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THE FALSE PROMISES OF EXPONENTIAL-WEIGHTING, PRO-CYCLICALITY AND A NEW MEASURE OF RISK: HYBRID VAR

With each financial failure, the credit crisis generates a new bout of criticism of Value-at-Risk (VaR) and practice of risk management in general.

This article does not seek to discuss how most of the criticism is often ill-placed, that VaR is *not* the only risk measure available in risk management, that VaR can be produced with models other than Gaussian Variance/Covariance, and so forth.

We want instead to focus our attention on two serious failures of VaR (and similar measures, including Expected Shortfall, or Conditional VaR) during this ongoing crisis:

- The significant failures in backtesting
- VaR pro-cyclicality[1]

These two criticisms are certainly valid and while, as risk managers, we can find justifications for the first (weather forecast tends to be less reliable when dealing with tornados), the second criticism is more serious and requires greater attention.

Exponentially Weighted (EW) risk models that give greater weight to more recent history are very popular and widely used, but they are also the most pro-cyclical of the models. In this article we will demonstrate that their pro-cyclicality is not balanced by better performance in backtesting terms, and will present a new measure of risk "Hybrid VaR".

The first part of the article looks at the methodology for computing Hybrid VaR, and the second part will demonstrate that it performs better in backtesting than pure historical VaR or EW VaR (both Gaussian and Historical-Adjusted).

In addition, Hybrid VaR is anti-cyclical in boom periods, discouraging risk-taking in good times, and much more stable than other measures. This significantly mitigates the problem of pro-cyclicality while also providing a measure that is more reliable than other risk measures.

[1] i.e. increases in risk measures during periods of market stress, increasing the likelihood that risk limits are violated, causing further liquidation of positions in a falling market. This pro-cyclicality thereby amplifies negative market movements

PART 1
Computing Hybrid VaR

Hybrid VaR is a combination of VaR and the worst of a selection of historical stress test scenarios, in the form:

Hybrid VaR = L * VaR + (1-L) * Worst Stress Test with 0<L<1 (1)

The term *Worst Stress Test* changes for each portfolio/asset analysed. First, we select a number of representative historical stress test scenarios. Each scenario produces a P&L and has a reference historical period. We normalise the P&L of each stress scenario to the holding period of VaR with the formula:

Normalised P&L = Scenario P&L * SQRT(VaR Holding Period / Stress Scenario Period) (2)

Once all the normalised P&Ls for the scenarios are computed for the chosen asset/portfolio, we select the scenario that produces the worst loss. This outcome becomes the *Worst Stress Test* term of formula (1).

The weight L is a function of the ratio Worst Stress Test/VaR (Ratio). If the ratio increases, the weight L decreases, or, in other words, the weight of VaR diminishes and the weight of the Worst Stress Test term increases accordingly. The rationale is that when the ratio increase during good times, Hybrid VaR retains the history of the worst market moment recorded, reducing the decrease of the risk measure.

If instead the Ratio converges to 1 or becomes lower than 1, the weight of VaR goes to 100%, as the current VaR is recording unprecedented bearish events, not included in the set of stress test scenarios.

The weight is defined as a linear function of the above mentioned Ratio, with a cap and a floor.

L =	For Ratio <= 1	100%	(3)
	For Ratio > 1	125% - 25% * Ratio	
	For Ratio >= 3	MIN	

Where

Ratio = Worst Stress Test / VaR
MIN = Minimum value of the weight L

The term MIN can take any value between 0% and 100%. In our simulations and backtesting exercises we have set MIN = 50%, as we want our risk measure to maintain a certain sensitivity to the decrease of risk during good times.

In the second part of the article we look at the statistics computed for Hybrid VaR for the period 2nd January, 2004 to 30th December, 2008. We will compare the performance of the model to a pure Historical VaR model and to two exponential-weighting methodologies, observing the number of violations recorded by each model during the above period.

PART 2
Comparison of Hybrid VaR to Historical and Exponentially-Weighted Historical Models

For our backtesting exercises, we have selected three indices: DJ Euro Stoxx 50, S&P 500 and Nikkei 225. We have used daily data ranging from 2nd January, 2002 to 30th December, 2008.

Settings and methodology used for each of the 4 models are explained below.

VaR Settings
For all models we used 500 rolling days of historical observations. VaR was always computed at 99%. The bi-weekly holding period has been obtained by multiplying the 1-day VaR by the square root of 10.

Historical Simulation
The i-th daily scenario of the underlying daily distribution has the traditional form (cfr. Cintioli-Marchioro, *StatPro Simulation Model Series 1*, AIFIRM newsletter No.2, 2007):

$$S_i = P_i / P_{i-1} \qquad \text{for } i = 1, 2, \ldots 500 \qquad (4)$$

Gaussian Exponential Weighting Moving Average (EWMA)
We have computed the EWMA volatility (${}^{EWMA}VOL_i$) for each i-th scenario using a factor $\lambda = 0.94$, utilising the same 500 scenarios used in the Historical simulation. To obtain the Gaussian VaR, we multiplied the EWMA volatility by a constant factor of 2.33.

Historical Adjusted by Exponential Weighting (HIST-EWMA)
We have modified the historically simulated scenarios S_i by multiplying them by the factor ${}^{EWMA}VOL_i / VOL_i$, where VOL_i is the historical standard deviation computed on the 500 rolling days of historical observations.

We used the Hull-White methodology for applying this correction (cfr. Hull-White *Incorporating Volatility Updating into the Historical Simulation Method for Value at Risk*, Journal of Risk, Fall 1998), but have ignored the process of normalising the distribution mean to zero, to simplify the analysis. We do not expect the lack of normalisation to have any relevant impact on the results of our analysis and on the evidence that we will present.

Hybrid VaR
The VaR used in formula (1) is historically simulated VaR with equal weighting.
The Worst Stress Test for the 3 indices is the performance from the following scenarios:

Euro Stoxx 50	=	September 11 (10-21 Sept 2001) – 9 observations
S&P 500	=	September 11 (10-21 Sept 2001) – 9 observations
Nikkei 225	=	Summer 1990 (17 July 1990 – 23 August 1990) – 28 observations

Backtesting Procedure
Backtesting was carried out over the observation period (2nd January, 2004 to 30th December, 2008) using different holding periods:

- Daily
- Bi-weekly with daily frequency
- True bi-weekly

The results of our backtesting analyses for each of the holding periods are presented below.

DAILY BACK-TESTING

We have computed the daily VaR for each i-th date and have compared the i-th daily performance of each index against the VaR computed at i-1.

The following four tables show the number of VaR violations for each model, in comparison them to the expected number of violations.

Effective Violations: number of violations in the observed historical period (2nd January, 2004 to 30th December, 2008).
Expected Violations: 1% of total number of observations used for each index.
Ratio of Violations: Effective Violations / Expected Violations.
Size of Violation: Average of the ratio (Effective Loss – VaR) / VaR in each "violation".

Table 1. VaR Violations for Historical Simulation

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	23	12,7	181%	44,40%
S&P 500	38	12,6	302%	33,85%
Nikkei 225	29	12,3	236%	35,14%
Total	***90***	***37,6***	***239%***	***36,96%***

Table 2. VaR Violations for Gaussian EWMA

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	27	12,7	213%	32,23%
S&P 500	28	12,6	222%	27,04%
Nikkei 225	24	12,3	195%	35,28%
Total	***79***	***37,6***	***210%***	***31,32%***

Table 3. VaR Violations for Historical EWMA VaR

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	17	12,7	134%	31,37%
S&P 500	20	12,6	159%	28,41%
Nikkei 225	18	12,3	146%	43,16%
Total	***55***	***37,6***	***146%***	***34,15%***

Table 4. VaR Violations for Hybrid VaR

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	12	12,7	94%	45,46%
S&P 500	24	12,6	190%	36,06%
Nikkei 225	20	12,3	163%	37,90%
Total	***56***	***37,6***	***149%***	***38,73%***

The above analysis returns some clear evidence:

- The Historical Simulation model presents a very high number of violations compared to expected violations (2.4 times).
- The EW Gaussian variance/covariance model produces somewhat better results than the historical simulation model, but by a modest amount: 79 violations (2.1 times the expected number).
- The Historical Simulation model with exponential weights (HIST-EWMA) consistently reduces the number of violations from 90 to 55 (1.5 times the expected number of violations).
- The Hybrid VaR is almost identical to the HIST-EWMA model, with 56 violations in the observed period.

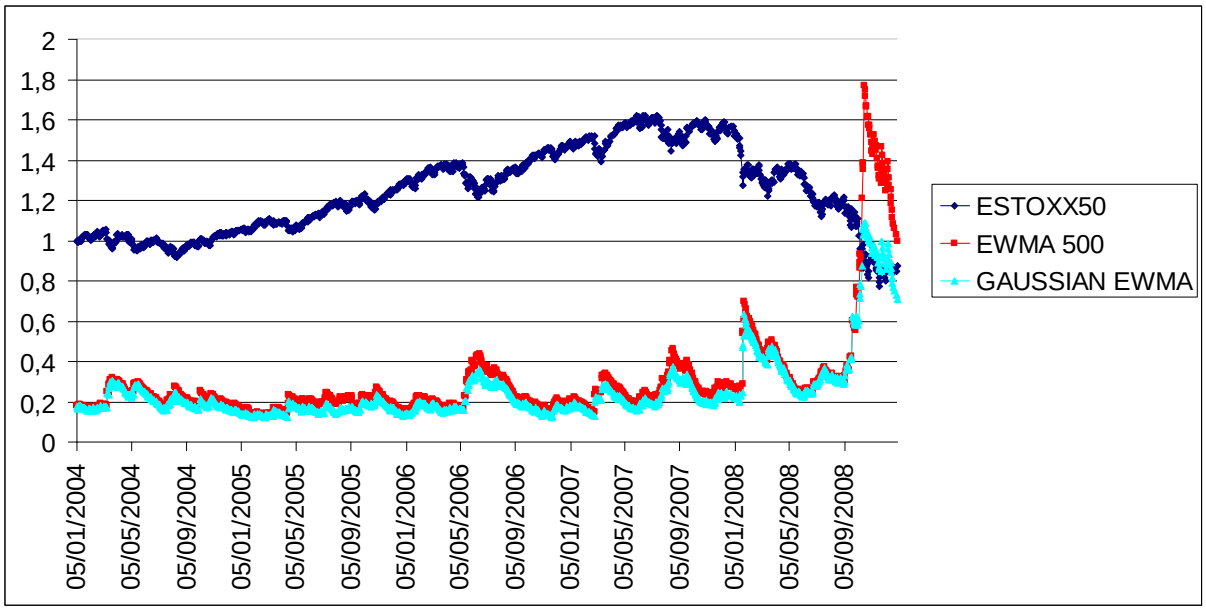
At the beginning of this article, we said that our initial objective was to address two major criticisms of VaR and similar risk measures:

- The significant failures in backtesting
- VaR pro-cyclicality

The evidence of the daily data represented above tells us that the failures in backtesting are most evident in the Historical Simulation and Gaussian EWMA models. EWMA Historical Simulation and the Hybrid VaR models consistently reduce the number of violations.

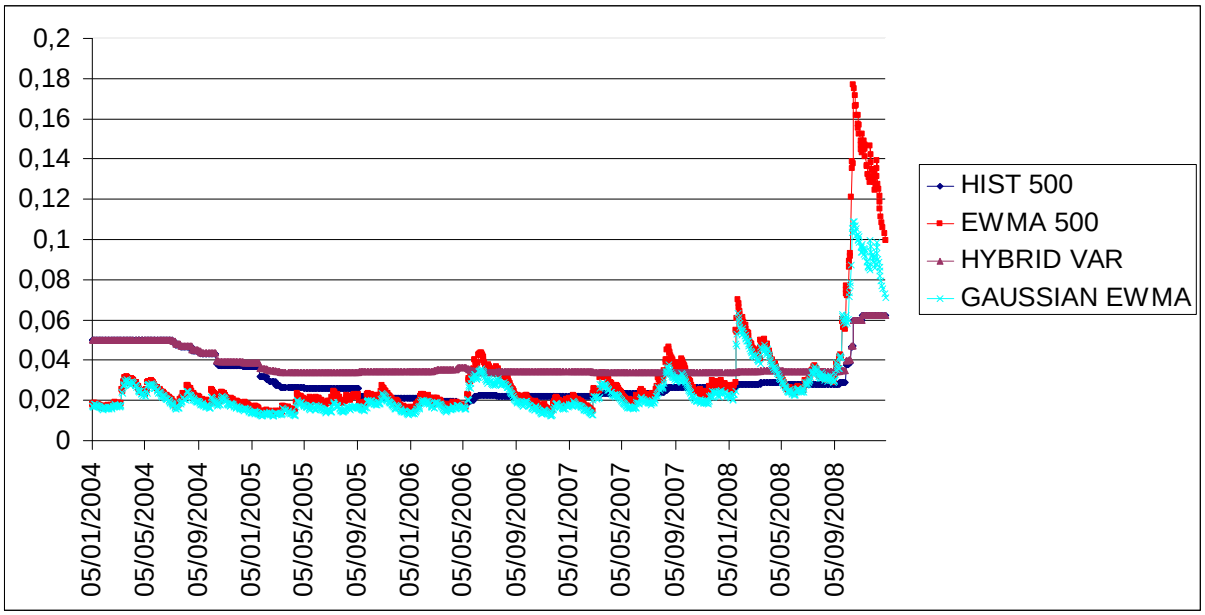
However, in terms of pro-cyclicality the exponential weighting methodology (both in Gaussian and Historical Models) has the worst impact. Graph 1 compares the evolution of the Euro Stoxx 50 with Gaussian EWMA and the HIST-EWMA VaR, showing the intimate relationship between drops in the market and increases in VaR (please note that the VaRs have been multiplied by 10 for scaling and presentation purposes).

Graph 1. Euro Stoxx50 and EWMA VaR



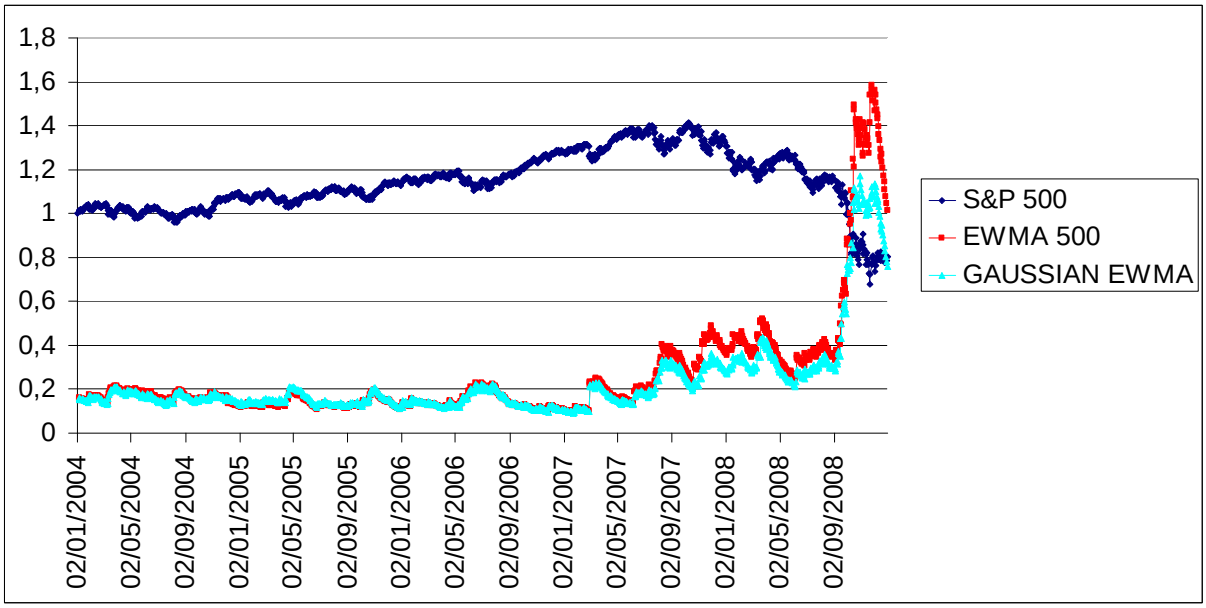
Graph 2 compares the four VaR models in the observed period, showing that Hybrid VaR is the most stable risk figure and the least pro-cyclical of the measures: during the bonanza years of 2005, 2006 and 2007, Hybrid VaR consistently remains above Historical VaR and often above the other VaRs.

Graph 2. Comparison of the four VaR models for Euro Stoxx 50

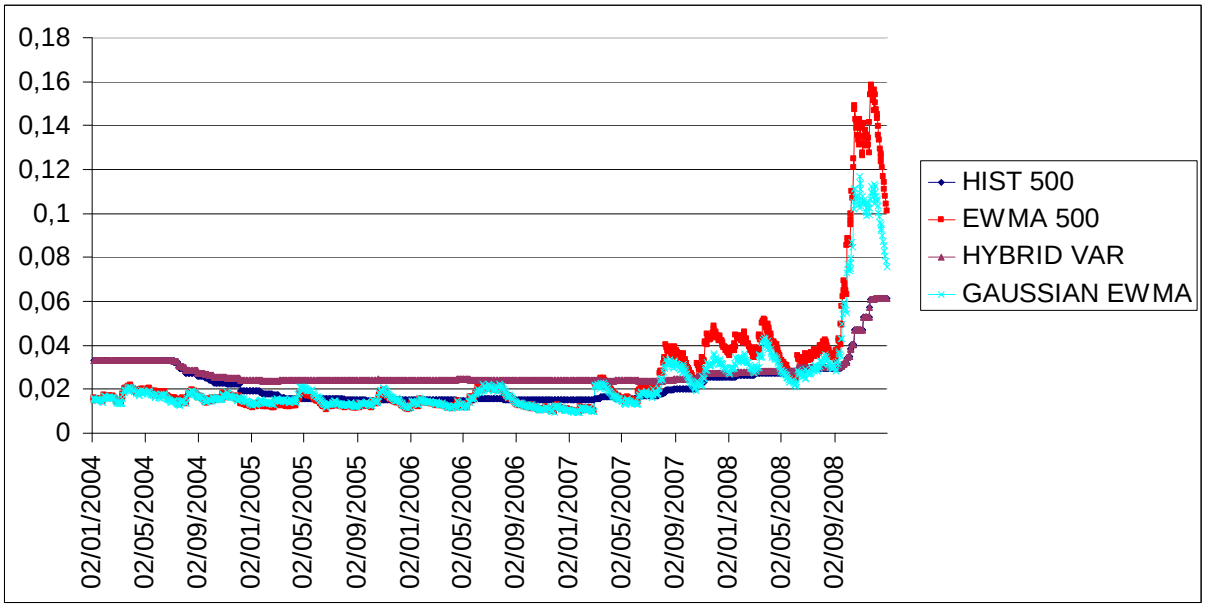


Graphs 3 – 6 show the same information for the S&P 500 and the Nikkei 225.

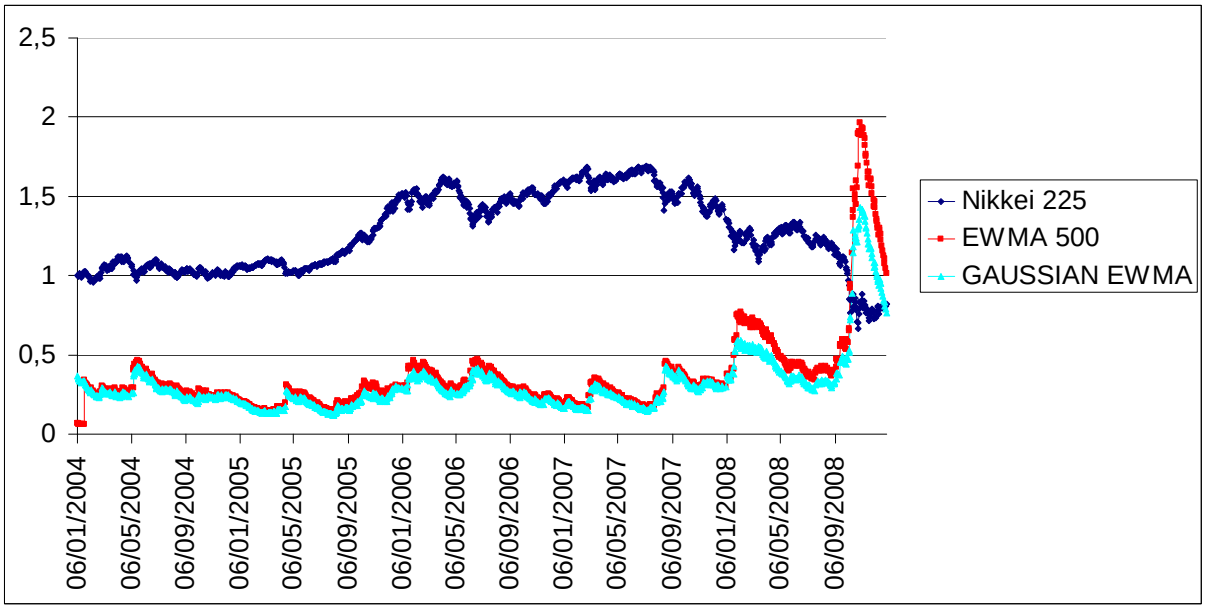
Graph 3. S&P 500 and EWMA VaR



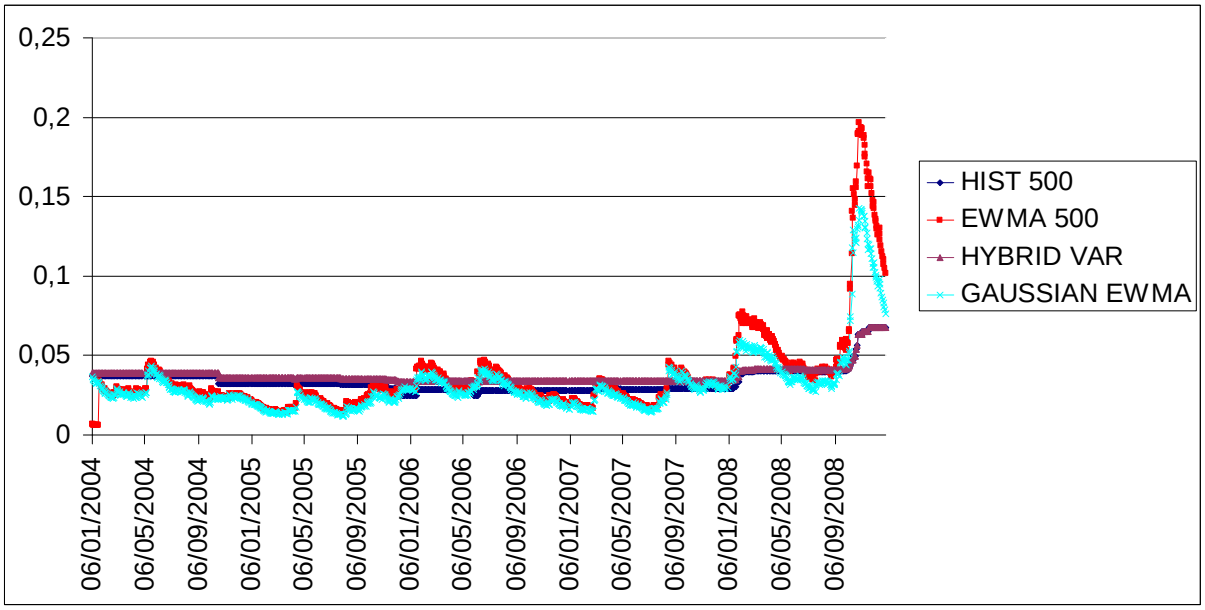
Graph 4. Comparison of the four VaR models for the S&P 500



Graph 5. Nikkei 225 and EWMA VaR



Graph 6. Comparison of the four VaR models for the Nikkei 225



As the above graphs show, Hybrid VaR is much less volatile than EWMA VaR, and significantly less pro-cyclical.

Hybrid VaR appears to be anti-cyclical compared to Historical VaR and to EWMA VaRs in "good" times. Table 5 shows VaR for the different methodologies on 4th May 2006, a bullish phase of the market.

Table 5. The four VaR models on 04 May 2006

	HIST	GAUSSIAN EWMA	HIST-EWMA	HYBRID
ESTOXX50	1,78%	1,68%	1,77%	3,62%
SP500	1,49%	1,21%	1,20%	2,44%
NIKKEI225	2,84%	2,52%	2,89%	3,41%

Note that Hybrid VaR is higher than the other VaR measures – in many case being almost double.

BI-WEEKLY DAILY BACKTESTING

While daily backtesting is the most appropriate verification process for risk figures with a daily holding period, most regulators require a longer holding period. A two-week holding period is the most frequent requirement. For this reason, we have also chosen a two-week holding period.

In this section, we compare ten-day performance periods with two-week VaR at t-11 on a rolling daily basis. This type of backtesting is the one most used by practitioners, under many regulations. Even if the holding period is bi-weekly, it is performed on a daily basis because the model is reviewed on a yearly basis and "true" two-week observations would not return enough statistics (only 26 observations).

As in the previous section, we report in the following four tables the VaR violations for the selected holding period.

Table 6. VaR Violations for Historical Simulation – Bi-weekly holding period with daily frequency

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	19	12,7	150%	40,96%
S&P 500	22	12,6	175%	30,98%
Nikkei 225	23	12,3	187%	34,70%
Total	***64***	***37,6***	***170%***	***35,28%***

Table 7. VaR Violations for Gaussian EWMA – Bi-weekly holding period with daily frequency

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	31	12,7	244%	31,70%
S&P 500	18	12,6	143%	19,05%
Nikkei 225	46	12,3	374%	24,41%
Total	***95***	***37,6***	***253%***	***25,77%***

Table 8. VaR Violations for HIST-EWMA VaR – Bi-weekly holding period with daily frequency

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	22	12,7	173%	27,10%
S&P 500	14	12,6	111%	21,71%
Nikkei 225	33	12,3	268%	20,41%
Total	***69***	***37,6***	***184%***	***22,81%***

Table 9. VaR Violations for Hybrid VaR – Bi-weekly holding period with daily frequency

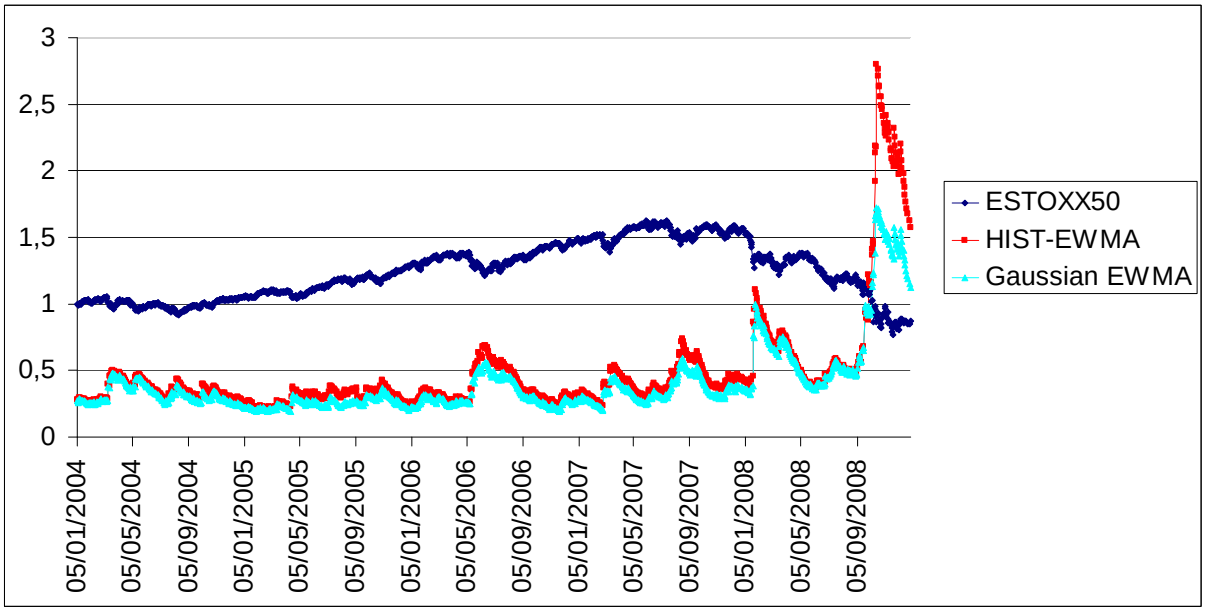
Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	10	12,7	79%	36,53%
S&P 500	13	12,6	103%	44,30%
Nikkei 225	16	12,3	130%	40,23%
Total	***39***	***37,6***	***104%***	***40,64%***

The backtesting results obtained in this case are striking. Here we list the most relevant evidence:

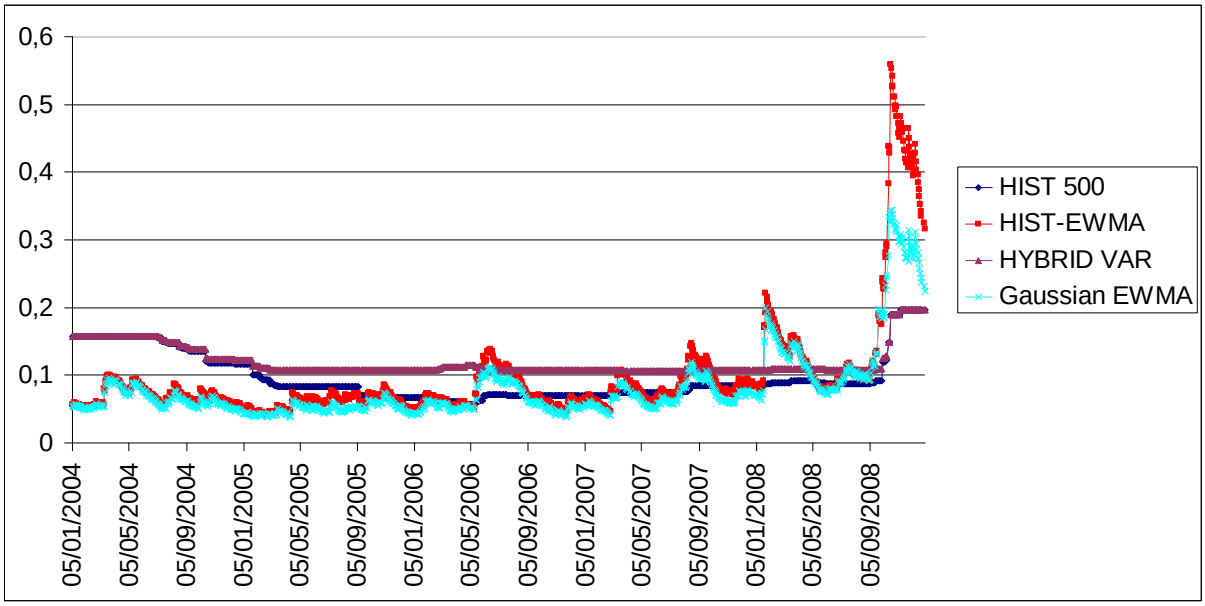
- While EWMA models (Gaussian and Historical) perform better in the daily holding period back-testing than Historical Simulation, this result is reversed in the two-week holding period. The Historical Simulation Model accumulates 64 violations, against the 95 (!) of the Gaussian EWMA and the 69 of the EWMA-Adjusted Historical Model. The only point in favour of the EWMA models is that the average size of the violation is lower than the Historical Model.
- Nonetheless, the Historical Model has a number of violations that remain high (1.7 times the expected number of violations).
- The Hybrid VaR performs extremely well, recording 39 violations out of the 38 expected!

The evidence of less pro-cyclicality for Hybrid VaR and of unsustainable variability for the EWMA models is confirmed in this case, by simply observing the following graphs.

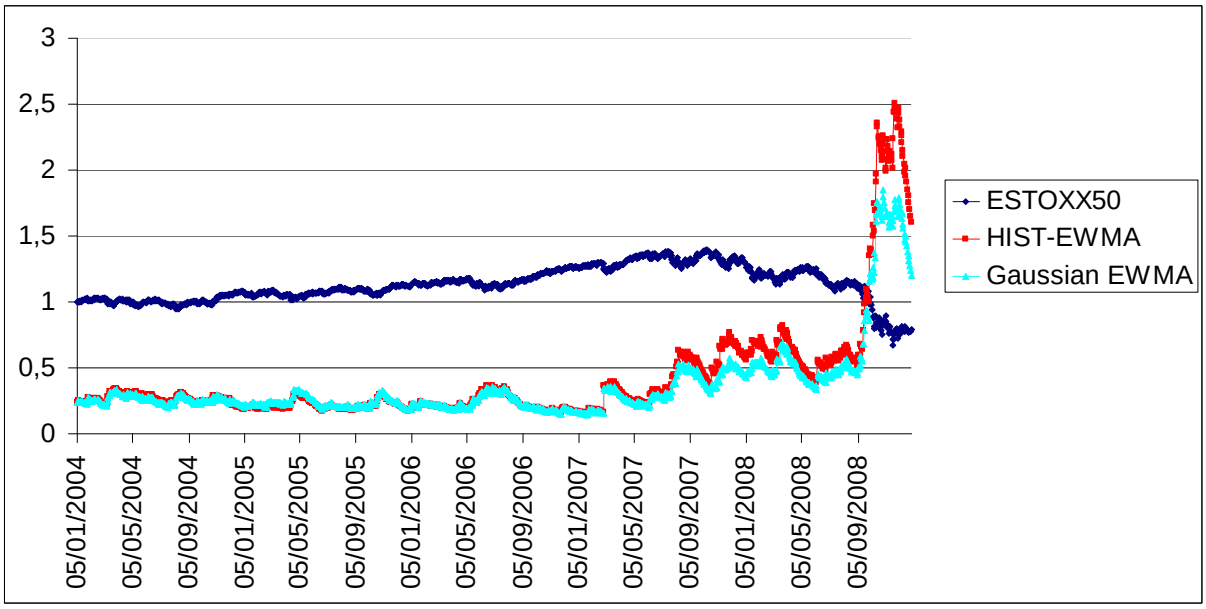
Graph 7. EuroStoxx50 and EWMA VaR



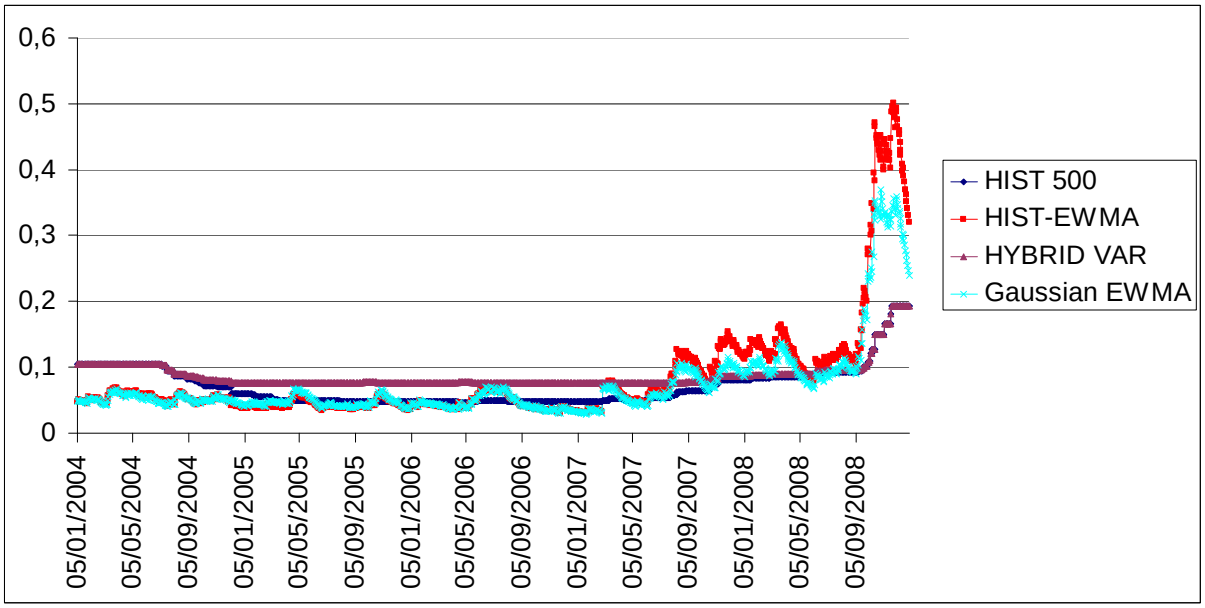
Graph 8. Comparison of the four VaR models for the Euro Stoxx 50



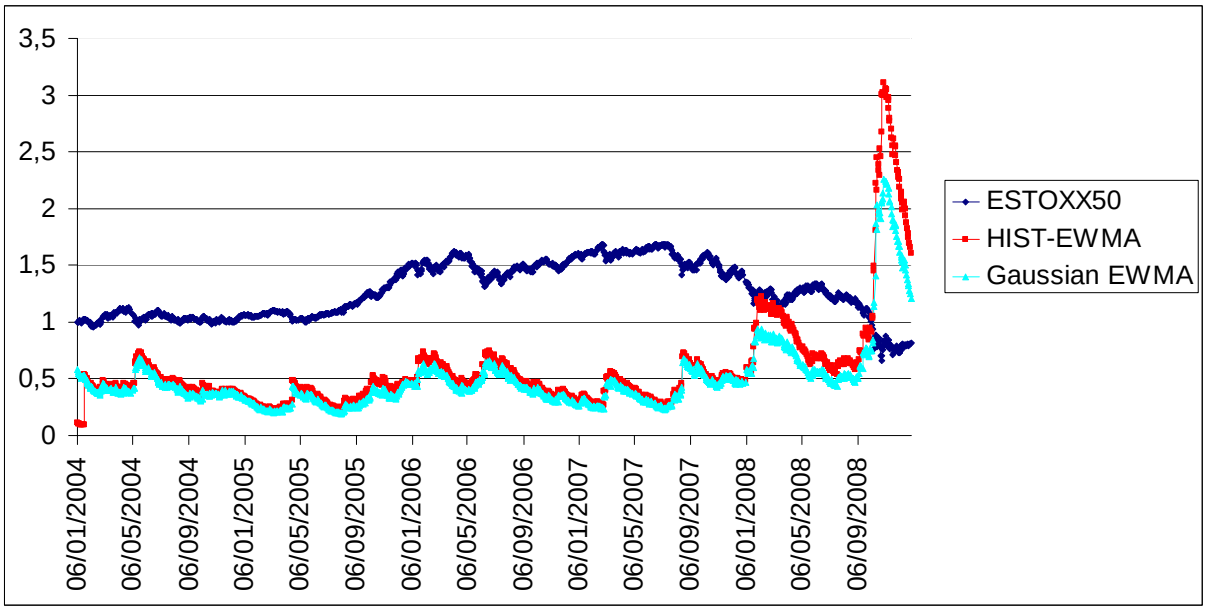
Graph 9. The S&P 500 and EWMA VaR



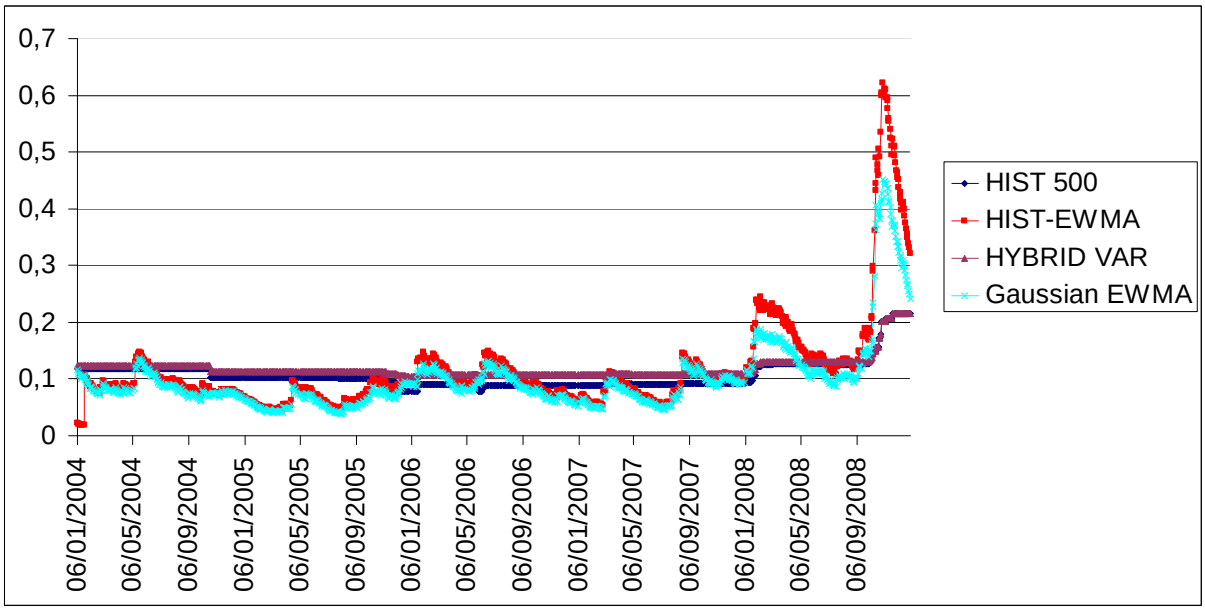
Graph 10. Comparison of the four VaR models for the S&P 500



Graph 11. The Nikkei 225 and EWMA VaR



Graph 12. Comparison of the four VaR models for the Nikkei 225



TRUE BI-WEEKLY BACK-TESTING

In this section we report the backtesting results of a two-week holding period with a "true" two-week (10 day) frequency. The number of observations is reduced by 10, but since we have 5 years of data, the total data is sufficiently representative.

The reader should now be familiar with our 4 back-testing tables.

Table 10. VaR Violations for Historical Simulation – True bi-weekly holding period

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	2	1,27	157%	61,62%
S&P 500	3	1,26	238%	23,91%
Nikkei 225	2	1,23	163%	53,73%
Total	***7***	***3,76***	***186%***	***43,20%***

Table 11. VaR Violations for Gaussian EWMA – True bi-weekly holding period

TRUE BI-WEEKLY - GEWMA	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	5	1,27	394%	29,58%
S&P 500	1	1,26	79%	30,62%
Nikkei 225	3	1,23	244%	45,49%
Total	***9***	***3,76***	***239%***	***35,00%***

Table 12. VaR Violations for HIST-EWMA VaR – True bi-weekly holding period

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	2	1,27	157%	37,25%
S&P 500	1	1,26	79%	27,26%
Nikkei 225	3	1,23	244%	26,12%
Total	***6***	***3,76***	***160%***	***30,02%***

Table 13. VaR Violations for Hybrid VaR – True bi-weekly holding period

Index	Effective Violations	Expected Violations	Ratio of Violations	Size of Violation
Euro Stoxx 50	2	1,27	157%	40,48%
S&P 500	2	1,26	159%	28,08%
Nikkei 225	1	1,23	81%	92,04%
Total	***5***	***3,76***	***133%***	***45,83%***

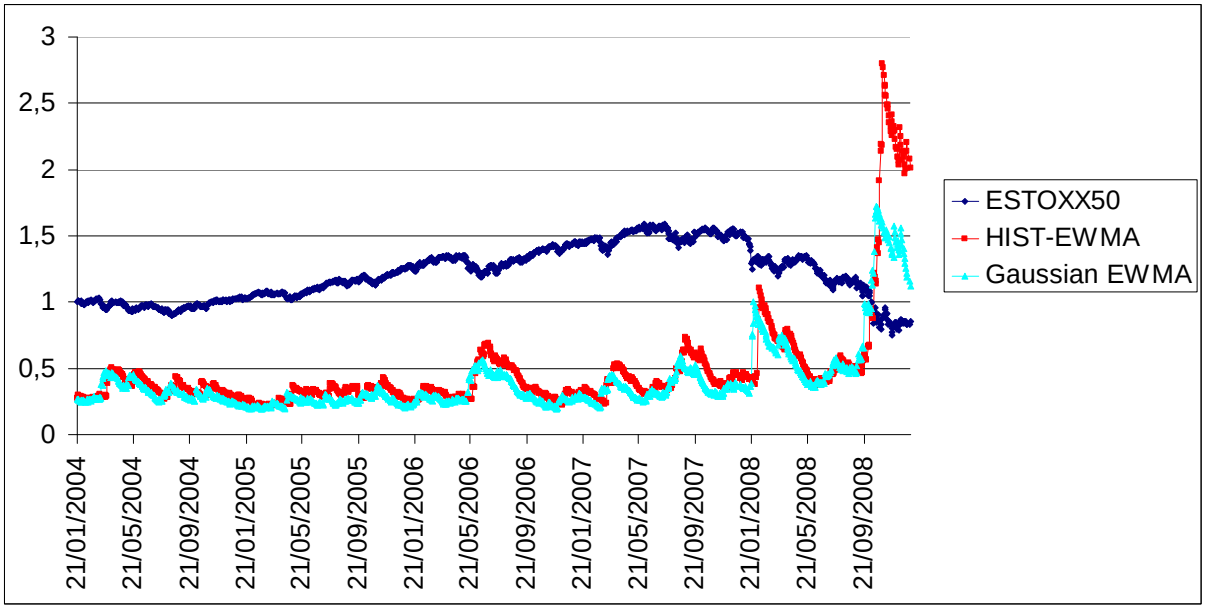
The analysis in the above tables confirms that Hybrid VaR has the lowest number of violations.

The Gaussian EWMA model continues to perform quite badly, and worse than Historical Simulation. The Ratio of Violations is not very different from the 250% recorded in the bi-weekly daily back-testing.

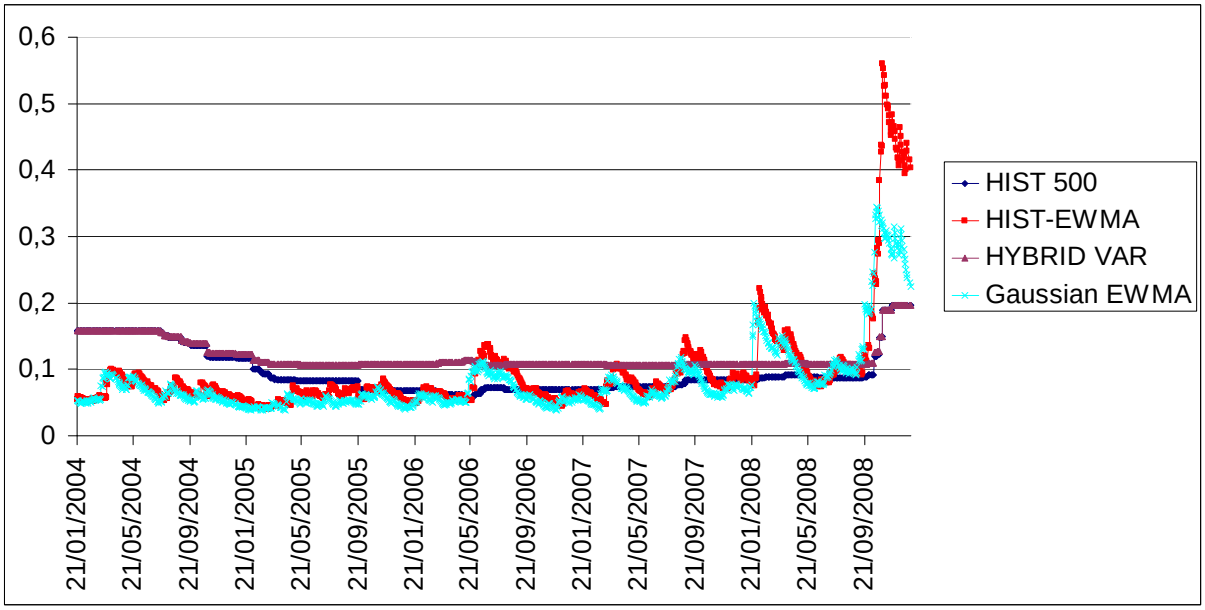
The EWMA-Adjusted Historical model performs better than the Historical model having one violation less and having the lowest size of violation.

From the point of view of pro-cyclicality, the graphs below again confirm the superiority of Hybrid VaR. The measure is more stable than any other measure – far more stable than the "hectic" EWMA measures. It is anti-cyclical in the good years of 2005-2006-2007, remaining consistently higher than Historical VaR in those periods.

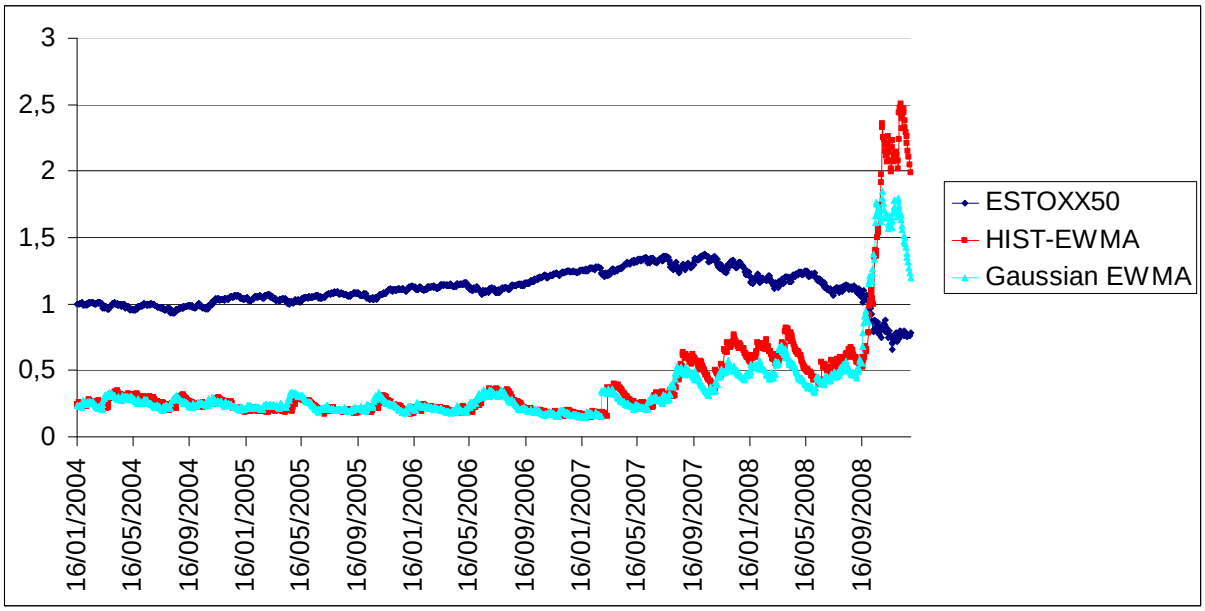
Graph 13. The Euro Stoxx50 and EWMA VaR



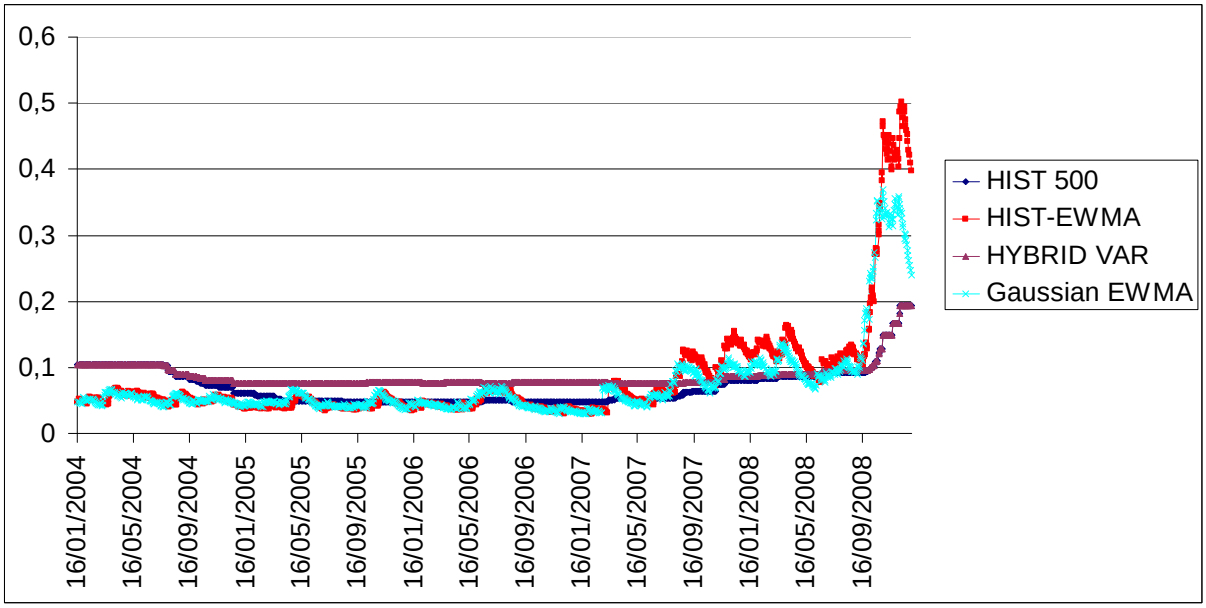
Graph 14. Comparison of the four VaR models for the Euro Stoxx 50



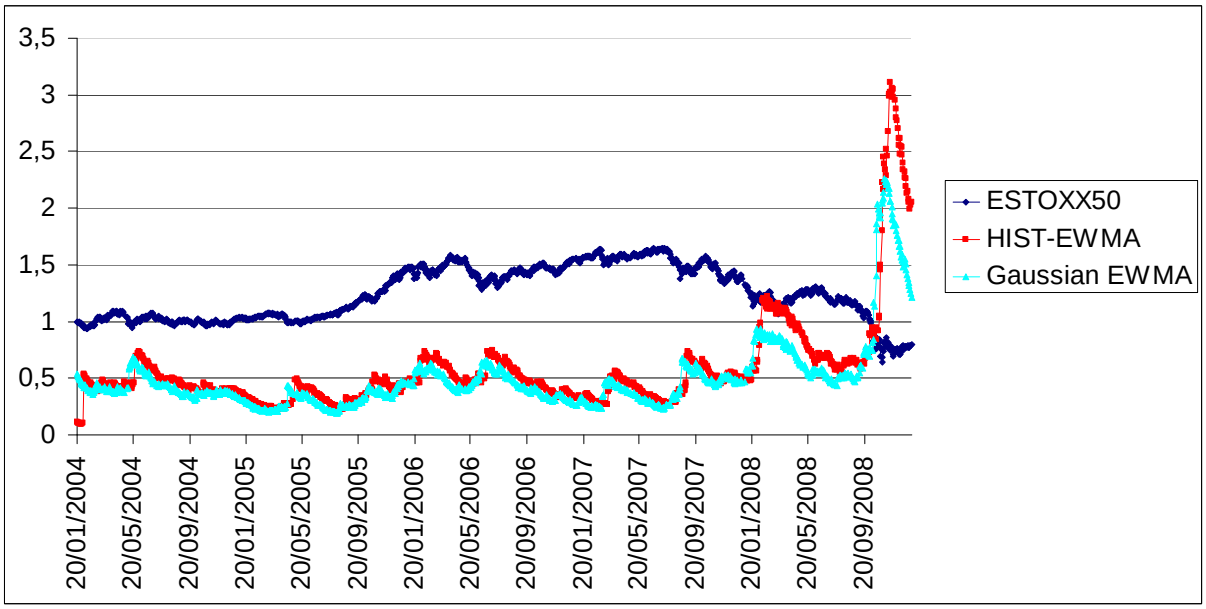
Graph 15.The S&P 500 and EWMA VaR



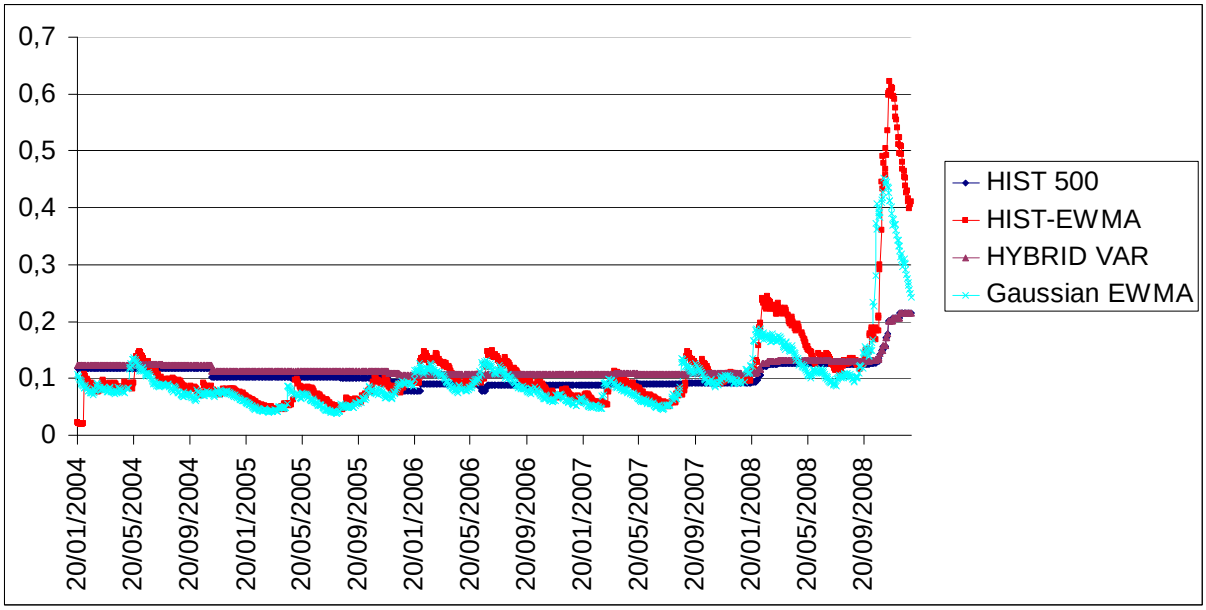
Graph 16. Comparison of the four VaR models for the S&P 500



Graph 17. The Nikkei 225 and EWMA VaR



Graph 18. Comparison of the four VaR models for the Nikkei 225



CONCLUSIONS

In this article we have introduced our new risk measure, Hybrid VaR, backtested it over the last 5 years for three major stock indices (Euro Stoxx 50, S&P 500, Nikkei 225), and compared it to other three models:

- Historical Model
- Gaussian Exponential Weighted
- Exponential-Weighting Adjustment of the Historical Model.

The results are significant:

1) Although there is consolidated opinion in the risk management arena that EWMA models work better than pure Historical models, this is not confirmed for the two-week holding period (it is confirmed for daily backtesting – modestly for Gaussian EWMA, more so for EWMA-Adjusted Historical).

2) Hybrid VaR exhibits the best performance in backtesting. Only the EWMA-Adjusted Model performs similarly in daily backtesting and in the *true* two-week backtesting. In two-week daily backtesting, Hybrid VaR remains largely superior.

3) When pro-cyclicality is considered, Hybrid VaR is the clear winner against the EWMA models and against the Historical Simulation model. The EWMA models are too volatile and unstable, especially in the last months of 2008 where risk literally exploded. The movements of VaR in EWMA models tend to follow market trends very closely, exaggerating them, which increases risk and leads to the liquidation of positions as risk limits are reached, thereby adding further market pressure. Hybrid VaR does not present the same variability. It was evidently anti-cyclical in the good years of 2005-2007, and though it increases unavoidably in 2008, it does so to a much lesser degree than the EWMA measures.

We can conclude that Hybrid VaR is the least pro-cyclical of the models considered in this paper, and the most reliable in terms of backtesting performance.

When VaR is used to assign risk limits it is essential that the measure is stable and possibly anti-cyclical. Stability is needed to avoid the frequent and potentially expensive re-adjustments of portfolio positions triggered by VaR swings.

The damage of pro-cyclicality has been widely discussed recently, and we cannot exclude that VaR methodologies have contributed to amplifying the market swings recorded in 2008.

Hybrid VaR seems to tackle these needs best and its anti-cyclicality in good times means that the measure discourages excessive risk-taking when *hubris* dominates, preventing problems, which is much more preferable than dealing with problems when it is too late and after *panic* has set in.