active contribution came from the Energy sector, where Value outperformed Growth by 19.2 percent.

In Table 7 we present the attribution of the Cross Product by currency. We assume that cash earns the risk-free rate, and therefore attribute the active Cross Product as in Equation 26. All of the markets were down sharply in October 2008, resulting in negative local excess return contributions. The euro and the pound, which both depreciated against the dollar, therefore contributed positively to the portfolio Cross Product. The yen, by contrast, contributed negatively. The active Cross Product is simply the difference between the Cross Product contributions for the portfolio and the benchmark. If we relax the assumption that cash earns the risk-free rate, then we must also include the contributions from cash in Table 7.

CONCLUSION

We have presented a comprehensive model for attributing the performance of multi-currency portfolios. Our approach is based on the Singer-Karnosky framework but makes certain refinements and generalizations. For instance, we explicitly account for the Cross Product, which was neglected in the Singer-Karnosky approach. Furthermore, our approach allows local excess return to be attributed according to any set of decision variables. We also show how to carry out multi-currency attribution for portfolios that have net cash positions. Finally, we discuss how to treat the case where cash earns a return different than the risk-free rate. These concepts were illustrated by a concrete example taken from the financial crisis of October 2008.

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ENDNOTES

[1] We implicitly assume a single-period buy-and-hold analysis, so that the portfolio and benchmark earn the same return on each asset. The results can be easily generalized to account for intra-period transactions, although this is beyond the scope of this article.

[2] The attribution model described by Equation 12 becomes problematic for long/short portfolios, since naive application of Equation 10 to sectors with net zero weight leads to infinite sector returns. However, the Brinson model can be generalized for long/short portfolios, while retaining the same intuition, as described by Menchero (2002/2003).