Oct 10, 2022

Theoretical Hypothesis Testing

Week 2: Hypothesis testing steps

Fundamental Setup

Let denote a sample of size n by $x_1, x_2, \ldots, x_n$, where the x_i are observations of identically independent distributed random variables X_i, $i = 1,2,...,n$. The hypotheses, null H_0 and alternative H_A, must be stated as follows:

$$H_0 : A = \ , \leq \ , \geq A_i$$

$$H_A : A \neq \ , > \ , < A_i$$

where the population parameter space of A is Θ is partitioned into disjoint sets Θ_0 and Θ_A with $\Theta_0 \cup \Theta_A$, corresponding to H_0 and H_A, respectively.

Fundamental Setup

The test statistics $T = f(X_1, X_2, \ldots, X_n)$ fulfills two criteria:

1. Its value must provide insight on whether or not the null hypothesis might be true

2. The distribution of it must be known, that the null hypothesis is true.

Hypothesis Testing Steps

1. Specify the null and alternative hypothesis
2. Set the significance (nominal) level α
3. Calculate the test statistic and p-value
4. Decision
5. Interpretation

1. Specifying the hypotheses

- **A population parameter** is used in the hypotheses.

- The combination of the H_0 and H_A **must consists the possible values of the interested population parameter** such as:

  1. $H_0 : \mu = 0$ vs $H_0 : \mu \neq 0$

  2. $H_0 : \mu \geq 0$ vs $H_0 : \mu < 0$

  3. $H_0 : \mu \leq 0$ vs $H_0 : \mu > 0$

2. Setting the nominal level

- Nominal level is used to compare with the p-value when deciding on the null hypothesis.
- The nominal level 0.05 means that the decision about the null hypothesis is true with 95% confidence, and there is a 5% chance that the decision is wrong.
- You can adjust the nominal level to increase the confidence of you decision on the null hypothesis. (**use domain knowledge**)

3. Calculation the test statistic and p-value

There are two approaches to decide on the null hypothesis:

1. p-value: Null hypothesis is rejected when the p-value is smaller than nominal level. (*Fisher style*)

2. critical region: The null hypothesis is rejected if the observed value of the test statistic lies in the critical region. (*Neyman-Pearson style*)

You can use either of them. In some of the procedures (e.g. resampling based tests), the p-value approach is only can be calculated.

4. Decision

Two possible decision in hypothesis testing:

1. Reject null hypothesis.
2. Do not reject the null hypothesis. (Please do not use the “accept null hypothesis”)

5. Interpretation

We should use two different (or complement) kind of interpretations:

1. **Technical**: According to the approaches that we used, we can interpret the results as follows:

- “there is enough evidence to reject the null hypothesis”
- “there is no enough evidence to reject the null hypothesis”

2. **General**: We can make the result more communicable for everyone as follows:

- “the population mean is equal to X, so the mean weight of the products is X kg.”
- “the population mean is not equal to X, so the mean weight of the products is not X kg.”

Application

Assume that we want to test that whether the mean life expectancy of Japan is 75 years. (data: gapminder dataset in R)

H_0 :

H_A :

Data (n = 12): 63.030, 65.500, 68.730, 71.430, 73.420, 75.380, 77.110, 78.670, 79.360, 80.690, 82, 82.603.

Application

Which test?

Application

Use following codes to conduct the process in R:s

```
library(gapminder)

data(gapminder)

t.test(gapminder$lifeExp[gapminder$country == “Japan”],

        mu = 75,

        alternative = “two.sided”) #(“greater” or “less”)
```

Application

Output:

One Sample t-test

data: gapminder$lifeExp[gapminder$country == “Japan”]

t = -0.092319, df = 11, p-value = 0.9281

alternative hypothesis: true mean is not equal to 75

95 percent confidence interval:

70.70043 78.95341

Application

Decision and Interpretation

Decision:

Interpretation (technical):

Interpretation (general):s