Titolo

Michele Castellana[1, 2]
[1] *Institut Curie, PSL Research University, Paris, France*
[2] *CNRS UMR168, 11 rue Pierre et Marie Curie, 75005, Paris,France*
(Dated: December 3, 2025)

This is the abstract

1. [**?**] study the steady state with arc-length gauge
2. In the dynamics, arc-length gauge does not work
3. We show how to use a generalized arc-length gauge and how to correctly incorporate it into the dynamics to obtain a proper dynamics
4. In this dynamics we implement the Crank-Nicolson method and prove it to be numerically stable
5. We couple the membrane dynamics to a bulk fluid. The membrane is a complex elastic body -¿ We proceed by steps:
   (a) Bulk fluid + rigid body
   (b) Elastic body with stable elastic model (stable under compression)
   (c) Bulk fluid + elastic body with stable elastic model
   (d) Bulk fluid + membrane
      - Bulk fluid and membrane both described with Crank-Nicolson method -¿ we prove that the resulting dynamics is numerically stable
      - It allows for overhangs
      - It is the first study that describes a bulk fluid coupled to a Helfrich membrane with ALE
      - It allows for turbulent behavior for both the bulk fluid and the membrane tangential flow

Perspectives:

1. Study nucleoid compaction in E. Coli with ALE

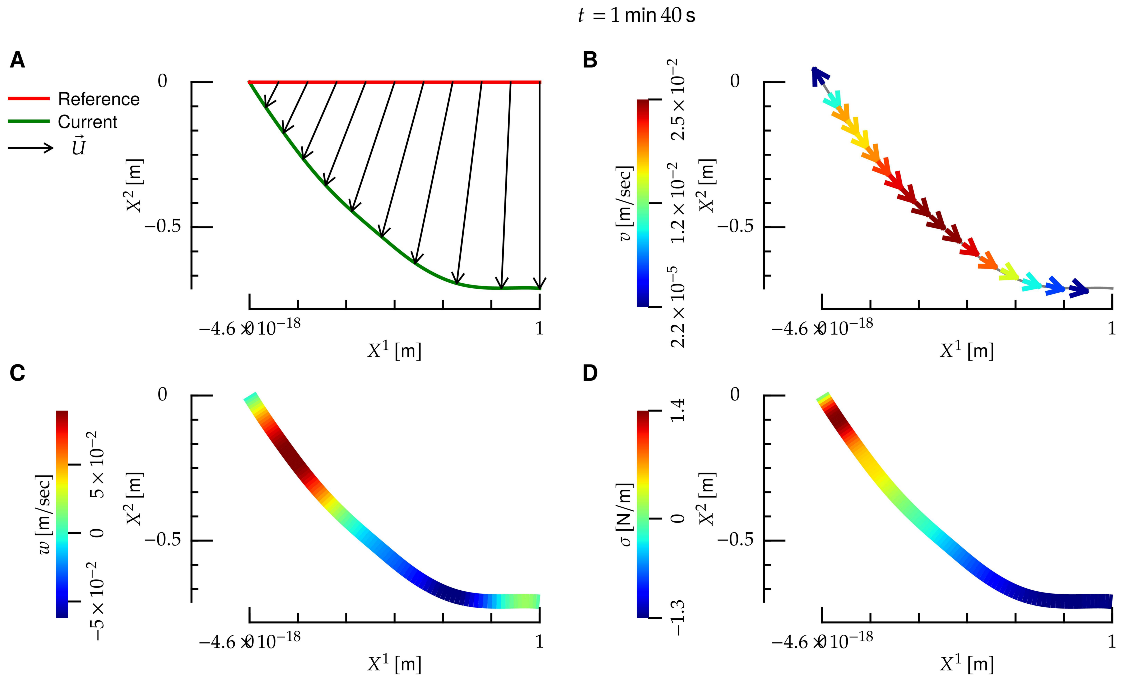

FIG. 1: Helfrich fluid layer described with the generalized arc length gauge. **A**) Displacement field $\vec{U}$, which relates the reference and the current configuration, shown in red and green, respectively. **B**) Tangential velocity. Arrows show the velocity direction, and the color code the velocity norm. **C**) Normal velocity, whose value is shown with the color code.**D**) Surface tension. All panels refer to the same instant of time, shown on top.

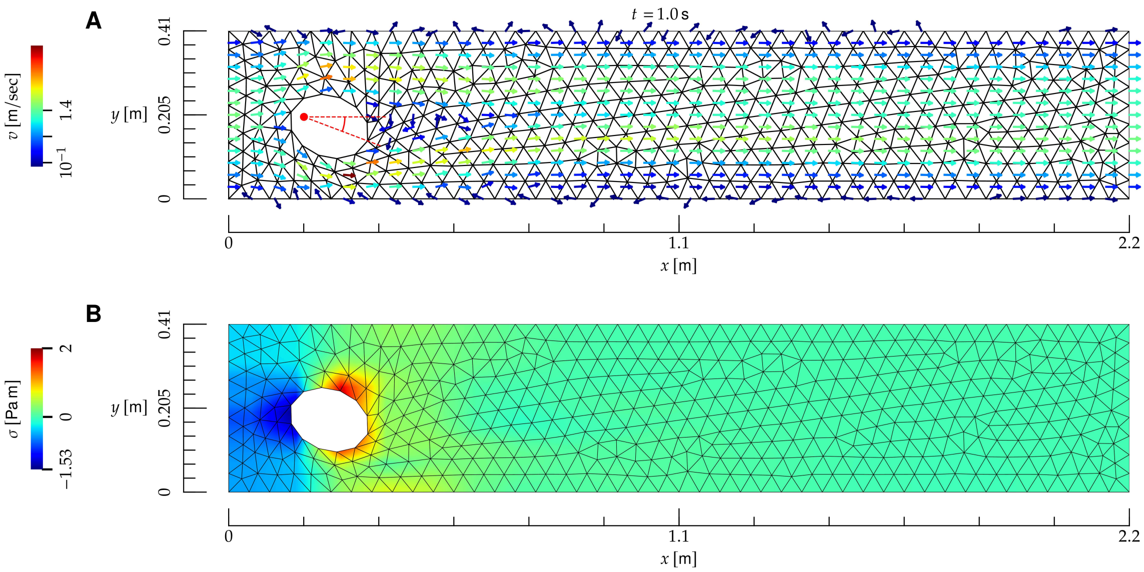

FIG. 2: Interaction between a bulk fluid and a rigid body. **A**) Temporal snapshot of the mesh (black lines) and velocity field (arrows). The direction of the velocity field is indicated by the arrows, and its norm by their color. The rigid body (ellipse) is allowed to pivot about its left focal point (red dot), and the related angle with respect to the x direction is denoted by the red arc. **B**) Color map of the surface tension. Both panels refer to the same instant of time, shown on top.

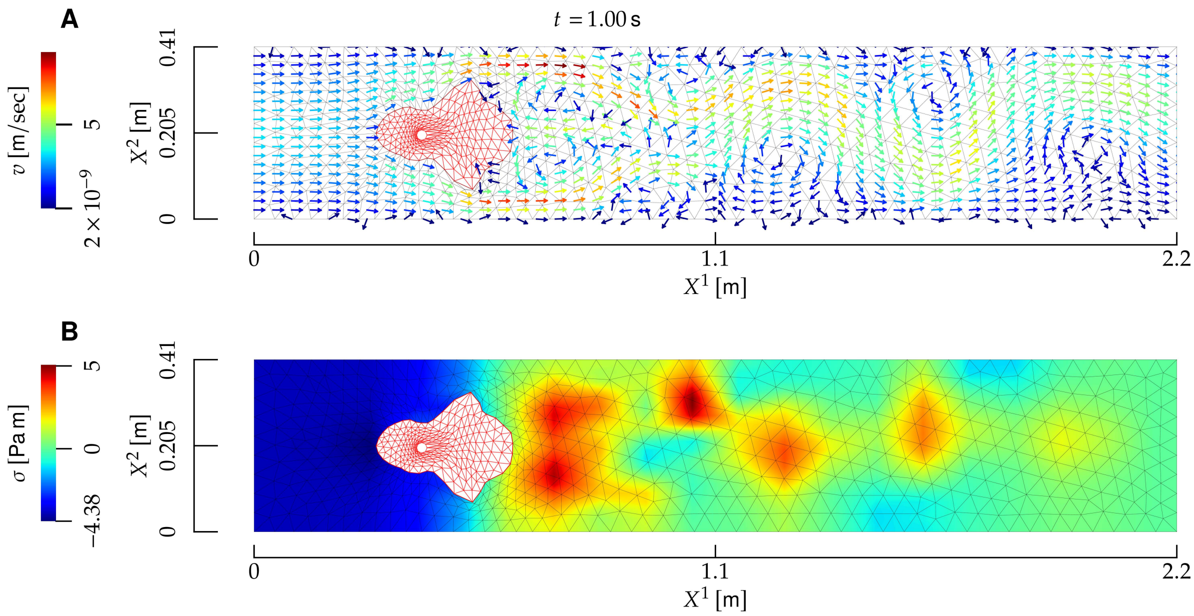

FIG. 3: Interaction between a bulk fluid and an elastic body. The notation is the same as in **??**, and the elastic body is depicted as a red mesh.

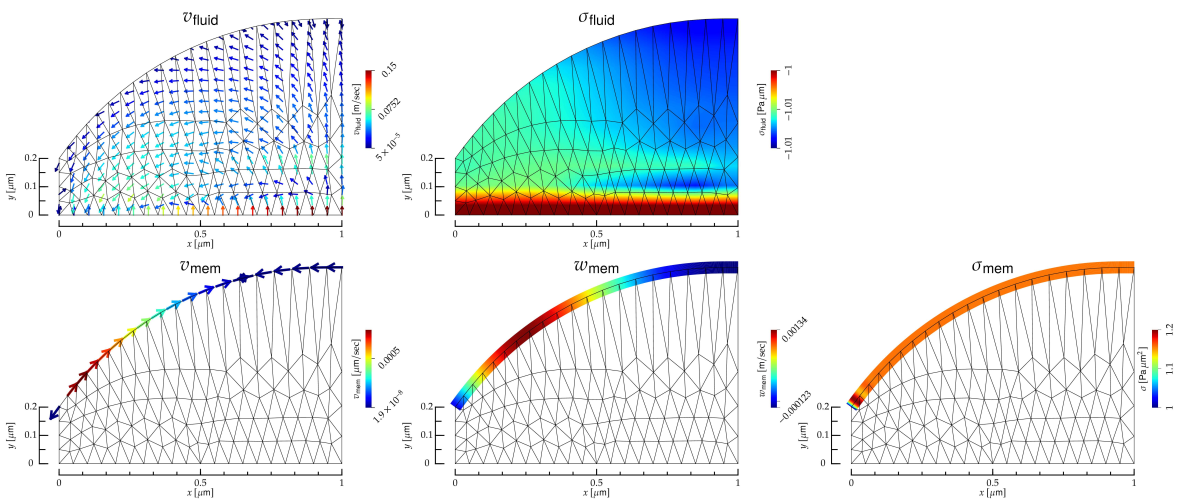

FIG. 4: Interaction between a bulk fluid and a Helfrich membrane. **A**) Bulk-fluid velocity; the notation is the same as in **??**A. **B**) Bulk-fluid tension. **C**), **D** and **E**): membrane tangential velocity, normal velocity and tension; the notation is the same as in **??**B, C and D, respectively. All panels refer to the same instant of time, shown on top.