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Research Insight

Pitfalls in Risk Attribution

Ben Davis
Jose Menchero

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Introduction

A central challenge in investment management is balancing the tradeoff between risk and return. Meeting this challenge requires tools that help managers measure the contribution to portfolio risk from each return source. On this basis, managers can assess whether the risk from an investment decision is justified by the expected return.

An intuitive framework for ensuring consistency between risk and return attribution was described by Menchero and Davis (2010). This framework identifies three distinct drivers of risk arising from a given return source. The first is the size of the source exposure; naturally, more aggressive positions lead to greater risk contributions. The second driver is the standalone volatility of the return source; more volatile return sources have a correspondingly larger impact on risk. The third driver is the correlation between the return source and the portfolio, that is, the extent to which a given return source increases or decreases risk depends on its tendency to move with or against the portfolio. Therefore, the critical first step in risk attribution is for managers to clearly identify the return sources that match the investment process.

For active portfolio management, most practitioners recognize that return sources should be defined on a benchmark-relative basis. Cash provides a clear illustration of the importance of properly distinguishing between absolute and benchmark-relative returns. In down-markets, the relative return of cash is positive, and a cash position contributes positively to active return. In up-markets, by contrast, cash drag can detract significantly from performance. In either case, the decision to hold cash is clearly a major driver of tracking error. Although this point is well understood and appreciated, many risk attribution models utilize absolute-return sources that treat cash as making zero contribution to active risk. This highlights a mismatch between the sources of risk in these models and the sources of return typically considered by active managers.

The need to distinguish between absolute and relative sources is widely recognized in the context of sector-based performance attribution. In the standard model of Brinson and Fachler (1985), the allocation effect is defined as the product of the sector active weight and the benchmark-relative sector return. In this model, positive allocation effect is achieved by overweighting sectors that outperform the overall benchmark, or by underweighting underperforming sectors. By contrast, in Brinson, Hood, and Beebower (1986), the authors describe a variant of the standard model wherein the allocation effect is given by the product of the sector active weight and the *absolute* sector return. A drawback of Brinson, Hood, Beebower from the standpoint of performance attribution is that a manager is awarded positive allocation effect by overweighting a sector with a positive return, *even if that sector underperforms the benchmark*. This drawback underscores the importance of using benchmark-relative returns for analyzing active portfolios.

Although less widely recognized, similar effects also appear in single-country factor models. In these models, industry factors are typically represented by portfolios with a 100 percent net long position in the given industry. In particular, a positive active exposure to an industry factor with positive return is awarded a positive return contribution, even if that industry factor portfolio underperforms the benchmark. Hence, these factors may represent non-intuitive return sources for attribution of active portfolios. As we show in this article however, such factor models can be modified to treat industry factors on a benchmark-relative basis.

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Our experience is that practitioners often conduct performance analysis on a benchmark-relative basis, while doing risk analysis on an absolute-return basis. This mismatch between sources of risk and return leads to a pitfall—active management decisions cannot be evaluated on a risk-adjusted basis. Even if absolute-return sources are applied consistently across risk and performance, as in the case of single-country industry factors, this often leads to non-intuitive results, as active management decisions are more appropriately described using relative-return sources.

In this article, we summarize the framework of risk-adjusted performance attribution. We stress the importance of attributing risk and return to the same underlying sources. We define and compare absolute and relative sources for securities, sectors, and factors. We provide detailed examples that illustrate risk attribution along these various dimensions, and argue that benchmark-relative return sources are more appropriate for analyzing active portfolios. In particular, several puzzling and non-intuitive effects associated with absolute-return sources disappear when switching to relative-return sources.

The Risk Attribution Framework

To assess the trade-off between risk and return, it is crucial to align the risk and performance attributions along the same investment decision variables. We summarize here the *x-sigma-rho* framework for aligning risk and performance, as described by Menchero and Davis (2010). In this approach, attribution of the active return is written in general form as

$$R^A = \sum_m x_m g_m, \tag{1}$$

where x_m is the portfolio exposure to source m, and g_m is the return of the source. The source exposures are known at the start of the analysis period, while the source returns are realized only at the end.

The risk attribution corresponding to the general return attribution of Equation (1) is given by the *x-sigma-rho* formula,

$$\sigma(R^A) = \sum_m x_m \sigma(g_m) \rho(g_m, R^A), \tag{2}$$

where $\sigma(g_m)$ is the stand-alone volatility of the source, and $\rho(g_m, R^A)$ is the correlation of the source with the active portfolio. The marginal contribution to active risk (MCAR) of source *m* can be easily computed in the *x-sigma-rho* framework by means of the formula

$$MCAR_m = \sigma(g_m) \rho(g_m, R^A), \tag{3}$$

from which we also obtain the well-known risk decomposition

$$\sigma(R^A) = \sum_m x_m MCAR_m \,, \tag{4}$$

as described by Litterman (1996).

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An important measure of risk-adjusted performance is the information ratio, which is the active return divided by the tracking error. In an *ex ante* analysis, the distribution of source returns are forecast, and the information ratio is quoted on the basis of expected returns,

$$IR = \frac{E[R^A]}{\sigma(R^A)}. \tag{5}$$

As described by Menchero (2006/2007), Equation (5) can be rewritten to gain insight into the sources of risk-adjusted performance,

$$IR = \sum_m \left(\frac{x_m MCAR_m}{\sigma(R^A)}\right)\left(\frac{E[g_m]}{MCAR_m}\right). \tag{6}$$

The first term is interpreted as the *risk weight* of source *m*; note that the risk weights sum to 100 percent. The second term is called the *component information ratio* of source *m*, and represents the ratio of the return contribution to the risk contribution coming from the source.

Conceptually, the information ratio can be maximized by progressively increasing the risk weight of sources with high component IR, while decreasing the risk weight of sources with low component IR. In the absence of constraints, one may proceed in this fashion until all sources have identical component information ratios, at which point no trade could increase the portfolio IR. Therefore, the expected return of each source is proportional to the MCAR with respect to the optimal portfolio,

$$E[g_m] = IR^{opt} \cdot MCAR_m^{opt}, \tag{7}$$

where the constant of proportionality IR^{opt} is the optimal portfolio information ratio.

If we assume that the actual portfolio is optimal, then Equation (7) can be used to compute the implied returns of each source. In other words, the manager must be bullish on sources with positive MCAR, and bearish on sources with negative MCAR. Otherwise, it would be possible to adjust source exposures to increase expected return and reduce risk, which would imply that the portfolio is not optimal, contrary to our initial assumption.

While this framework is thus very useful for understanding the sources of risk-adjusted performance, it is contingent upon using the same sources for both return and risk. In particular, the framework for analyzing risk-adjusted return fails when risk and return are attributed to different sources.

Sector-Based Attribution

A top-down sector-based investment process consists of two types of decisions by the managers. The first is sector allocation, where the manager seeks to outperform the benchmark by active sector weighting. The second is security selection within sectors, where the objective is to overweight the outperforming assets within the sector, and underweight the underperformers.

The Brinson, Hood, Beebower performance attribution is given by

$$R^A = \sum_i (w_i^P - w_i^B) R_i^B + \sum_i w_i^P (R_i^P - R_i^B), \tag{8}$$

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where w_i^P, R_i^P, w_i^B, and R_i^B are the portfolio and benchmark sector weights and returns, respectively. As previously noted, a drawback to this model is that positive allocation effect is earned by overweighting a sector with a positive return, even when that sector underperforms the benchmark.

Applying the *x-sigma-rho* formula to Equation (8) immediately produces the risk attribution

$$\sigma(R^A) = \sum_i (w_i^P - w_i^B)\sigma(R_i^B)\rho(R_i^B, R^A) + \sum_i w_i^P \sigma(R_i^P - R_i^B)\rho(R_i^P - R_i^B, R^A). \tag{9}$$

The first sum represents the total tracking error due to allocation decisions; the second sum gives the tracking error due to security selection.

To compare and contrast the various risk attribution models, we consider the following example. The analysis period is October 2009, and the US dollar is the base currency. The benchmark is MSCI *World Value Index* (World Value). The portfolio is 95 percent MSCI *World Growth Index* (World Growth), with a 5 percent cash position in US dollars. Annualized risk forecasts are obtained using the Barra Global Equity Model (GEM2L), a multi-factor model described by Menchero, Morozov, and Shepard (2010). The forecast benchmark risk is 30.74 percent, the predicted tracking error is 6.73 percent, and the active beta with respect to the benchmark is -0.18 (i.e., the portfolio beta is 0.82).

An *x-sigma-rho* risk attribution report using the Brinson, Hood, Beebower model is presented in Table 1. We segment the market according to Global Industry Classification Standard (GICS®) economic sectors and cash. In this model, the 5 percent active cash position makes zero contribution to tracking error. This is because the absolute return of cash is zero, and hence has zero volatility. Allocation to Financials makes by far the greatest contribution to active risk (879 bps), which is seen via the *x-sigma-rho* formula as resulting from the large underweight (-26%) of the highly volatile (38.66%) Financials sector, which is negatively correlated (-0.86) with the active portfolio. By contrast, the large overweight of the Information Technology (IT) sector actually reduced tracking error by 3.0 percent within this model.

Sector Name	Portfolio Weight	Bench Weight	Active Weight	Absolute Return Vol	Absolute Return Corr	Alloc Risk Contrib	Active Return Vol	Active Return Corr	Selec Risk Contrib	Total Risk Contrib
Energy	0.07	0.14	-0.06	31.32%	-0.70	1.41%	12.64%	-0.23	-0.21%	1.20%
Materials	0.08	0.05	0.03	38.24%	-0.73	-0.87%	12.09%	0.38	0.39%	-0.48%
Industrials	0.09	0.11	-0.02	32.84%	-0.79	0.45%	7.84%	0.53	0.39%	0.84%
Cons Disc	0.11	0.07	0.03	31.48%	-0.76	-0.81%	6.44%	0.30	0.21%	-0.61%
Cons Stpls	0.15	0.05	0.10	22.21%	-0.74	-1.68%	6.86%	0.21	0.22%	-1.46%
Health Care	0.13	0.07	0.06	22.58%	-0.70	-0.93%	8.15%	0.16	0.16%	-0.77%
Financials	0.08	0.34	-0.26	38.66%	-0.86	8.79%	9.61%	0.61	0.46%	9.25%
IT	0.19	0.03	0.16	29.32%	-0.66	-3.00%	10.34%	0.07	0.13%	-2.86%
Telecom	0.02	0.06	-0.04	24.37%	-0.76	0.70%	14.39%	0.02	0.01%	0.71%
Utilities	0.02	0.07	-0.05	23.38%	-0.74	0.86%	9.28%	0.26	0.05%	0.91%
Cash	0.05	0.00	0.05	0.00%	0.00	0.00%	0.00%	0.00	0.00%	0.00%
Total	1.00	1.00	0.00	5.65%	0.87	4.92%	3.31%	0.55	1.81%	6.73%

Table 1: Brinson, Hood, Beebower risk attribution (absolute sources).

Note also that the absolute return of every sector is negatively correlated with the active portfolio. This is because the sectors have positive beta, whereas the active portfolio has a negative beta. Assuming optimality, the negative correlation implies, via Equation (3) and Equation (7), that the expected return of every sector is negative. This view is consistent with the fact that the active portfolio, which has positive expected return, has negative active beta. While this explanation is internally consistent, it is

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not useful, since active managers are primarily concerned with the performance of sectors *relative to* the overall benchmark.

Most practitioners recognize the shortcoming of the Brinson, Hood, Beebower model, and instead use the Brinson and Fachler sector-based model. This model attributes active returns as

$$R^A = \sum_i \left(w_i^P - w_i^B\right)\left(R_i^B - R^B\right) + \sum_i w_i^P\left(R_i^P - R_i^B\right), \qquad (10)$$

where R^B is the overall benchmark return. The only difference between Equation (10) and Equation (8) is the use of relative benchmark sector returns to compute the sector allocation effects. Nevertheless, this seemingly minor difference has a major impact on interpreting the sources of active risk, as we now demonstrate.

An example of risk attribution using the Brinson and Fachler model is presented in Table 2. In contrast to Table 1, the allocation to the Cash sector now makes a large contribution to tracking error (127 bps). This is due to the positive exposure (5%) to the highly volatile (30.74%) relative return of cash, which is positively correlated (0.83) with the active portfolio. Note that since cash relative return is simply the negative of benchmark return, these two sources have precisely the same volatility. The positive correlation of cash with the active portfolio can be understood by noting that when cash outperforms the benchmark, the active return also tends to be positive due to the negative beta of the active portfolio.

Sector Name	Portfolio Weight	Bench Weight	Active Weight	Relative Return Vol	Relative Return Corr	Alloc Risk Contrib	Active Return Vol	Active Return Corr	Selec Risk Contrib	Total Risk Contrib
Energy	0.07	0.14	-0.06	13.19%	0.26	-0.22%	12.64%	-0.23	-0.21%	-0.43%
Materials	0.08	0.05	0.03	12.43%	-0.19	-0.07%	12.09%	0.38	0.39%	0.31%
Industrials	0.09	0.11	-0.02	7.15%	-0.06	0.01%	7.84%	0.53	0.39%	0.40%
Cons Disc	0.11	0.07	0.03	8.16%	0.17	0.05%	6.44%	0.30	0.21%	0.25%
Cons Stpls	0.15	0.05	0.10	13.43%	0.68	0.93%	6.86%	0.21	0.22%	1.15%
Health Care	0.13	0.07	0.06	15.68%	0.62	0.58%	8.15%	0.16	0.16%	0.74%
Financials	0.08	0.34	-0.26	10.22%	-0.77	2.08%	9.61%	0.61	0.46%	2.55%
IT	0.19	0.03	0.16	13.16%	0.47	0.96%	10.34%	0.07	0.13%	1.10%
Telecom	0.02	0.06	-0.04	13.80%	0.51	-0.27%	14.39%	0.02	0.01%	-0.26%
Utilities	0.02	0.07	-0.05	13.80%	0.59	-0.40%	9.28%	0.26	0.05%	-0.35%
Cash	0.05	0.00	0.05	30.74%	0.83	1.27%	0.00%	0.00	0.00%	1.27%
Total	1.00	1.00	0.00	5.65%	0.87	4.92%	3.31%	0.55	1.81%	6.73%

Table 2: Brinson and Fachler risk attribution (relative sources).

The allocation decision to underweight Financials still makes the greatest contribution to active risk (208 bps), although it is far less than the 879 bps reported in Table 1. Also in contrast to Table 1, many of the sector relative returns are now positively correlated with the active return. For instance, the IT sector now has a positive correlation (0.47) with the active return, meaning the portfolio tends to outperform the benchmark when IT performs well on a relative basis. This is perfectly intuitive, considering the portfolio is 16 percent overweight of the IT sector. In fact, most sectors with positive (negative) active weights tend to be positively (negatively) correlated with active returns, thus leading to positive risk contributions. A notable exception is the Utilities sector, where the portfolio is 5 percent underweight and yet the correlation is positive (0.59). The positive correlation can be explained in terms of beta. More specifically, Utility stocks tend to have low beta, which means that the *relative return* of Utilities has negative beta. The active portfolio also has a negative beta, so it tends to co-move with the relative return of Utilities.

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Security-Based Attribution

In a bottom-up investment process, the portfolio manager seeks to outperform the benchmark by weighting individual securities. One way to attribute active return is on the basis of security absolute return,

$$R^A = \sum_n w_n^A r_n \, , \tag{11}$$

where r_n is absolute return of security n. However, a drawback to this model is that positive security selection is earned by overweighting any security with a positive return, even if that security underperforms the benchmark.

Applying the *x-sigma-rho* formula to Equation (11), we obtain

$$\sigma(R^A) = \sum_n w_n^A \sigma(r_n) \rho(r_n, R^A). \tag{12}$$

Here, $\sigma(r_n)$ represents the volatility of the absolute return, and $\rho(r_n, R^A)$ is the correlation between security absolute return and active portfolio return. By Equation (3), the MCAR of security absolute return is given by

$$MCAR_{n,abs} = \sigma(r_n) \rho(r_n, R^A), \tag{13}$$

which gauges the sensitivity of active portfolio risk to small changes in exposure to the absolute return source.

An example of risk attribution based on security absolute return is presented in Table 3. Since the absolute return of the US Dollar is zero, it has zero volatility, and therefore the 5 percent cash position makes no contribution to active risk. Among stocks, Bank of America is reported as the riskiest position, Exxon Mobil as second riskiest, and Microsoft is the most diversifying.

Asset Name	Port Weight	Bench Weight	Active Weight	Absolute Vol	Absolute Corr	Absolute MCAR	Active Risk Contrib
BANK OF AMERICA	0.00%	1.38%	-1.38%	76.10%	-0.65	-49.64%	0.68%
EXXON MOBIL	0.00%	3.15%	-3.15%	31.98%	-0.59	-18.79%	0.59%
GENERAL ELECTRIC	0.00%	1.64%	-1.64%	51.47%	-0.68	-35.08%	0.57%
JPMORGAN CHASE	0.00%	1.55%	-1.55%	54.08%	-0.65	-35.15%	0.55%
HSBC	0.00%	1.87%	-1.87%	42.17%	-0.67	-28.40%	0.53%
…	…	…	…	…	…	…	…
CISCO	1.26%	0.00%	1.26%	36.00%	-0.55	-19.67%	-0.25%
IBM	1.47%	0.00%	1.47%	33.78%	-0.51	-17.16%	-0.25%
APPLE	1.54%	0.00%	1.54%	37.67%	-0.46	-17.26%	-0.27%
BHP BILLITON	1.04%	0.00%	1.04%	55.84%	-0.47	-26.35%	-0.27%
MICROSOFT	1.93%	0.00%	1.93%	34.87%	-0.46	-15.97%	-0.31%
Total							6.73%

Table 3: Security-based risk attribution using absolute return sources.

Notice that the correlation and absolute MCAR of every stock is negative. This behavior is identical to what we saw in the Brinson, Hood, Beebower example in Table 1, and can be understood in terms of beta: All the stocks have a significant positive beta, meaning the stock absolute returns tend to move

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with the benchmark. On the other hand, the active portfolio has a negative beta, meaning the active returns tend to move against the benchmark. Thus, the stock absolute returns are negatively correlated with the active portfolio, and hence the stock absolute MCARs must be negative as well by Equation (13).

Assuming optimality, Equation (7) shows that the negative MCARs imply that all stock expected returns are negative, which further implies that the expected return of the benchmark is negative. However, this analysis gives no insight into the expected return of each security relative to the benchmark, which is of central importance to the active manager.

In order to address this shortcoming, we attribute the active return R^A for a bottom-up process as

$$R^A = \sum_n w_n^A (r_n - R^B), \tag{14}$$

where w_n^A is the active weight of security n, and $r_n - R^B$ is the *relative return of the security*. Equation (14) expresses the intuitive result that positive active return is achieved by overweighting securities that outperform the benchmark.

Applying the *x-sigma-rho* formula to Equation (14), we immediately obtain

$$\sigma(R^A) = \sum_n w_n^A \sigma(r_n - R^B) \rho(r_n - R^B, R^A). \tag{15}$$

Here, $\sigma(r_n - R^B)$ represents the volatility of the relative return, and $\rho(r_n - R^B, R^A)$ is the correlation between security relative return and active portfolio return. By Equation (3), the relative MCAR of security *n* relative return is given by

$$MCAR_{n,rel} = \sigma(r_n - R^B) \rho(r_n - R^B, R^A). \tag{16}$$

Equation (16) gauges the sensitivity of active portfolio risk to small changes in exposure to the relative return source. For a security with a negative relative MCAR, a small increase in weight will reduce active portfolio risk.

An example of risk attribution based on security relative return is presented in Table 4. The securities are sorted by tracking error contribution, resulting in a seemingly random permutation of Table 3. The US Dollar is reported as the riskiest position, contributing 127 bps to tracking error, in precise agreement with the Cash allocation effect in Table 2. As in Table 3, Bank of America is the riskiest position, but Microsoft jumps from most diversifying to fifth riskiest, while Exxon Mobil drops from second riskiest to most diversifying.

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Asset Name	Port Weight	Bench Weight	Active Weight	Relative Vol	Relative Corr	Relative MCAR	Active Risk Contrib
US Dollar	5.00%	0.00%	5.00%	30.74%	0.83	25.48%	1.27%
BANK OF AMERICA	0.00%	1.38%	-1.38%	59.24%	-0.41	-24.16%	0.33%
CITIGROUP	0.00%	0.73%	-0.73%	80.49%	-0.42	-33.85%	0.25%
NESTLE	1.44%	0.00%	1.44%	26.64%	0.51	13.52%	0.19%
MICROSOFT	1.93%	0.00%	1.93%	27.16%	0.35	9.51%	0.18%
…	…	…	…	…	…	…	…
PFIZER	0.00%	1.05%	-1.05%	26.13%	0.32	8.31%	-0.09%
TOYOTA MOTOR	0.00%	1.04%	-1.04%	29.19%	0.30	8.76%	-0.09%
BP	0.00%	1.56%	-1.56%	25.16%	0.23	5.85%	-0.09%
AT&T	0.00%	1.50%	-1.50%	25.28%	0.27	6.72%	-0.10%
EXXON MOBIL	0.00%	3.15%	-3.15%	24.34%	0.27	6.69%	-0.21%
Total							6.73%

Table 4: Security-based risk attribution using relative return sources.

The differences in rankings between the two security-based models can be understood intuitively by comparing the betas of the underlying sources of return. As of October 2009, energy stocks tended to be low beta. In particular, Exxon Mobil had an absolute return beta of less than 1 (though still positive), implying that the relative return beta was *negative*. This explains why wildly different risk impacts can be attributed to the Exxon Mobil position: The absolute and relative return contributions tend to move in opposite directions. On the other hand, Bank of America was a high-beta stock, so the absolute and relative returns both have *positive beta*, thus explaining the positive risk contribution of the Bank of America position in both Tables 3 and 4.

Assuming optimality, Equation (7) can be used to obtain implied relative returns. In particular, the US Dollar relative MCAR (25.48%) implies that the Dollar is expected to outperform the benchmark, which is equivalent to the view that the benchmark expected return is negative. The negative Bank of America relative MCAR implies that this stock is expected to underperform the benchmark. This is intuitive because Bank of America is a high beta stock, and the benchmark has a negative expected return.

There is a precise relation between security absolute and relative MCAR, namely,

$$MCAR_{n,abs} = MCAR_{n,rel} - MCAR_{cash,rel}. \tag{17}$$

For instance, the Exxon Mobil absolute MCAR (-18.79%) is precisely the difference between the Exxon Mobil relative MCAR (6.69%) and the US Dollar relative MCAR (25.48%). This relation may be understood by considering the corresponding return sources: Security absolute return is precisely equal to the difference between security and cash relative returns,

$$r_n = (r_n - R^B) - (0 - R^B), \tag{18}$$

and Equation (17) follows immediately from the portfolio property of MCAR.

There is an interesting subtlety in risk-adjusted attribution that is absent from simple performance attribution. From Equation (14), it seems that active management is solely a matter of overweighting outperformers, and underweighting underperformers. The reality is a bit more complex. For instance, from Table 4 we see that Exxon Mobile has a positive MCAR, indicating that the portfolio manager believes this stock will outperform. Nevertheless, the manager is *underweighting* the stock! The reader might ask how this is optimal. The key here is to recognize that Exxon Mobil serves as a *hedge* for other

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parts of the portfolio. In other words, the manager is willing to accept an expected loss on the stock because holding the stock actually reduces the tracking error by 21 bps.

Factor-Based Attribution

Factor models decompose excess return into a systematic component that is explained by factors and a residual component that is not,

$$r_n = \sum_k X_{nk} f_k + u_n. \tag{19}$$

Here, the factor exposure matrix X_{nk} is the exposure of asset n to factor k, f_k is the factor return, and u_n is the asset residual return. Portfolio active return is attributed by rolling up contributions from the asset level,

$$R^A = \sum_k X_k^A f_k + \sum_n w_n^A u_n \ , \tag{20}$$

where w_n^A is the weight of asset *n*, and X_k^A is the exposure to factor *k*,

$$X_k^A = \sum_n w_n^A X_{nk}. \tag{21}$$

Positive contributions to active return are earned through positive exposure to factors with positive returns and by overweighting assets with positive residual returns.

Factor exposures are developed from fundamental characteristics of the assets. For instance, the industry exposures are usually taken as (0,1) indicator variables that reflect the industry membership of the securities. The portfolio industry factor exposure is simply the portfolio weight held in that industry. Style factor exposures are constructed from intuitive stock attributes called descriptors. For instance, price-to-book ratio, earnings yield, and dividend yield are all characteristics that can be used to identify value stocks. For the Volatility factor, the dominant descriptor is historical beta, whereas for Size it is log of market cap. Momentum descriptors include relative strength of the stock over the trailing 6-12 months. We standardize style factor exposures to be benchmark cap-weighted mean zero and standard deviation 1. As a consequence, the benchmark has zero exposure to all styles, and other portfolios have positive or negative exposures to the extent that they tilt on a particular factor. For instance, a portfolio tilted to high-beta stocks will have positive exposure to the Volatility factor.

Factor returns are estimated by cross-sectional regression at the end of the period. Factor returns can be written in the general form

$$f_k = \sum_n \Omega_{kn} r_n \, , \tag{22}$$

where Ω_{kn} is the weight of stock n in *factor replicating portfolio* k. When the factor exposure matrix has full rank, each factor replicating portfolio has unit exposure to the underlying factor, and net exposure zero to all other factors. In particular, since a style factor replicating portfolio has net exposure zero to all industries, it must be a long-short portfolio with net zero weight within each industry. On the other hand, since an industry factor-replicating portfolio has unit exposure to the

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underlying industry, it must be a net 100 percent long portfolio. The precise relationship between sector and factor models is detailed in Davis and Menchero (2010).

Applying the *x-sigma-rho* formula to Equation (20) immediately produces the risk attribution

$$\sigma(R^A) = \sum_k X_k^A \sigma(f_k) \rho(f_k, R^A) + \sum_n w_n^A \sigma(u_n) \rho(u_n, R^A). \tag{23}$$

A portfolio manager following a factor-based process may make investment decisions along a set of customized factors that differ from those used to estimate a risk model. In this case, the custom factor volatilities and correlations are obtained by applying the risk model to the custom factor replicating portfolios, as described by Menchero and Poduri (2008).

In Table 5, we present a custom factor risk attribution for a model comprising 10 industry factors based on the GICS sectors, and 4 style factors based on the Barra Global Equity (GEM2) risk model. The industry factor replicating portfolios are 100 percent net long, and hence represent absolute sources of return. Note that every industry factor has a strong negative correlation and negative MCAR. This non-intuitive effect is exactly analogous to the negative correlations seen in Table 1 and Table 3, which also employ absolute return sources. For instance, in Table 5, the Utilities factor contributes 115 bps to active risk; this is in qualitative agreement with the 86 bps contribution reported in Table 1 using the Brinson, Hood, Beebower model.

Source	Port Exp	Bench Exp	Active Exposure	Factor Volatility	Factor Corr	Factor MCAR	Active Risk Contrib
Energy	0.07	0.14	-0.06	34.35%	-0.71	-24.35%	1.56%
Materials	0.08	0.05	0.03	34.05%	-0.73	-24.69%	-0.77%
Industrials	0.09	0.11	-0.02	32.33%	-0.79	-25.55%	0.45%
ConsDscr	0.11	0.07	0.03	30.43%	-0.77	-23.50%	-0.79%
ConsStpls	0.15	0.05	0.10	30.00%	-0.79	-23.79%	-2.44%
HealthCare	0.13	0.07	0.06	28.72%	-0.75	-21.49%	-1.28%
Financials	0.08	0.34	-0.26	33.38%	-0.86	-28.81%	7.58%
IT	0.19	0.03	0.16	30.14%	-0.67	-20.12%	-3.13%
Telecom	0.02	0.06	-0.04	30.58%	-0.77	-23.65%	0.90%
Utilities	0.02	0.07	-0.05	29.32%	-0.79	-23.25%	1.15%
Momentum	-0.05	0.00	-0.05	7.71%	0.17	1.33%	-0.07%
Volatility	-0.34	0.00	-0.34	10.67%	-0.72	-7.63%	2.59%
Value	-0.48	0.00	-0.48	4.24%	-0.21	-0.90%	0.43%
Size	-0.07	0.00	-0.07	3.93%	-0.24	-0.93%	0.07%
Residual	1.00	1.00	1.00	3.24%	0.15	0.48%	0.48%
Total				6.73%	1.00	6.73%	6.73%

Table 5: Factor-based risk attribution (absolute sources for industries).

From the perspective of active management however, active return is best described using benchmark-relative return sources. This can be accomplished by augmenting the model with a *World factor*, as we now describe. All equities have unit exposure to this factor, while cash has an exposure of zero. The portfolio exposure to this factor therefore records the portfolio weight held in equities. The World factor introduces an exact collinearity into the factor structure: namely, the World exposure is equal to the sum of the industry exposures. To obtain a unique regression solution, we impose the constraint that the benchmark cap-weighted industry factor returns sum to zero,

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$$\sum_{k \in Ind} w_k^B \tilde{f}_k = 0, \tag{24}$$

where we use $\tilde{f}_k$ to distinguish factor returns in the augmented model from the factor returns f_k in the model lacking the World factor. The introduction of the World factor offsets the industry factor returns

$$\tilde{f}_k = f_k - \tilde{f}_{World}, \tag{25}$$

but has no impact on style factor returns. Consequently, the MCARs of the industry factors in the presence of the World factor are offset:

$$MCAR_{k,rel} = MCAR_{k,abs} - MCAR_{World}. \tag{26}$$

Moreover, when the regression weights are equal to the cap-weights of the benchmark, the return of the World factor is precisely equal to the benchmark return, i.e., $\tilde{f}_{World} = R^B$. Therefore, the MCAR of the World factor is equal in magnitude to the relative MCAR of cash, but opposite in sign. As a result, the introduction of the World factor causes the MCARs of the industry factors to become offset by the relative MCAR of cash, just as in Equation (17) in the case of securities:

$$MCAR_{k,abs} = MCAR_{k,rel} - MCAR_{cash,rel}. \tag{27}$$

In Table 6, we augment the model used in Table 5 with a World factor. As expected, the MCAR of the World factor (-25.48%) is precisely the opposite of the US Dollar relative MCAR reported in Table 4. Furthermore, the active risk contribution of the World factor (127 bps) is precisely equal to the Cash Allocation effect in the Brinson and Fachler attribution in Table 2. Following Equation (27), the Utilities factor absolute MCAR (-23.25%) is the difference of the Utilities factor relative MCAR (2.23%) and the Cash relative MCAR (25.48%). In the augmented model, the Utilities factor reduces risk by 11 bps, in qualitative agreement with the Brinson and Fachler result reported in Table 2.

Source	Port Exp	Bench Exp	Active Exposure	Factor Volatility	Factor Corr	Factor MCAR	Active Risk Contrib
World	0.95	1.00	-0.05	30.74%	-0.83	-25.48%	1.27%
Energy	0.07	0.14	-0.06	13.41%	0.08	1.13%	-0.07%
Materials	0.08	0.05	0.03	10.70%	0.07	0.79%	0.02%
Industrials	0.09	0.11	-0.02	6.89%	-0.01	-0.07%	0.00%
ConsDscr	0.11	0.07	0.03	7.82%	0.25	1.98%	0.07%
ConsStpls	0.15	0.05	0.10	9.08%	0.19	1.69%	0.17%
Health Care	0.13	0.07	0.06	11.54%	0.35	3.99%	0.24%
Financials	0.08	0.34	-0.26	7.54%	-0.44	-3.33%	0.88%
IT	0.19	0.03	0.16	12.81%	0.42	5.36%	0.83%
Telecom	0.02	0.06	-0.04	11.29%	0.16	1.83%	-0.07%
Utilities	0.02	0.07	-0.05	9.79%	0.23	2.23%	-0.11%
Momentum	-0.05	0.00	-0.05	7.71%	0.17	1.33%	-0.07%
Volatility	-0.34	0.00	-0.34	10.67%	-0.72	-7.63%	2.59%
Value	-0.48	0.00	-0.48	4.24%	-0.21	-0.90%	0.43%
Size	-0.07	0.00	-0.07	3.93%	-0.24	-0.93%	0.07%
Residual	1.00	1.00	1.00	3.24%	0.15	0.48%	0.48%
Total				6.73%	1.00	6.73%	6.73%

Table 6: Factor-based risk attribution with a World factor (relative sources for industries).

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Conclusion

An intuitive attribution of risk requires that the sources of risk be aligned with the sources of return. For active managers, these return sources are typically defined on a benchmark-relative basis. Improper use of absolute return sources may lead to a number of pitfalls, including a mismatch between risk and return sources, non-intuitive MCARs, and flagging aggressive positions as risk reducing. These pitfalls can be addressed using relative MCARs, which result in a set of consistent and intuitive effects across securities, sectors, and factors.

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