A8 – Morphological Operations

Introduction

Morphology refers to shape or structure. In image processing, classical morphological operations are treatments done on binary images, particularly aggregates of 1's that form a particular shape, to improve the image for further processing or to extract information from it. All morphological operations affect the shape of the image in some way, for example, the shapes may be expanded, thinned, internal holes could be closed, disconnected blobs can be joined. In a binary image, all pixels which are "OFF" or have value equal to zero are considered background and all pixels which are "ON" or 1 are foreground.

Morphological operations make use of Set Theory. We review a few basic definitions. Let A be a set in 2-D integer space Z^2. The elements of Z^2 in our case are the x-y location of pixels in the image. If a = (x,y) is an *element* of A then

$$a \in A \quad . \tag{1}$$

If b is *not an element* of A we denote the fact as

$$b \notin A \quad . \tag{2}$$

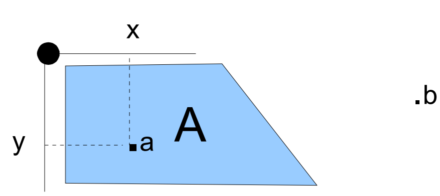

***Illustration 1**: The point a is an element of the set A while b is not an element of A.*

If A is a *subset* of another set B then we write

$$A \subset B \tag{3}$$

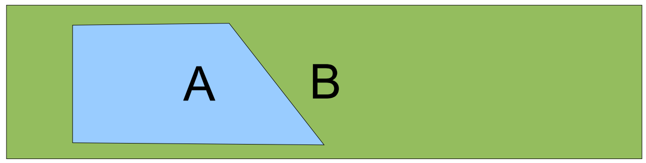

Illustration 2*: Set A is a subset of Set B.*

The *union* of two sets is the set of all elements that belong to either A or C and the set is written as

$$A \cup C. \tag{4}$$

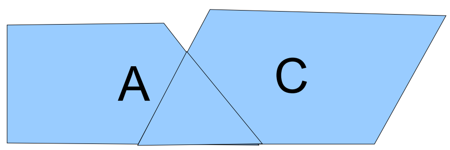

Illustration 3*: The union of Sets A and C are the points in blue.*

The *intersection* of two sets is the set of all elements that are both in A and C only and is denoted as

$$A \cap C \quad . \tag{5}$$

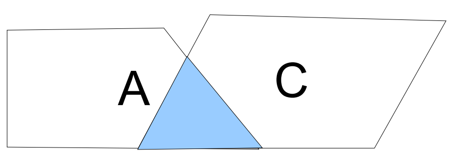

Illustration 4*: The intersection of Sets A and C are the points in blue.*

Two sets are said to be mutually exclusive if they have no common elements. In this case their intersection is the *empty set*, Ø. This fact is expressed as,

$$A \cap D = \emptyset \quad . \tag{6}$$

The *complement* of a set A is the set of elements not contained in A:

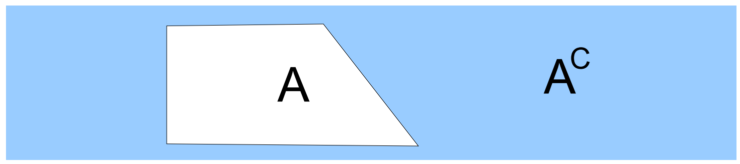

***Illustration 5**: The complement of Set A are the points in Set A^C.*

$$A^c = \{w : w \notin A\}. \tag{7}$$

The *difference* of two sets A and C denoted by A-C is defined as all elements of A excluding all elements of C,

$$A - C = \{w : w \in A, w \notin C\} = A \cap C^c \tag{8}$$

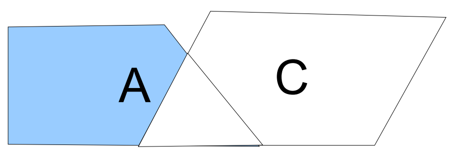

***Illustration 6**: The difference between A and C is the set in blue.*

The *reflection* of set A is defined as

$$\hat{A} = \{w : w = -a, \text{ for } a \in A\} \tag{9}$$

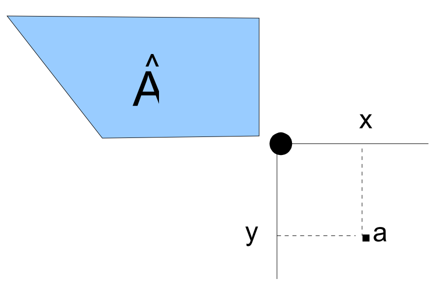

***Illustration 7**: The reflection of A is shown in blue.*

The *translation* of set A by point z = (z_1, z_2) denoted $(A)_z$ is defined as

$$(A)_z=\{c: c=a+z\ ,\ for\, a \in A\} \quad . \tag{10}$$

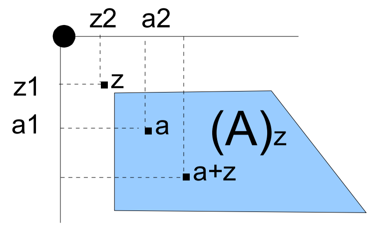

***Illustration 8**: The translation of A are the set of points in A translated by z.*

Logical operations are complementary to morphological operations on binary images.

The NOT operator is the same as complement. The AND is equal to intersection.

Question: What is the XOR operator equal to? How about [NOT (A)] AND B?

Dilation and Erosion

The *dilation* of A by B denoted by A dilation B is defined as

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \} \tag{11}$$

This involves all z's which are translations of a reflected B that when intersected with A is not the empty set. B is known as a structuring element. The effect of a dilation is to expand or elongate A in the shape of B.

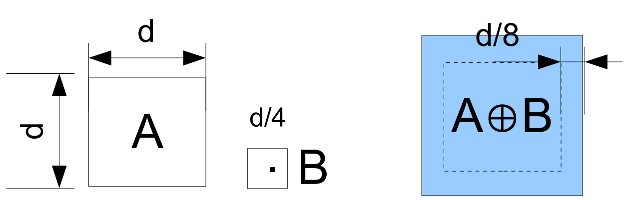

***Illustration 9:** The dilation of A by the structuring element B results in an expanded version of A.*

The *erosion* operator is defined as

$$A \ominus B = \{z | (B)_z \subseteq A\}. \tag{12}$$

The erosion of A by B is the set of all points z such that B translated by z is contained in A. The effect of erosion is to reduce the image by the shape of B.

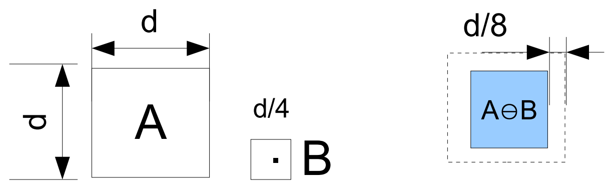

Illustration 10: *The erosion of A by the structuring element B results in a reduced version of A.*

Erosion and dilation are duals of each other and the following set operations are true:

$$(A \ominus B)^c = A^c \oplus B \tag{13}$$

Procedure

1. Predict-Observe-Explain. This is the only activity where you need to hand-draw your output. On a piece of graphing paper, draw the following shapes:
    1. A 5×5 square
    2. A triangle, base = 4 boxes, height = 3 boxes
    3. A hollow 10×10 square, 2 boxes thick
    4. A plus sign, one box thick, 5 boxes along each line.
2. Predict the resulting image if the following structuring elements are used to a) erode and b) dilate the images above. Scan your hand-drawn predictions and post on your blog.
    1. 2×2 ones
    2. 2×1 ones
    3. 1×2 ones

4. cross, 3 pixels long, one pixel thick.
5. A diagonal line, two boxes long, i.e. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

3. Create the above binary images and structuring elements in Scilab, Matlab or Python and perform erosion and dilation. See if you predicted the output correctly. For those using Scilab install the module Image Processing Design (IPD). This toolbox has morphological and blob analysis functions not found in SIVP. For those using Matlab, use the **bwmorph** or **imerode** or **imdilate** function. For python, make sure you have the **opencv** package.

4. For Scilab users with IPD installed:

- To create a structuring element, for example, a 3x3 cross, use

```
SE = CreateStructureElement('custom',[ %f %t %f; %t
%t %t; %f %t %f]);
```

- To perform erosion or dilation, use

```
Result = ErodeImage(BW, SE);
```