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Theoretical Hypothesis Testing

Week 5: Normality tests

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Warm-up example

In a refrigerator manufacturing company, the breakdown times of a component are tested and 50 observation values are collected on a decade (ten-years) scale.

The production engineer wants to test whether the average breakdown time of this component is 5 years.

H_0 :

H_A :

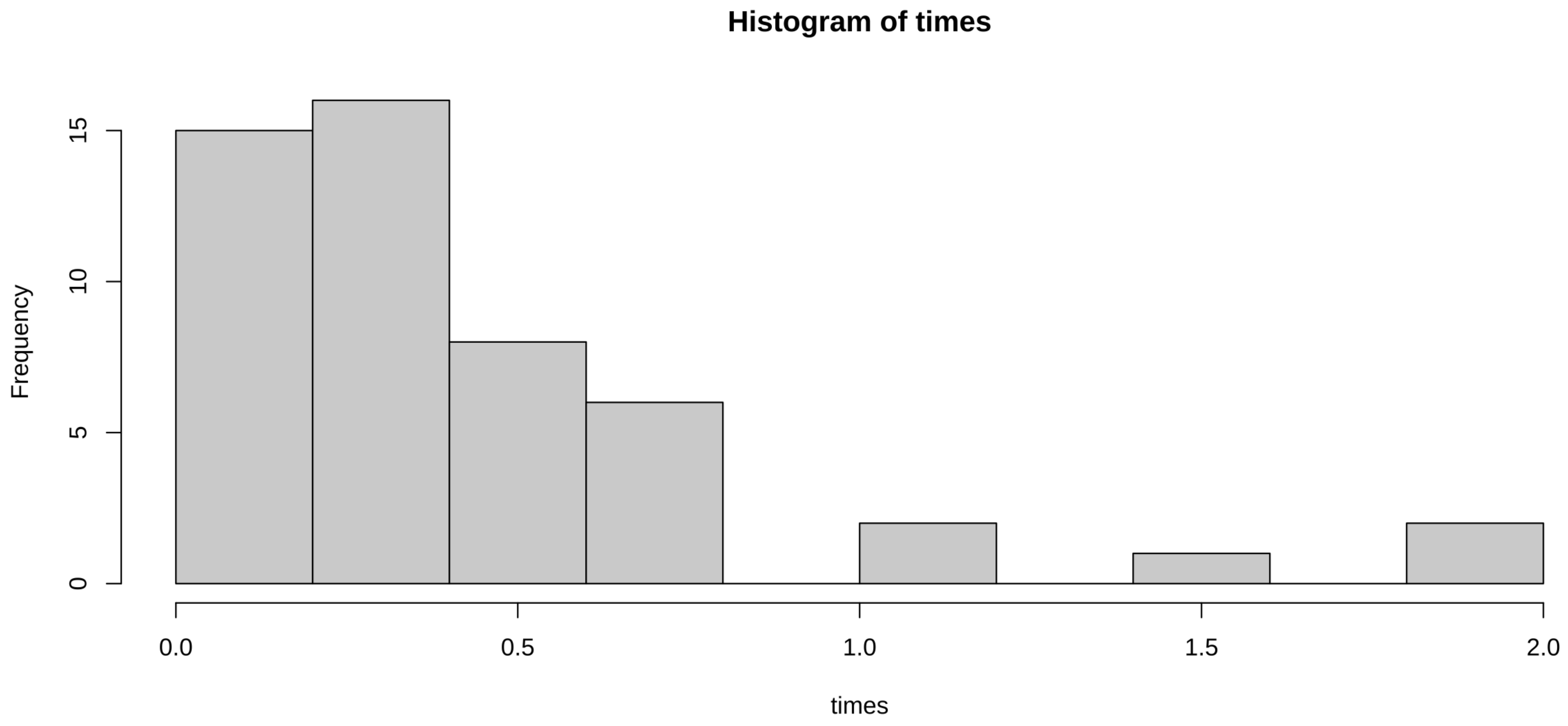
Motivation Example

```
times <- c(0.364, 0.137, 0.234, 0.381, 0.126, 0.355, 0.683,
0.285,      0.030, 0.077, 0.797, 0.292, 0.141, 0.258, 1.012, 0.687,
            0.197, 0.096, 0.075, 1.817, 0.516, 0.652, 0.268, 0.403,
            0.485, 0.406, 0.351, 0.668, 1.023, 0.031, 0.076, 0.122,
            0.271, 0.402, 0.132, 0.263, 0.320, 1.930, 0.032, 0.492,
            0.265, 0.502, 0.220, 0.511, 0.377, 0.001, 0.029, 0.286,
            1.431, 0.642)

t.test(times, mu = 0.5)

One Sample t-test

data:  times
t = -1.3123, df = 49, p-value = 0.1955
alternative hypothesis: true mean is not equal to 0.5
95 percent confidence interval:
 0.30514 0.54090
sample estimates:
mean of x
  0.42302
```

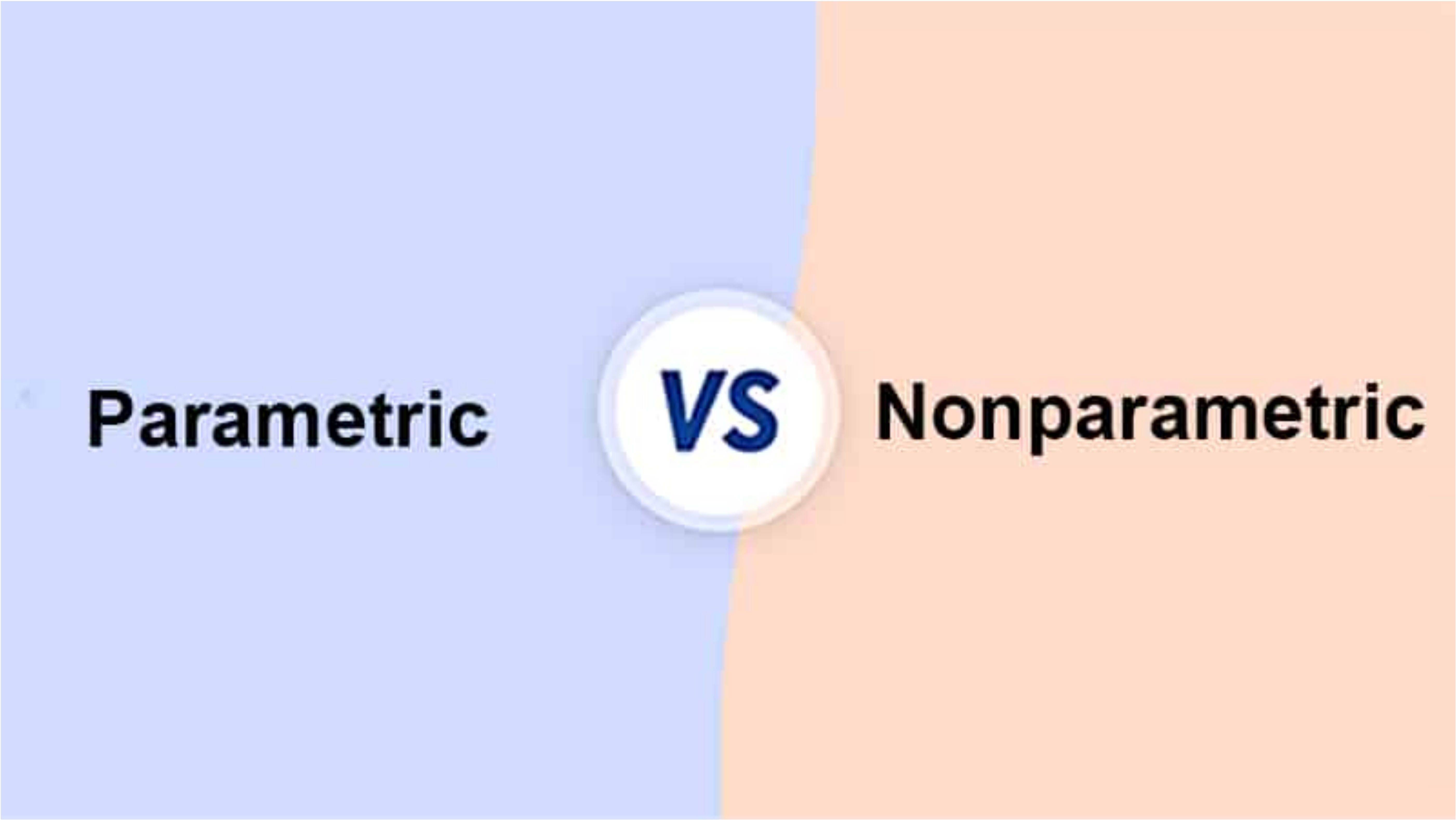
Decision:

Interpretation:

Are you sure?



Focus



Focus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Focus

The church of normal distribution*

Hallgrímskirkja, is a Lutheran (Church of Iceland) parish church in Reykjavík, Iceland: https://en.wikipedia.org/wiki/Hallgr%C3%ADmskirkja

Normality tests

- Kolmogorov-Smirnov (Lilliefors) test
- Anderson-Darling test
- Shapiro-Wilk test
- Jarque-Bera test
- …

For more see the following papers:

- Berna Yazici & Senay Yolacan (2007) A comparison of various tests of normality, Journal of Statistical Computation and Simulation, 77:2, 175-183, DOI: 10.1080/10629360600678310
- Mustafa Cavus, Berna Yazici & Ahmet Sezer (2021) Penalized power properties of the normality tests in the presence of outliers, Communications in Statistics - Simulation and Computation, DOI: 10.1080/03610918.2021.1938124

Normality test

- Normality tests are **used to check whether the data follow the normal distribution**.
- A normality test is **a kind of goodness-of-fit test**.
- There are **many normality tests** based on empirical distribution function or not.
- The hypotheses are:

$H_0 : X \sim N(\mu, \sigma^2)$ - *The considered data (X) follows the normal distribution.*

$H_A : X \nsim N(\mu, \sigma^2)$ - *The considered data does not follows the normal distribution.*

Kolmogorov-Smirnov test

This test evaluates the greatest vertical distance between the EDF and the CDF of the standard normal distribution.

$$T_{KS} = sup_x \,|\, F(x) - S_n(x) \,|$$

where $S_n(x)$ is the sample cumulative distribution function and $F(x)$ is the cumulative distribution function of the null hypothesis. The null hypothesis is rejected when $T_{KS} > Z_\alpha$.

Anderson-Darling test

Anderson-Darling test is a modification of the Cramer-von-Mises* test and it gives more weight to the tails of the distribution.

$$T_{AD} = -n - \frac{1}{n} \sum_{i=1}^{n} (2j-1)(lnF(x_{(i)} + ln[1 - F(x_{(n-i+1)})]))$$

The null hypothesis is rejected when $T_{AD} > Z_{\alpha}$.

* For more info about von-Mises, see https://twitter.com/mustafa__cavus/status/1476884521110970369?s=20&t=zR7hJrj68C0mZENPPLhpIA

Shapiro-Wilk test

Shapiro-Wilk test is based on expected value of order statistics from a standard normal distribution

$$T_{SW} = \frac{1}{D}\Big[\sum_{i=1}^{m} a_i(x_{(n-i+1)}, x_{(i)})\Big]^2$$

where $m = n/2$ if n is even while $m = (n-1)/2$ if n is odd. The values of T_{SW} lie between 0 and 1, and small values indicate that the departure from normality.

One sample non-parametric tests

- In this part, we deal with the question that if the median of a population differs from a predefined value.
- The most straightforward test is the sign test.
- However, if a symmetric distribution can be assumed the Wilcoxon signed-rank test is a better alternative.

One-sample non-parametric tests

Tests if the location (median M) of a population differs from a specific value m_0.

Hypotheses:

1. $H_0 : M = M_0$ vs. $H_A : M \neq M_0$
2. $H_0 : M \leq M_0$ vs. $H_A : M > M_0$
3. $H_0 : M \geq M_0$ vs. $H_A : M < M_0$

Wilcoxon signed-rank test

The one-sample Wilcoxon signed-rank test is non-parametric equivalent of the t-test.

$$T_W = \sum_{i=1}^{N} sign(X_i)R_i$$

where R_i is the rank of the observations. The null hypothesis is rejected when $T_W > T_\alpha$.

Two sample non-parametric tests

In this part, we deal with the question that if the median of two population equal to each other or the difference between them differs from a predefined value.

<u>Hypotheses:</u>

1. $H_0 : M_1 = M_2$ vs. $H_A : M_1 \neq M_2$ (or $H_0 : M_1 - M_2 = 0$ vs. $H_A : M_1 - M_2 \neq 0$)
2. $H_0 : M_1 \leq M_2$ vs. $H_A : M_1 > M_2$ (or $H_0 : M_1 - M_2 \leq 0$ vs. $H_A : M_1 - M_2 > 0$)
3. $H_0 : M_1 \geq M_2$ vs. $H_A : M_1 < M_2$ (or $H_0 : M_1 - M_2 \geq 0$ vs. $H_A : M_1 - M_2 < 0$)

Two sample Wilcoxon signed-rank test

The two sample Wilcoxon signed-rank test is non-parametric equivalent of the two sample t-test.

$$T_{W2} = \sum_{i=1}^{n} \sum_{j=1}^{m} S(X_i, Y_j)$$

where $S(X, Y) = 1$ if $X > Y$, $S(X, Y) = \frac{1}{2}$ if $X = Y$, and $S(X, Y) = 0$ if $X < Y$.

The null hypothesis is rejected when $T_{W2} > T_\alpha$.