Oct 24, 2022

Theoretical Hypothesis Testing

Week 4: Tests for normal distributed population mean(s)

Focus

Dealing with the questions related to the mean parameter of the normal distributed population(s).

One-sample tests

Tests for the population mean.

One-sample Z-test

To test a population mean μ differs from a specific value μ_0.

Assumptions:

- data are measured on an interval or ratio scale.
- data are randomly sampled from a normal distribution.
- standard deviation of the population σ is known.

Test statistic:

$$T_Z = \frac{\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) - \mu_0}{\sigma/\sqrt{n}} \sim Z(0,1)$$

One-sample t-Test

To test a population mean μ differs from a specific value μ_0.

Assumptions:

- data are measured on an interval or ratio scale.
- data are randomly sampled from a normal distribution.
- standard deviation of the population σ is unknown, and estimated by the population standard deviation s.

Test statistic:

$$T_t = \frac{\bar{X} - \mu_0}{s}\sqrt{n} \sim t_{(n-1)} \text{ where } s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

One-sample Z-test vs. t-test

The condition for using Z-test is to know **population variance**, and t-test can be used without knowing the population variance.

Two-sample tests

Tests for two populations’ means.

Two-sample Z-test

To test two population means μ_1 and μ_2 differ less than, more or by a value d_0.

Assumptions:

- data are measured on an interval or ratio scale.
- data are randomly sampled from two independent normal distribution.
- standard deviation of the populations σ_1 and σ_2 are known.

Test statistic:

$$T_{Z2} = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}} \sim Z(0,1)$$

Two-sample pooled t-test

To test two population means μ_1 and μ_2 differ less than, more or by a value d_0.

Assumptions:

- data are measured on an interval or ratio scale.
- data are randomly sampled from two independent normal distribution.
- standard deviation of the populations σ_1 and σ_2 are unknown but equal and estimated through the pooled population standard deviation s_p.

Test statistic:

$$T_{Z2} = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}$$

Welch Test

To test two population means μ_1 and μ_2 differ less than, more or by a value d_0.

Assumptions:

- data are measured on an interval or ratio scale.
- data are randomly sampled from two independent normal distribution.
- standard deviation of the populations σ_1 and σ_2 are unknown and not necessarily equal; estimated through the population standard deviation of each sample.

Test statistic:

$$T_W = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_{(v)} \text{ where } v = \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2 \Big/ \left(\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}\right)$$

Two sample Z-test vs. t-test and Welch-test

- If **population variances are known**, two sample Z-test can be used.
- If **population variances are unknown but assumed as equal**, two sample t-test can be used.
- If **population variances are not unequal**, Welch-test is used.

Paired two-sample tests

Tests for paired two populations' means.

Question #1

Unpaired or paired?

A professor wants to determine whether or not two different studying techniques lead to different mean exam scores. To perform this, he could 10 students and have them use one studying technique for one month and take an exam, then have them use the second studying technique for one month and take another exam of equal difficulty.

Question #2

Unpaired or paired?

A professor wants to determine whether or not two different studying techniques lead to different mean exam scores. To perform this, he could recruit 20 total students and randomly split them into two groups of 10. He could assign one group to use one studying technique for one month and assign the other group to use the second studying technique for one month and have all students take the same exam.

Examples

Unpaired

- Effect of drug on ½ of the patients assigned to a treatment group and other ½ assigned to a control group
- Measuring the level of glucose for two independent groups like men and women
- Comparing the time taken by trains following different routes and New York

Paired

- Effect of a drug on the same sample of people
- Different courses for the same subject on a group of students
- Standard exam results for a group of students before and after prelims

Paired two-sample Z-Test

To test the difference between two population means μ_1 and μ_2 less than, more or by a value d_0 in the case that observations are collected in pairs.

Assumptions:

- data are measured on an interval or ratio scale and randomly sampled in pairs (X_1, X_2).
- X_1 follows a normal distribution with mean μ_1 and variance σ_1^2. X_2 follows a normal distribution with mean μ_2 and variance σ_2^2. The covariance of X_1 and X_2 is σ_{12}.
- The standard deviation $\sigma_d = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$ of the differences $X_1 - X_2$ is known.

Test statistic:

$$T_{PZ} = \frac{\bar{D} - d_0}{\sigma_d}\sqrt{n} \sim Z(0,1) \text{ where } D = \frac{1}{n}\sum_{i=1}^{n}(X_{1i} - X_{2i})$$

Paired two-sample t-Test

To test the difference between two population means μ_1 and μ_2 less than, more or by a value d_0 in the case that observations are collected in pairs.

Assumptions:

- data are measured on an interval or ratio scale and randomly sampled in pairs (X_1, X_2).
- X_1 follows a normal distribution with mean μ_1 and variance σ_1^2. X_2 follows a normal distribution with mean μ_2 and variance σ_2^2. The covariance of X_1 and X_2 is σ_{12}.
- The standard deviations are unknown. The standard deviation σ_d of the differences is estimated through the population standard deviation s_d of the differences.

Test statistic:

$$T_{PZ} = \frac{\bar{D} - d_0}{s_d}\sqrt{n} \sim t_{(n-1)} \text{ where } s_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2}\ , \bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i \text{ and } D_i = X_{1i} - X_{2i}$$

Unpaired vs. paired

- A paired situation is conducted to compare/test the mean of **same populations <u>under two separate scenarios</u>**.
- An unpaired is conducted to compare/test the mean of two **independent of unrelated populations**.

R Application #1

According to the measurement values of the mean blood pressures of 25 patients with the same disease, it is desired to investigate whether the average blood pressure of people who have this disease is 140 mmHg. Test the null hypothesis suitable for this research using the appropriate method.

```
kan_basinci <- c(120, 115, 94, 118, 111, 102, 102, 131, 104, 107, 115, 139, 115,
                 113, 114, 105, 115, 134, 109, 109, 93, 118, 109,106, 125)
```

H_0 :

H_A :

R Application #1

Decision:

Interpretation (Technical):

Interpretation (General):

R Application #2

After using a drug, it is claimed that the mean blood pressure of patients is lower than 140 mmHg. To test this claim, 25 of patients is given drug, and 30 of them is given placebo. The standard deviation of the patients are known and it is 20. **Test the clam using the appropriate method.**

$H_0:$

$H_A:$

R Application #2

Decision:

Interpretation (Technical):

Interpretation (General):

R Application #3

Run a simulation to see the power difference between one sample Z and t test.

Go GitHub!