# Statistical inference III

## Data Science in Practice

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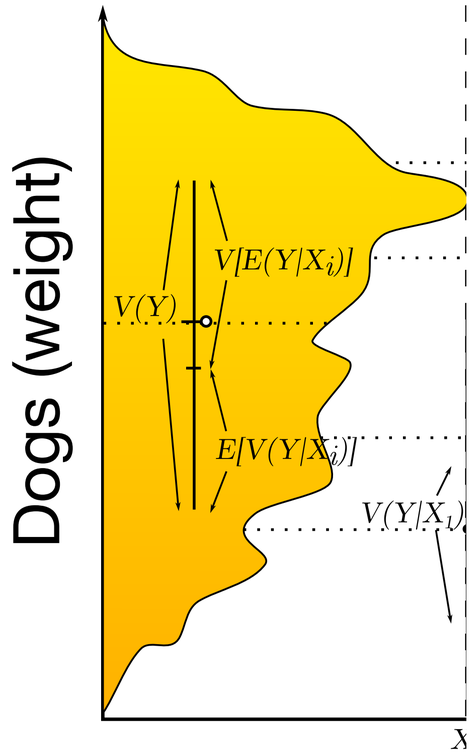
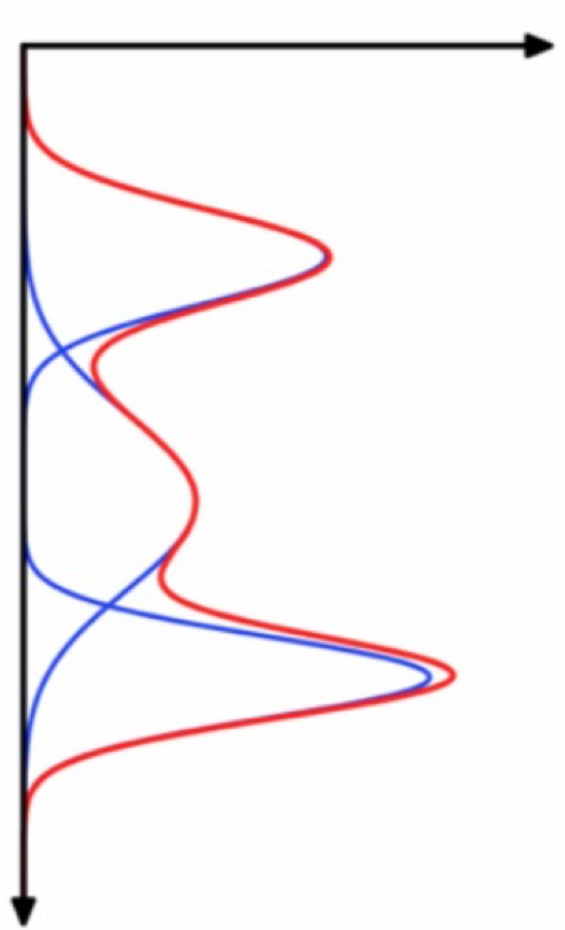
# Beyond Pairwise testing: ANOVA

## Same assumptions as t-test




Pretty clearly not a single normal distribution

But what if its a bunch of normals slammed together?



https://en.wikipedia.org/wiki/Analysis_of_variance

# Beyond Pairwise testing: ANOVA

## Same assumptions as t-test

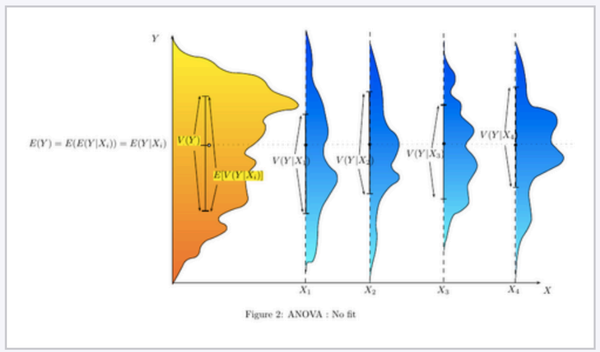
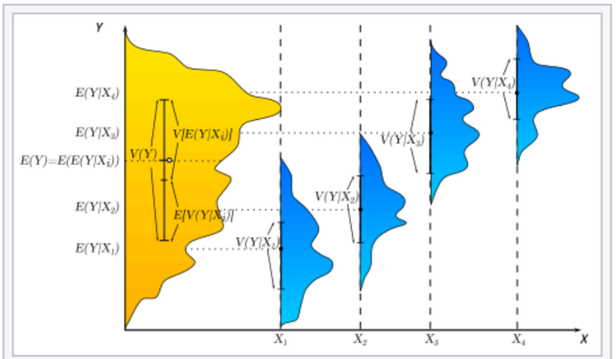



Figure 2: ANOVA : No fit

No fit: Young vs old, and short-haired vs long-haired




Fair fit: Pet vs Working breed and less athletic vs more athletic

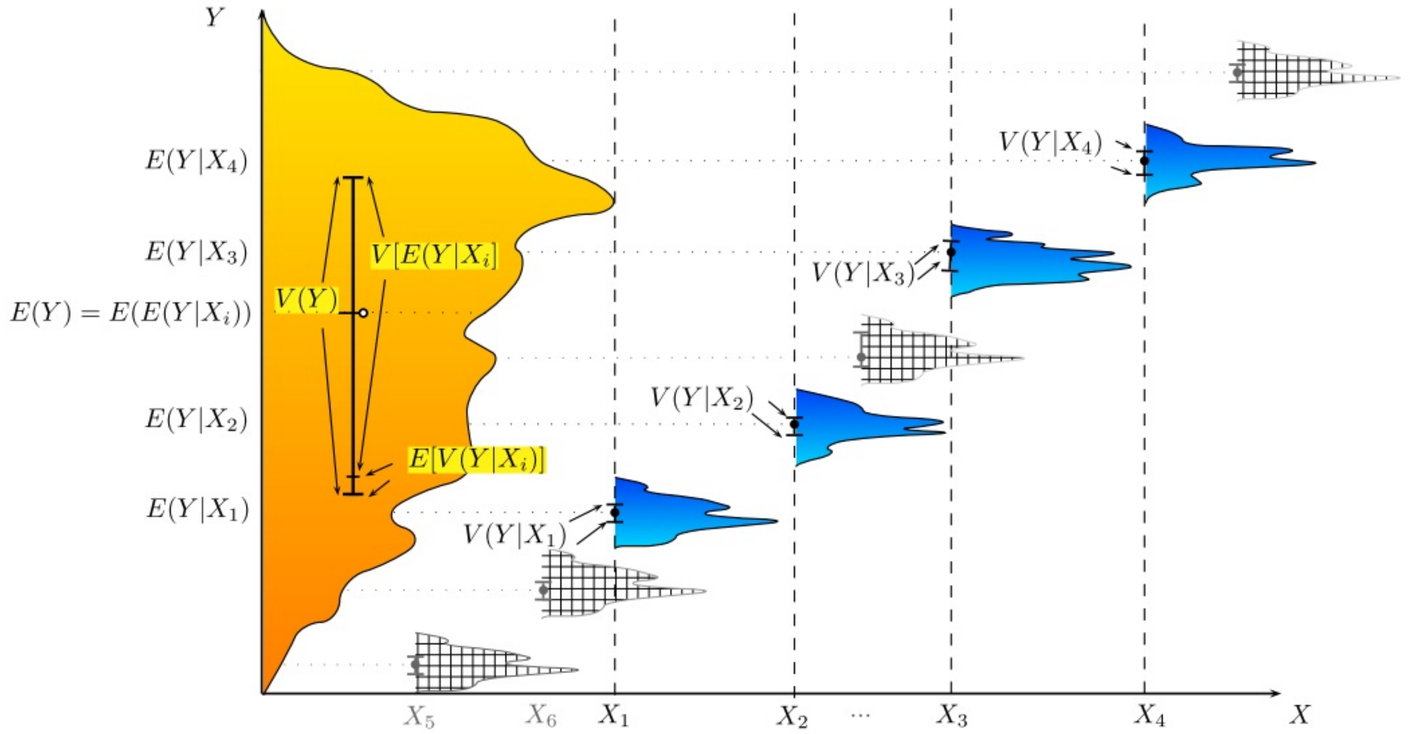
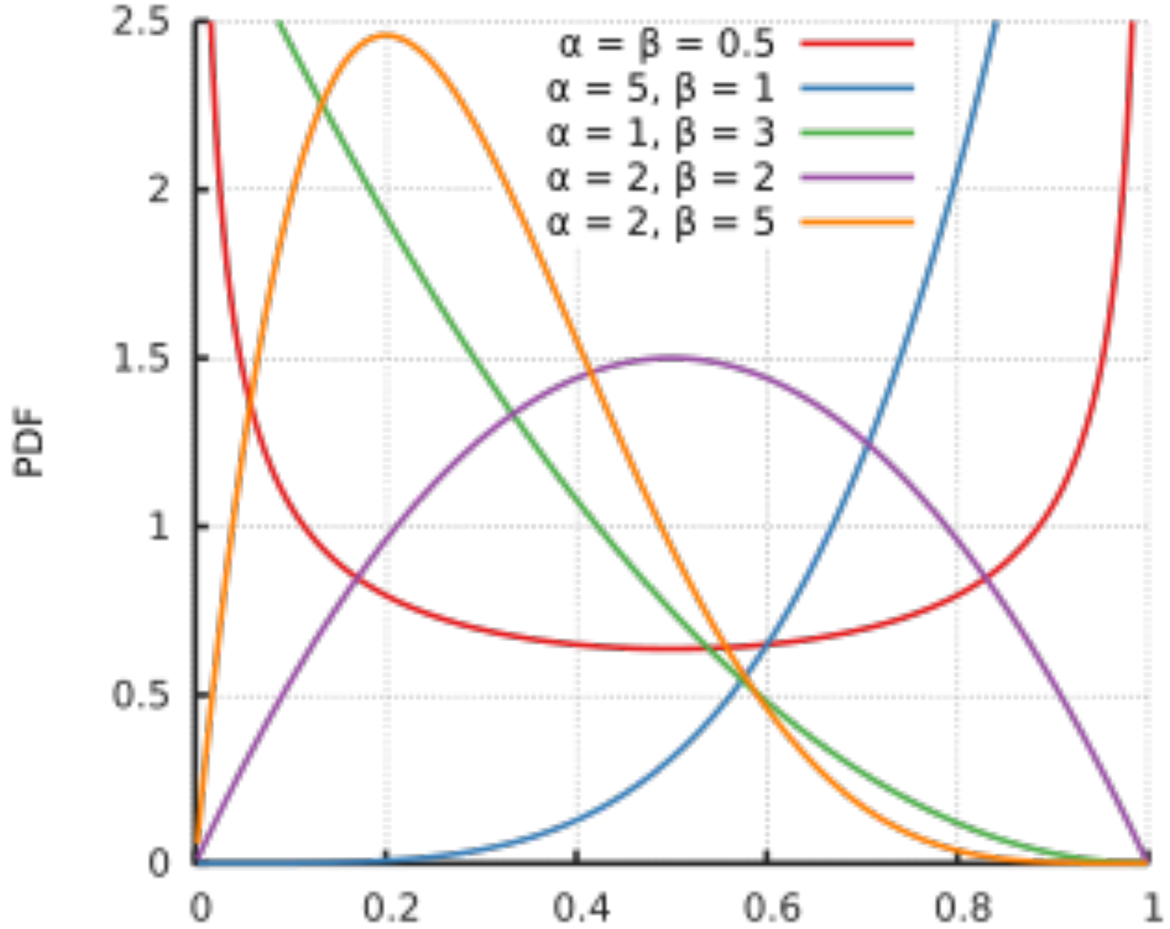



Figure 3: ANOVA : very good fit

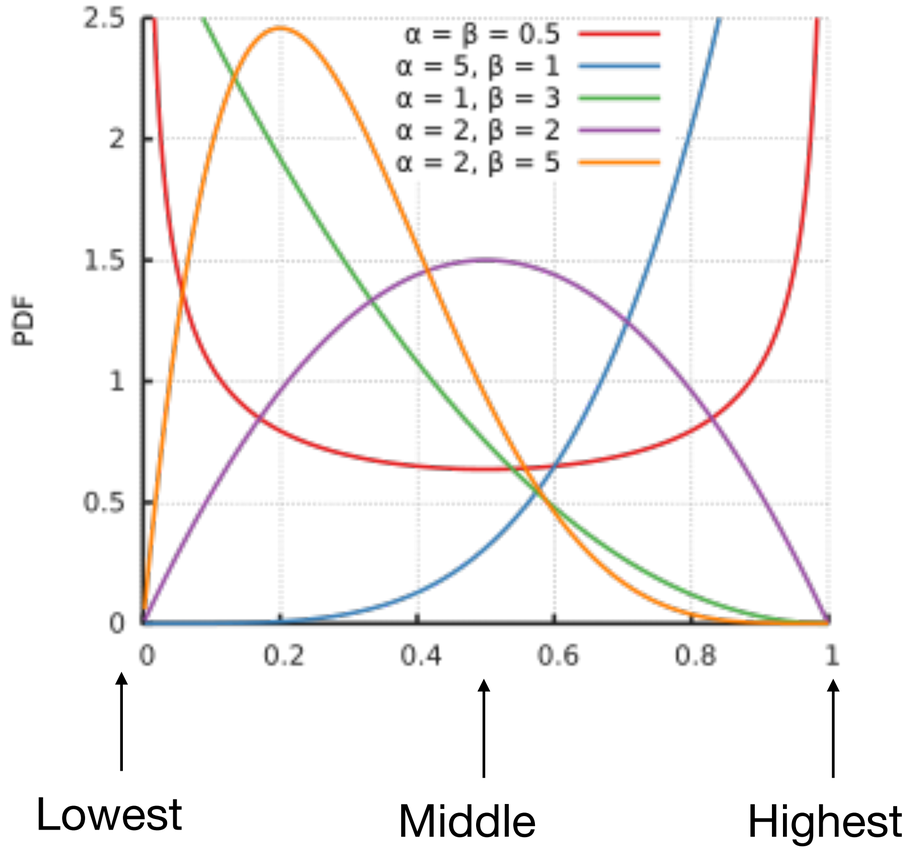
Fit by dog breed

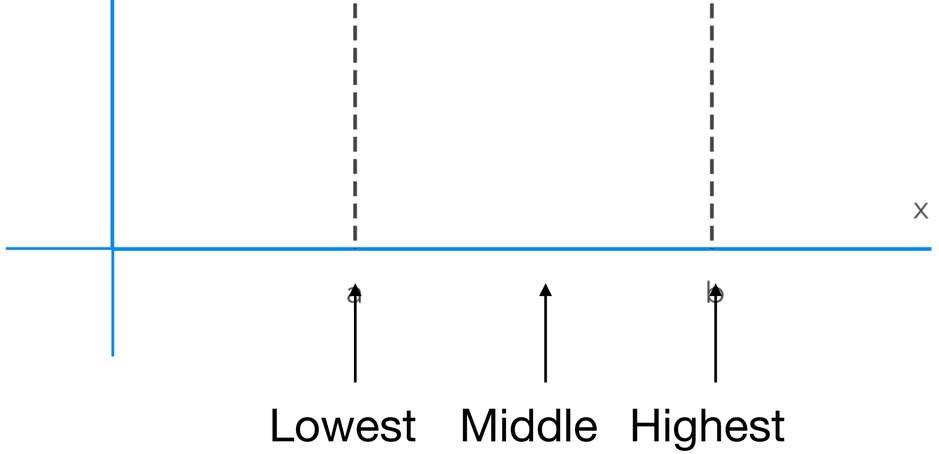
https://en.wikipedia.org/wiki/Analysis_of_variance

# Non-parametric Statistics: What if your distribution looks like this?



# Non-parametric Statistics: What if your distribution looks like this? Then rank transform!



# Rank Statistics

| quantitative data | | ordinal data |
|---|---|---|
| 1, 4.5, 6.6, 9.2 | → | 1, 2, 3, 4 |

**Data are transformed from their quantitative value to their rank.**

**Ordinal data** - categorical, where the variables have a natural order

Particularly helpful when data have a ranking but no clear numerical interpretation (i.e. movie reviews)

# Ordinality

**Which of the following variables contains ordinal data?**

https://forms.gle/UUbbQd9nyz8tgzVh6

# Wilcoxon rank-sum test (Mann Whitney U test)

- Determine whether two independent samples were selected from the same populations, having the same distribution

- Similar to t-test (but does not require normal distributions) & tests median

•Assumptions:

- Observations in each group are independent of one another

- Responses are ordinal

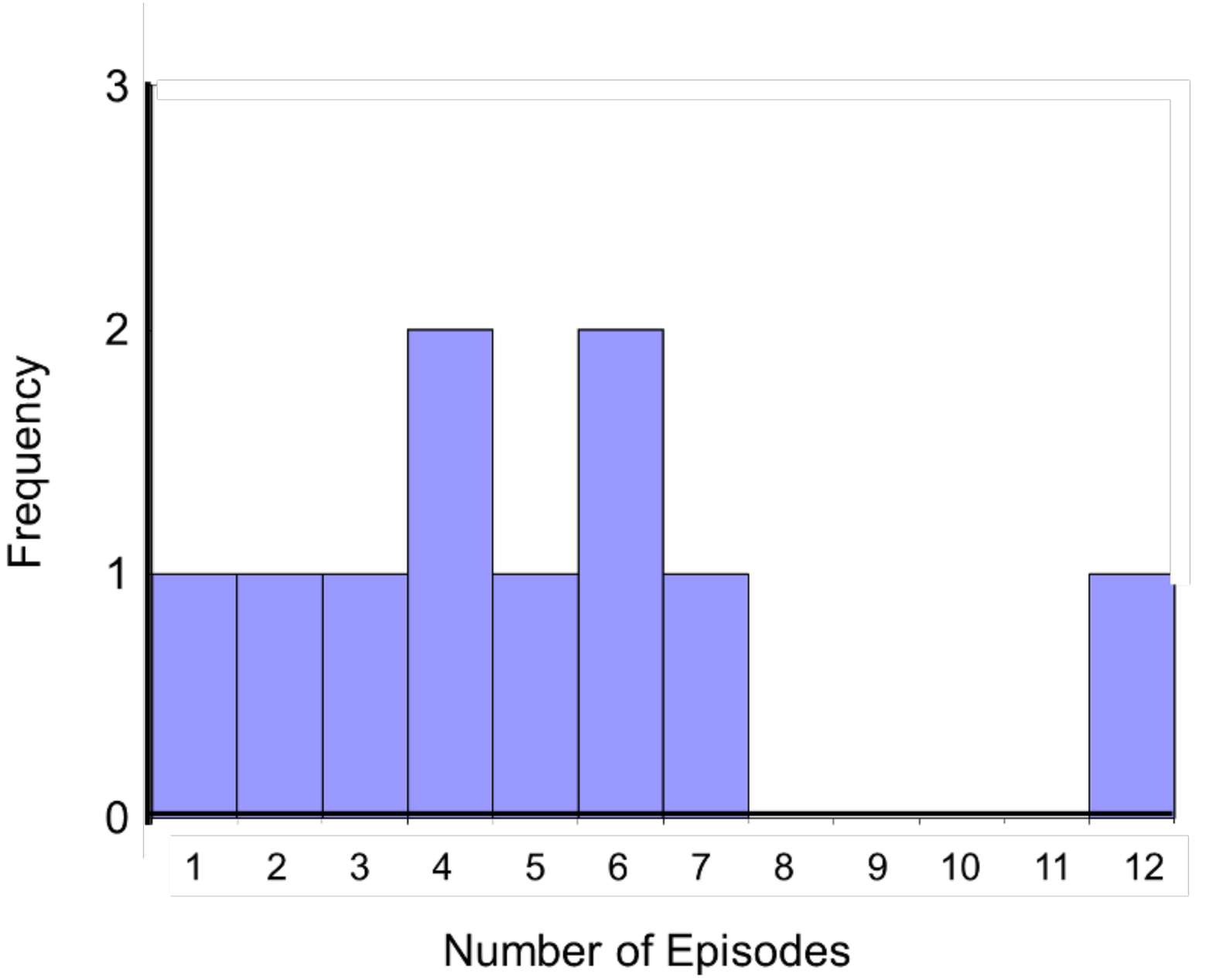
•$H_o$: distributions of both populations are equal

•$H_a$: distributions are *not* equal

# Mann-Whitney U: question example

- In a clinical trial, is there a difference in the number of episodes of shortness of breath between placebo and treatment?

- Step 1: Participants record number of episodes they have.
- Step 2: Episodes from both groups are combined, sorted, and ranked
- Step 2: Resort the ranks into separate samples (placebo vs. treatment)
- Step 3: Carry out statistical test




http://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/BS704_Nonparametric4.html

| | | Total Sample (Ordered Smallest to Largest) | Ranks |
|---|---|---|---|
| Placebo | New Drug | | |
| 7 | 3 | | |
| 5 | 6 | | |
| 6 | 4 | | |
| 4 | 2 | | |
| 12 | 1 | | |
| | | | |
| | | | |
| | | | |

**Sum of ranks:**
**Placebo = 37**
**New Drug = 18**

http://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_

# Mann-Whitney *U*

**Ho**: low and high scores are approximately evenly distributed in the two groups

**Ha:** low and high scores are NOT evenly distributed in the two groups (U <= 2)

$$U_A = n_a n_b + \frac{n_a(n_a+1)}{2} - T_A$$

The max possible value of TA

The observed sum of ranks for sample A

$n_a$ = number of elements in group A

$n_b$ = number of elements in group B

$U_{Placebo} = 3$

$U_{treatment} = 22$

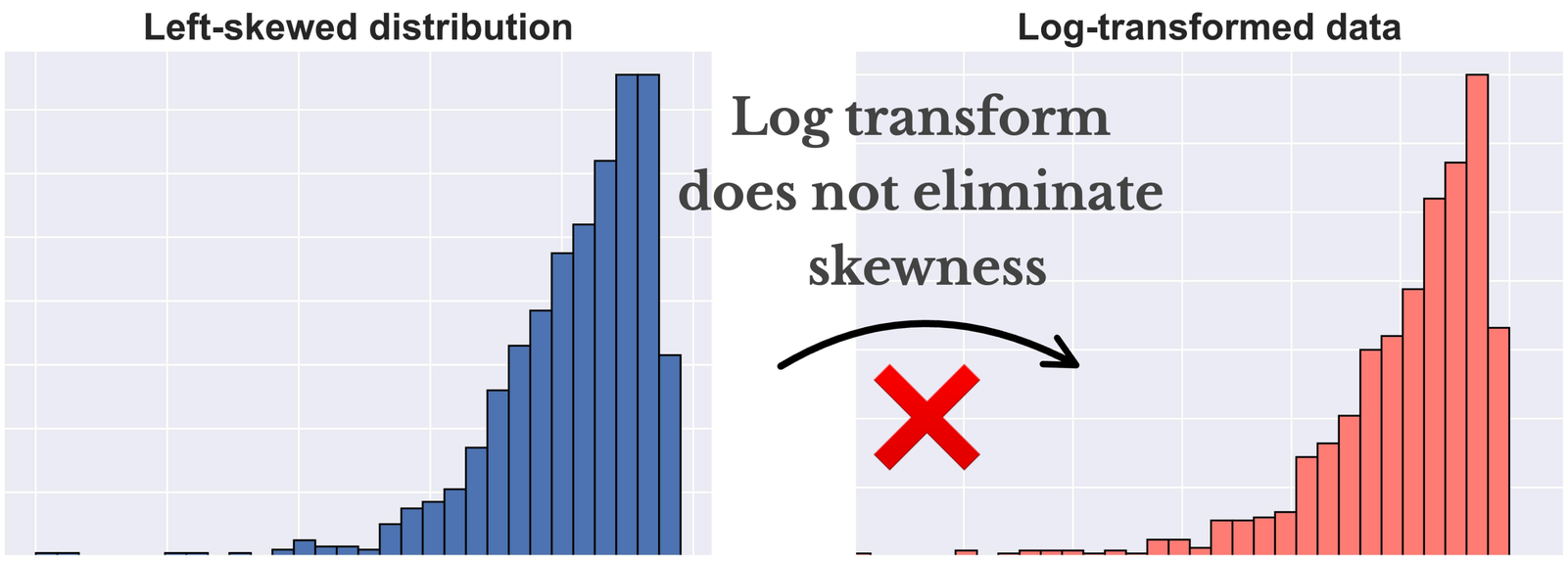
$0 < U < n_1 * n_2$

Complete separation → no separation

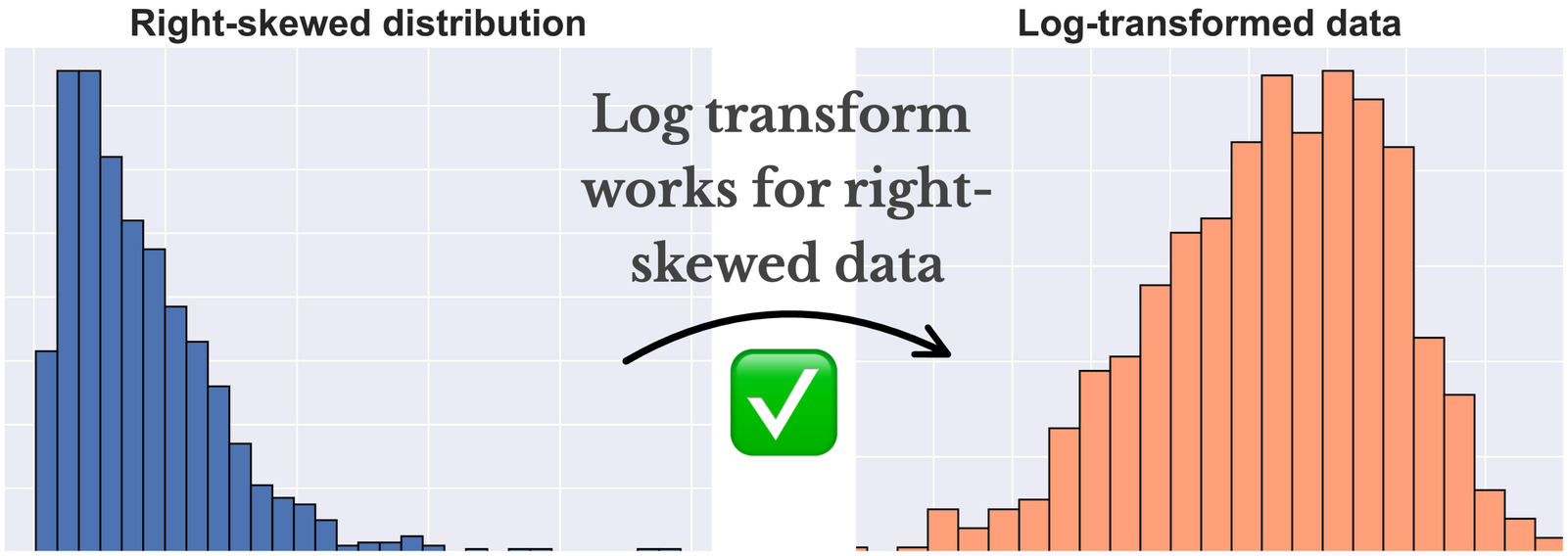
We reject the null if U is small.

# Anything can be transformed!

- Rank transforms are useful with any data which has ordinality!
- Why don’t we use rank based statistics all the time, instead of wasting time with Student’s t etc??
  - We lost statistical power by doing the transform and also from the lack of strong distributional assumptions in the test
- Other transforms can force normality on something non-normal and then we can use normal distribution assumptions and tests.  Like, e.g. the log transform…



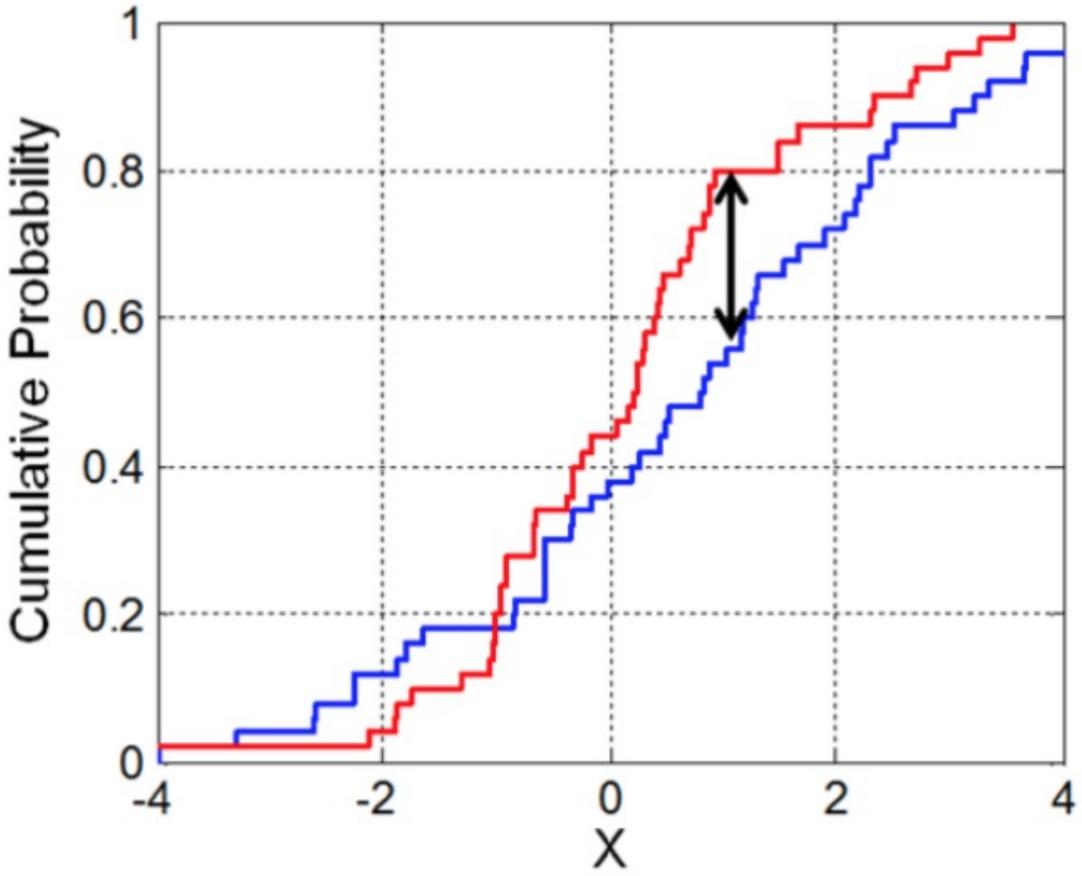

log_data = np.log(data+1)



# Kolmogorov-Smirnov (KS) test

## Comparing cumulative distributions regardless of their form

Find the maximum difference between the CDFs.



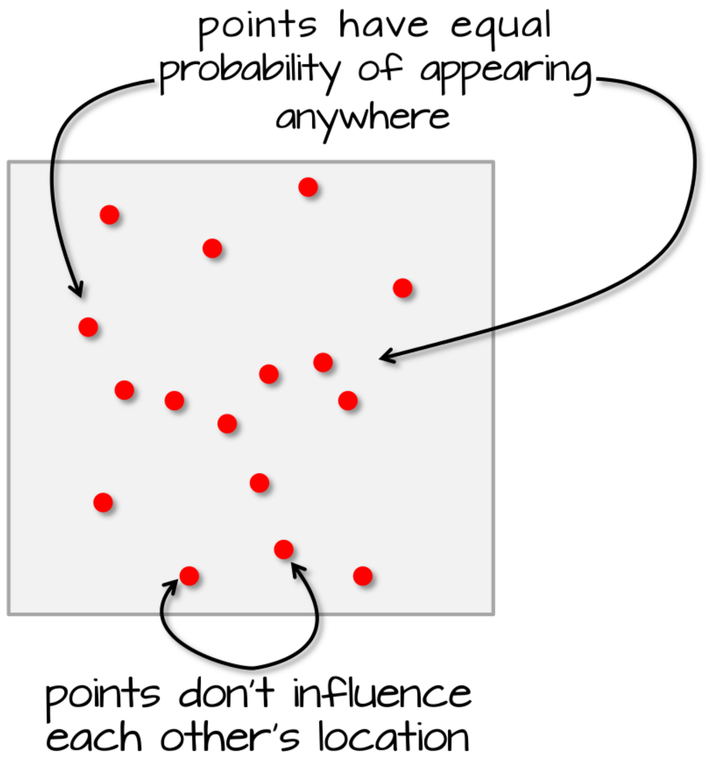

Tests:

- whether a sample is drawn from a given distribution
- Whether two samples are drawn from the same distribution

# Geospatial statistical inference

## What is a null distribution?

Compare observed point patterns to ones generated by an independent random process (IRP), aka complete spatial randomness (CSR).



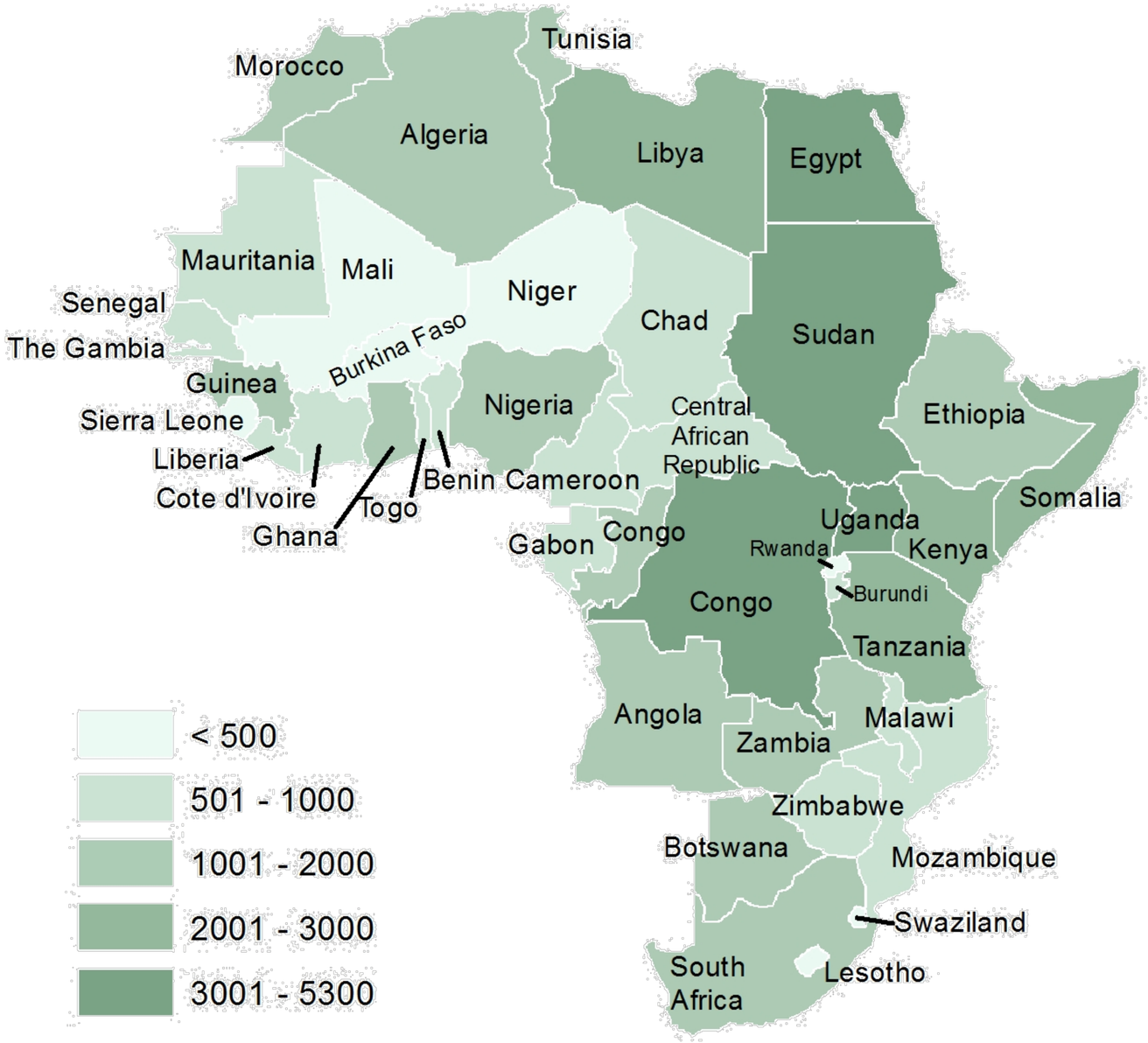

CSR/IRP satisfy two conditions:

1. Any event has equal probability of being in any location, a 1st order effect.
2. The location of one event is independent of the location of another event, a 2nd order effect

https://mgimond.github.io/Spatial/introGIS.html

# What metrics are we doing inference on in geospatial applications?

Are countries with a high conflict index score geographically clustered?

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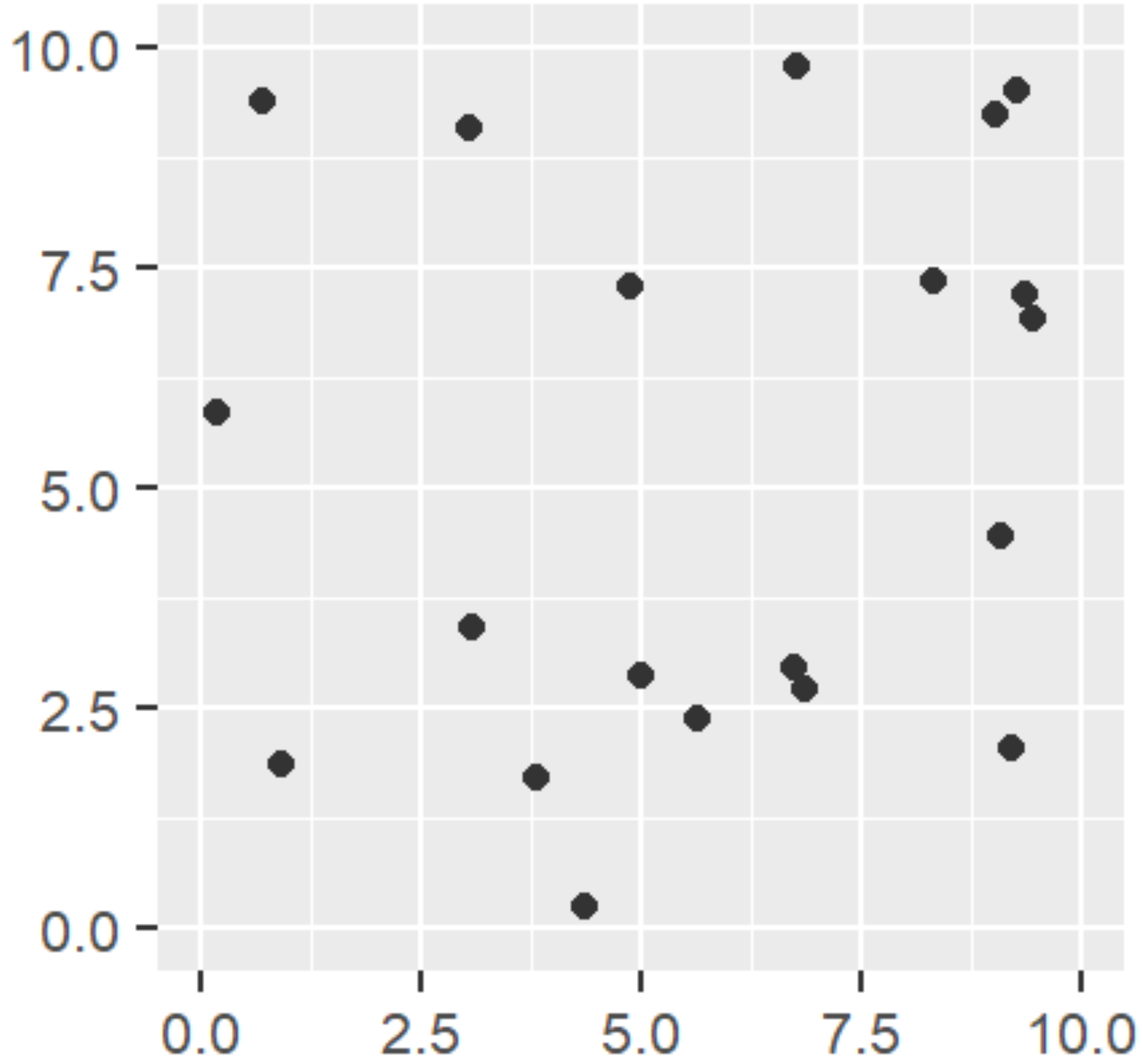
# Global Point Density

the ratio of observed number of points to the study region's surface area

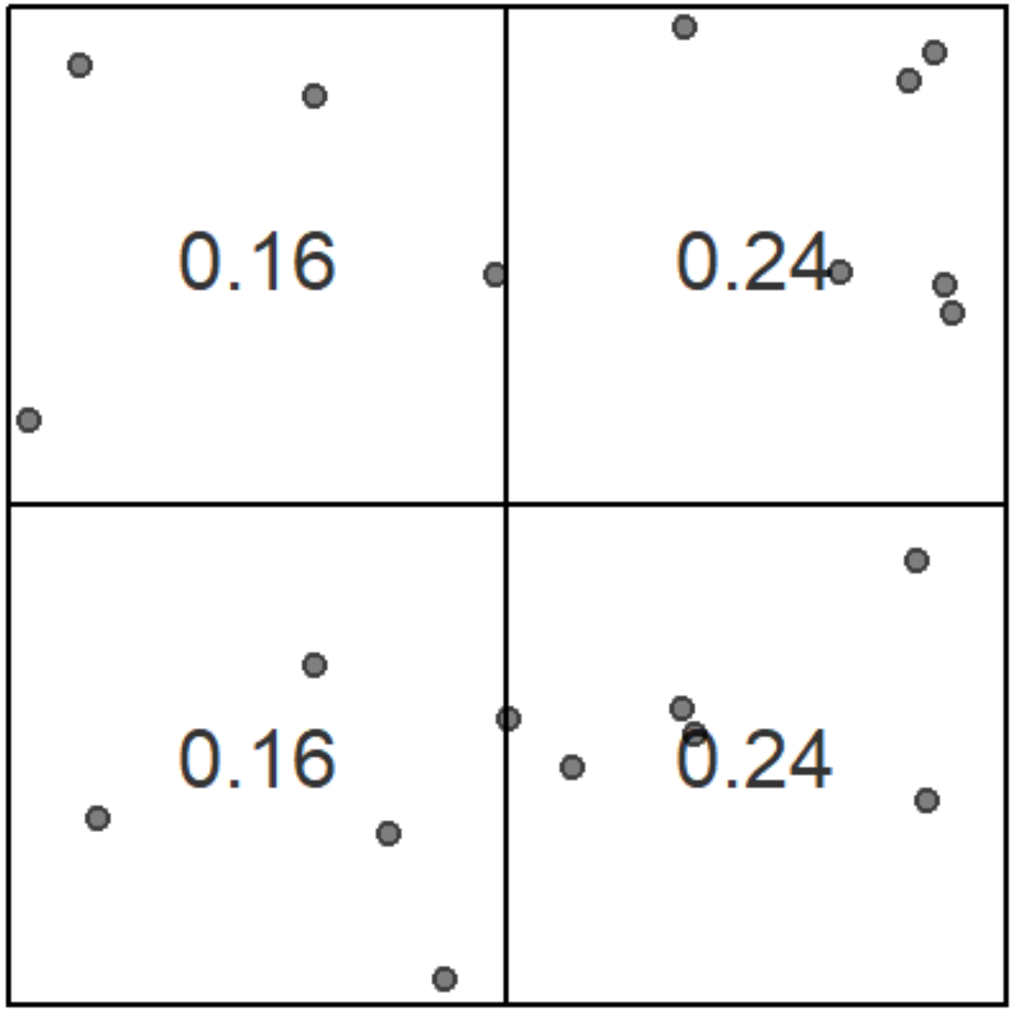



https://mgimond.github.io/Spatial/introGIS.html

# Quadrat Density (local)

Surface is divided and then point density is calculated within quadrat

Note: quadrat number and shape will affect measurement estimate. Suffers from MAUP.

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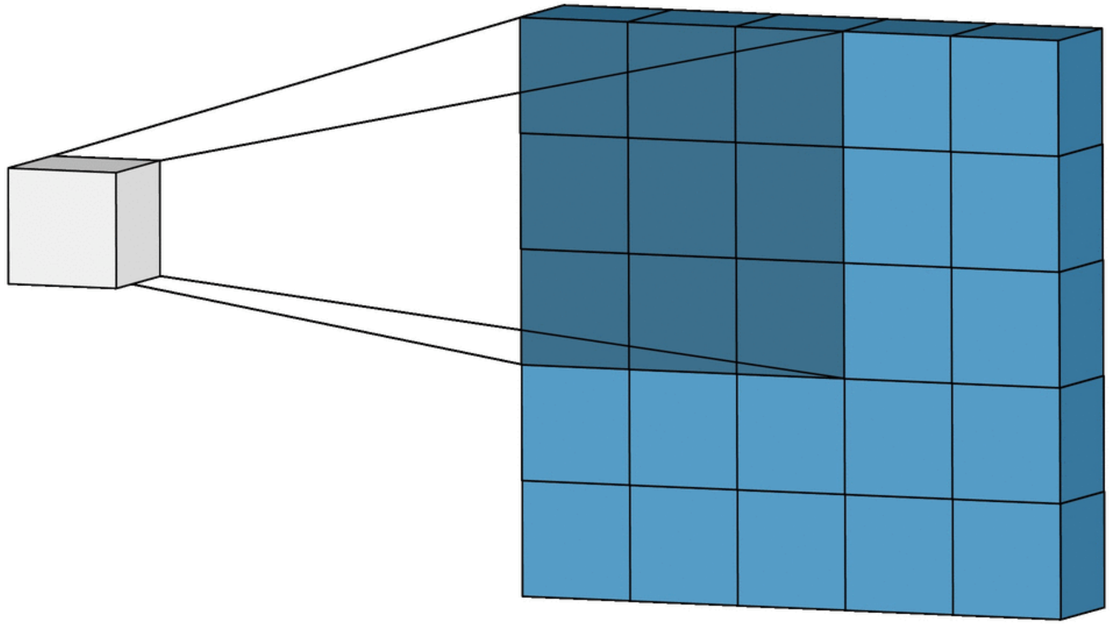
# Kernel Density (local)

Point density is calculated within sliding windows (window size = kernel)

Note: kernel will affect measurement estimate, but this is less susceptible to MAUP.



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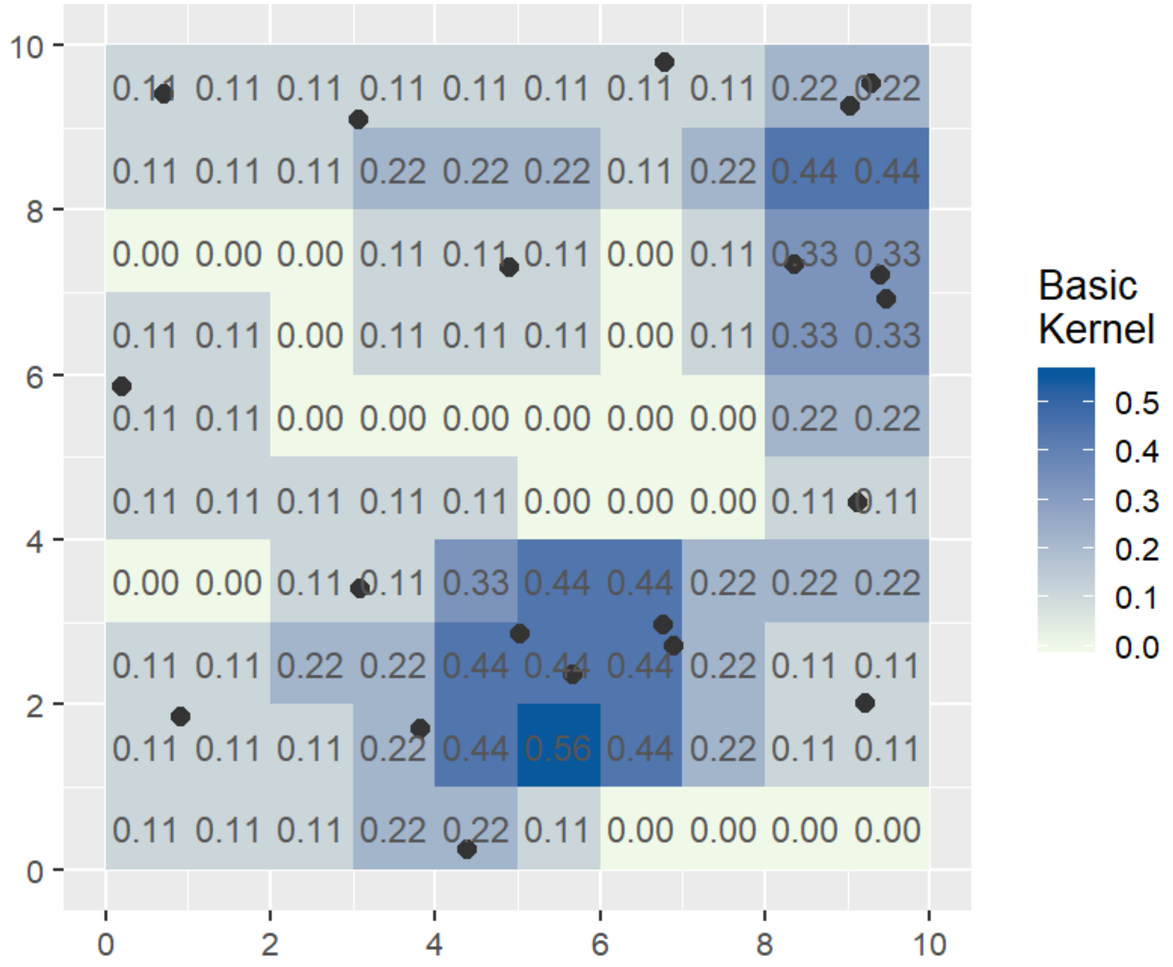


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# Modeling these data: Poisson Point Process

(Density-based Methods - - how the points are distributed relative to the study space)

$$\lambda(i) = e^{\alpha+\beta Z(i)}$$

λ(i) is the modeled intensity at location *i*

$e^{\alpha}$ is the base intensity when the covariate is *zero*

$e^{\beta}$ is the multiplier by which the intensity increases (or decreases) for each 1 unit increase in the covariate

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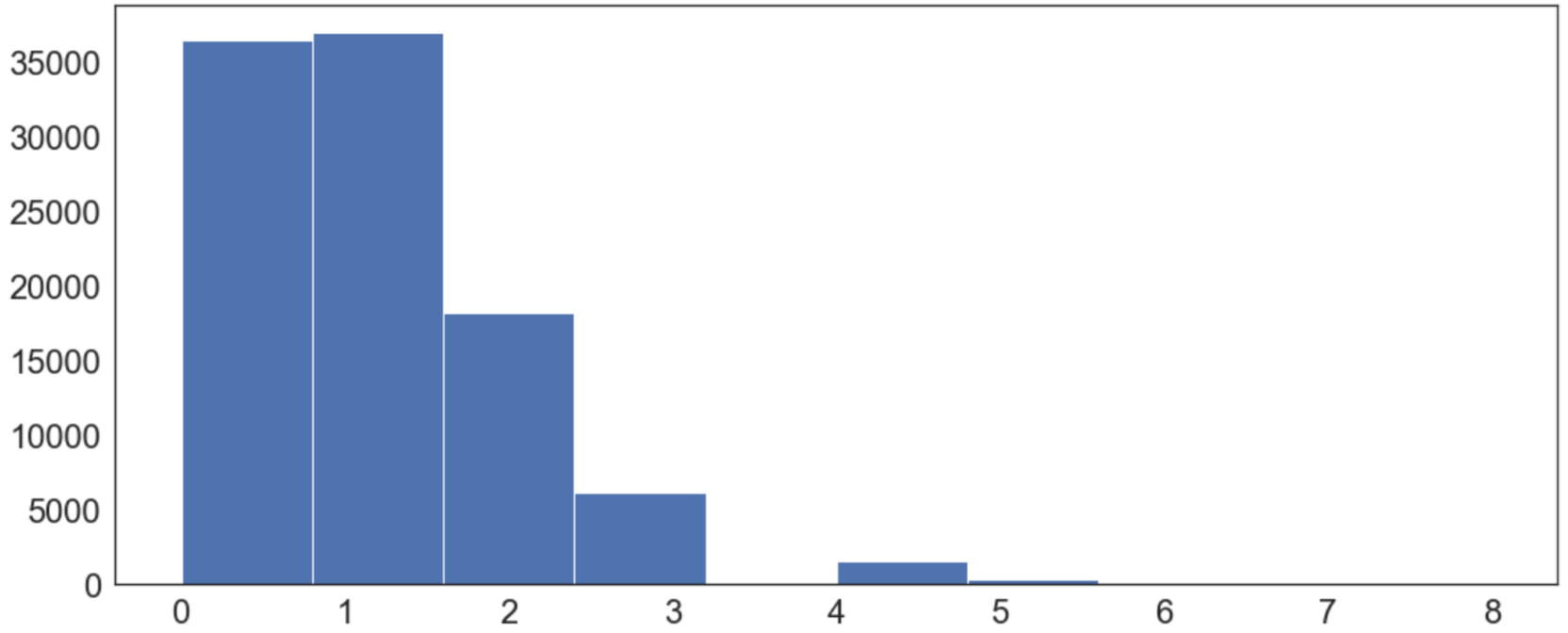
## Poisson Distribution

The Poisson Distribution models events in fixed intervals of time, given a known average rate (and independent occurences).

In [55]:

Slide Type Fragment

```
dat = poisson.rvs(mu=1, size=100000)
plt.hist(dat);
```

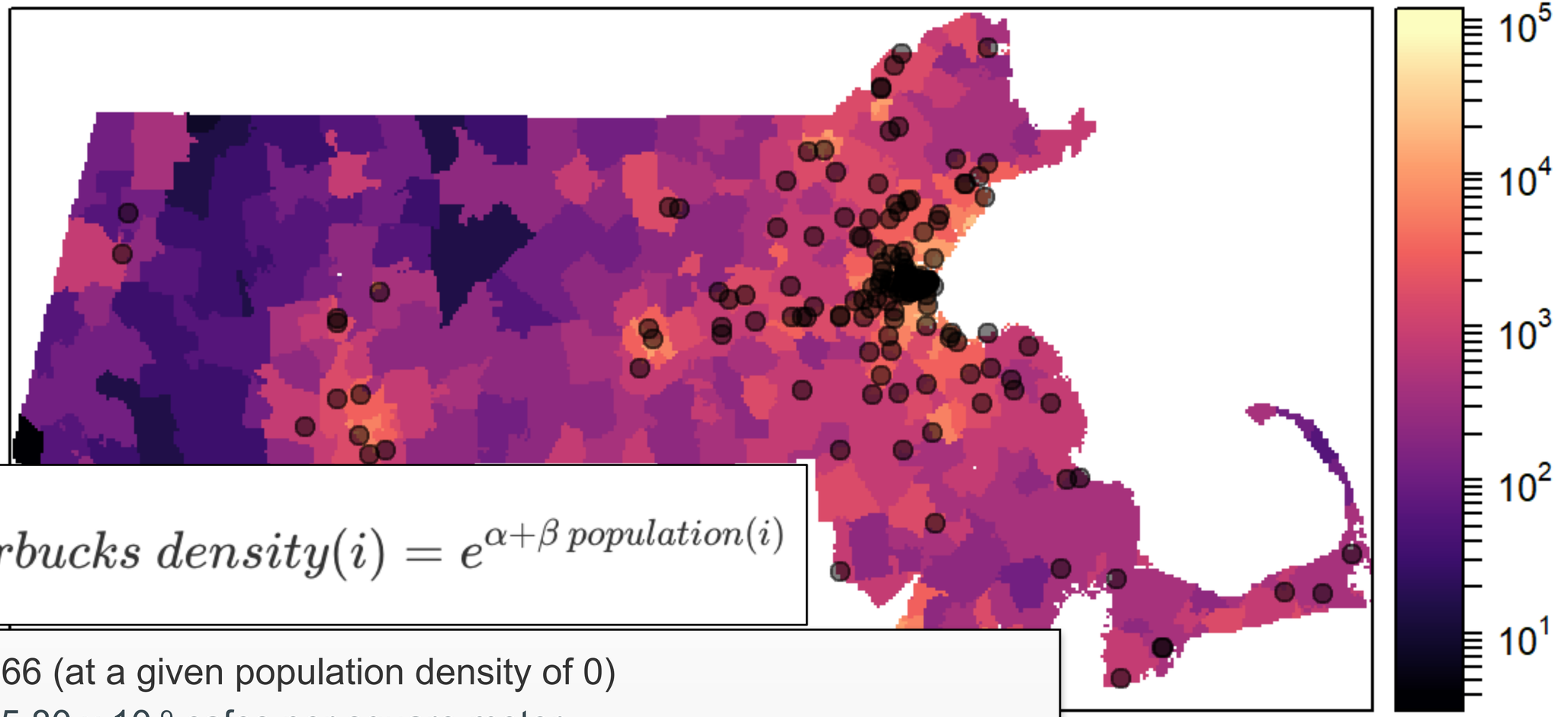



Slide Type Fragment

The **number of visitors a fast food drive-through gets each minute** follows a Poisson distribution. In this case, maybe the average is 3, but there's some variability around that number.

A Poisson distribution can help calculate the probability of various events related to customers going through the drive-through at a restaurant. It will predict lulls (0 customers) and flurry of activity (5+ customers), allowing staff to plan and schedule more precisely.

# Location of Starbucks relative to population density in MA




$$Starbucks\ density(i) = e^{\alpha + \beta\ population(i)}$$

$\alpha$ = -18.966 (at a given population density of 0)

$e^{-18.966}$ = 5.80 x $10^{-9}$ cafes per square meter

$\beta$ = 0.00017; $e^{0.00017}$ or 1.00017

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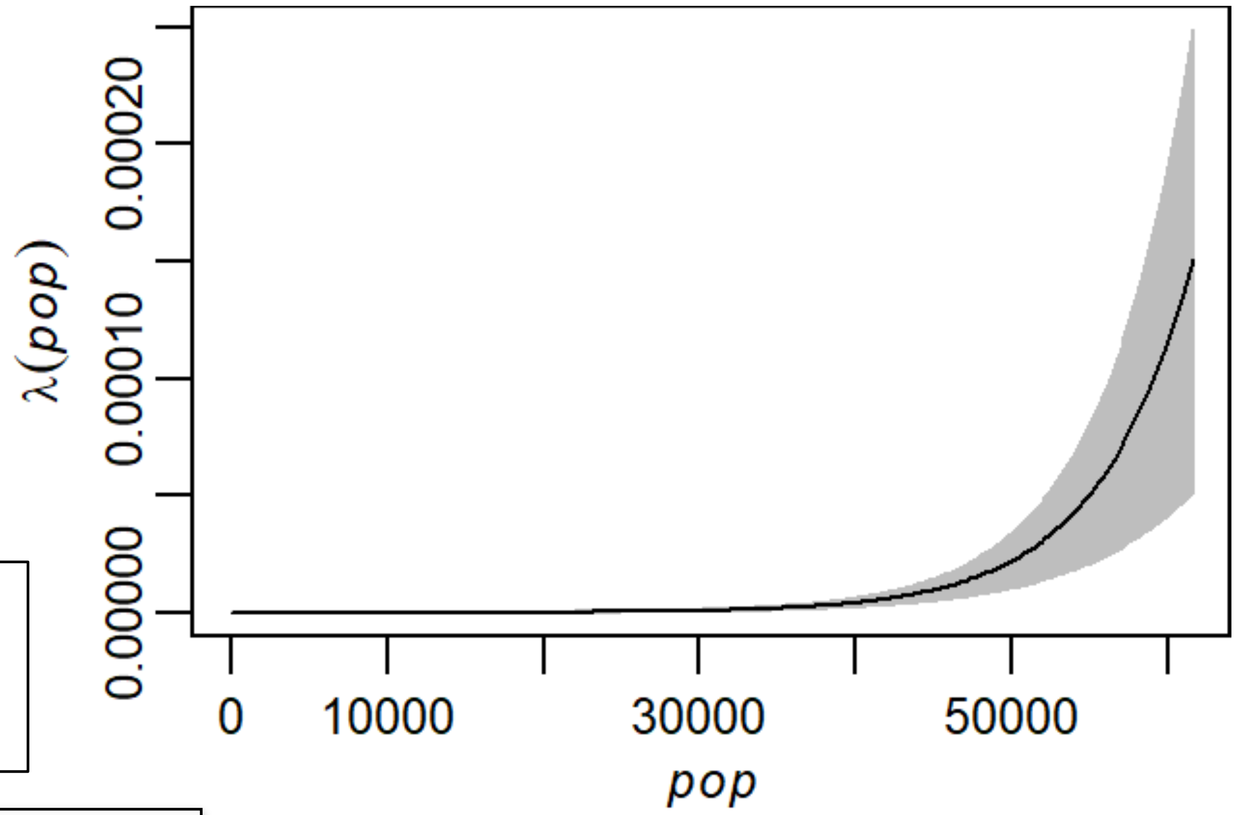


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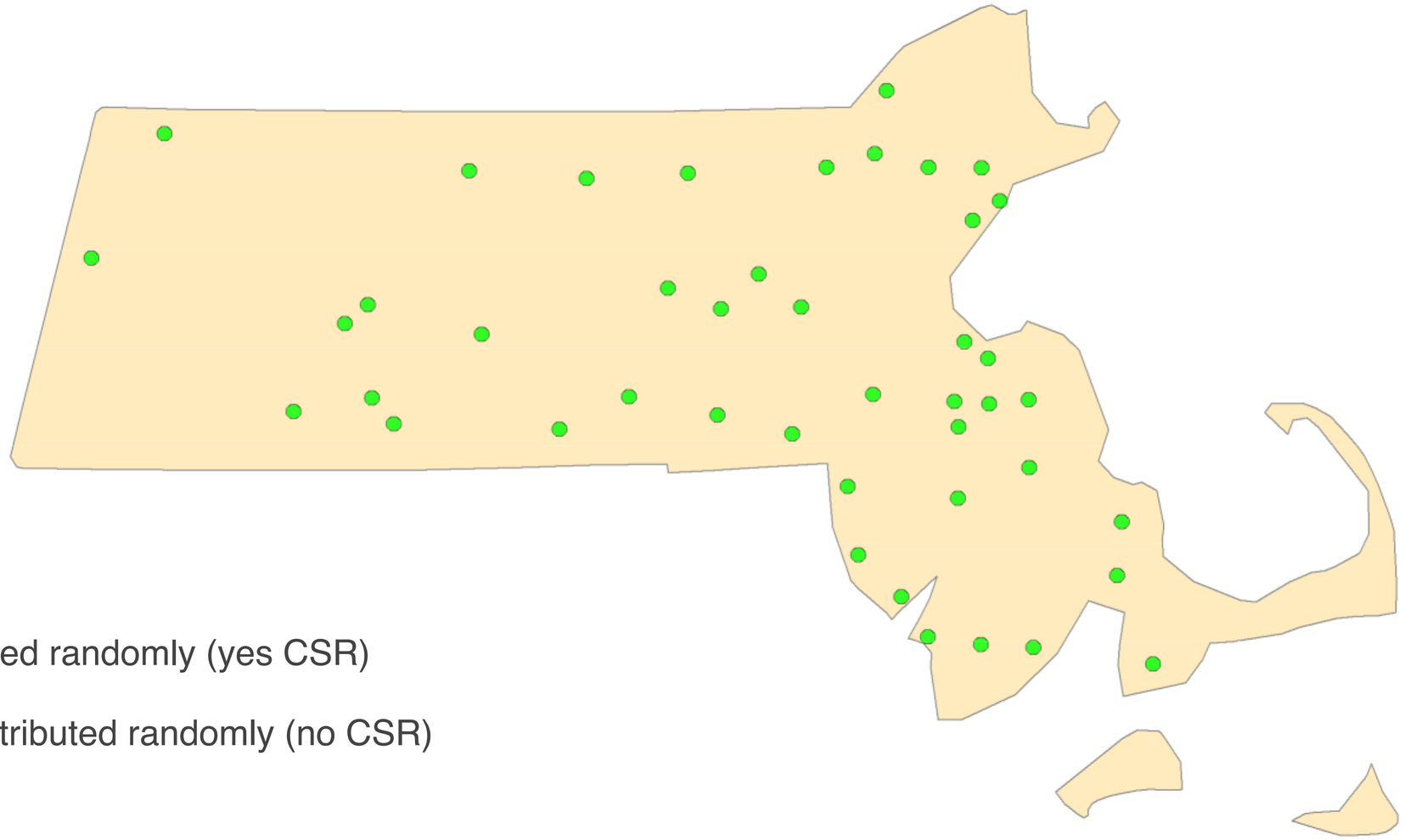
# Is this distribution of Walmarts in MA the result of CSR?



$H_o$: Distributed randomly (yes CSR)

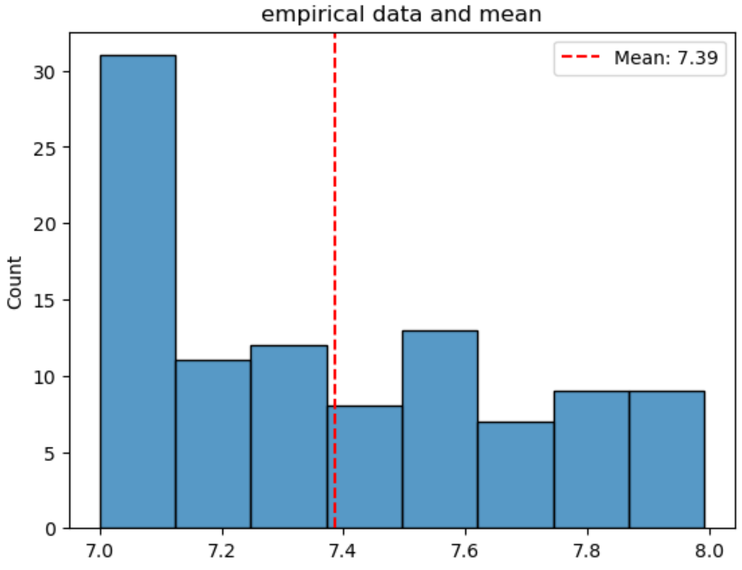
$H_a$: NOT distributed randomly (no CSR)

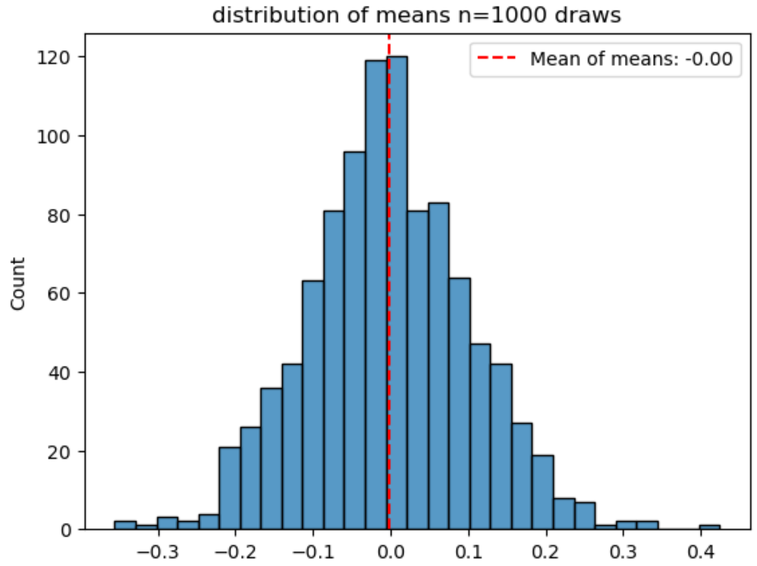
https://mgimond.github.io/Spatial/introGIS.html

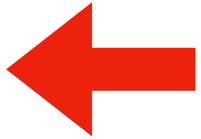
# Monte Carlo simulation

## Making an empirical null distribution

1. First, we postulate a null hypothesis $H_o$
2. Next, we simulate many draws from of our postulated process and compute a statistic on each one
3. Finally, we calculate the p-value of our real observed data in comparison to the simulated null distribution

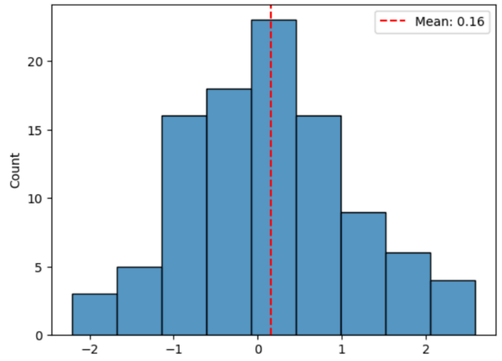


```
from scipy import stats
import seaborn as sns
```

```
data=stats.norm.rvs(size=100); ax=sns.histplot(data); ax.axvline(np.mean(data), color=
```

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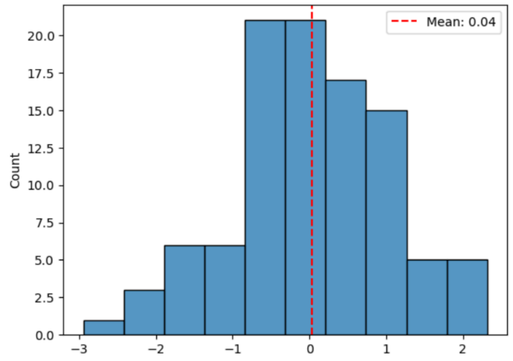


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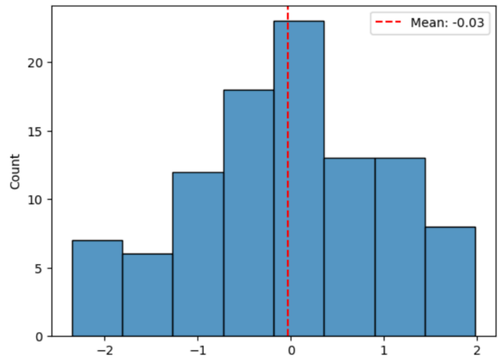

# MONTE CARLO simulations

1. First, we postulate a process–our null hypothesis, $H_o$. For example, we hypothesize that the distribution of Walmart stores is consistent with a completely random process (CSR).
2. Next, we simulate many realizations of our postulated process and compute a statistic (e.g., mean distance to nearest neighbor) for each realization.
3. Finally, we compare our observed data to the patterns generated by our simulated processes and assess (via a measure of probability) if our pattern is a likely realization of the hypothesized process.

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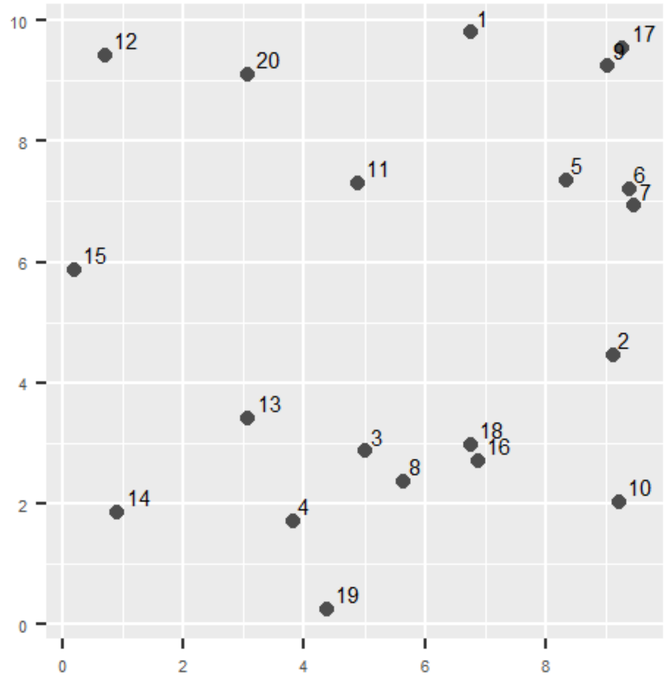
# Modeling these data: Average Nearest Neighbor

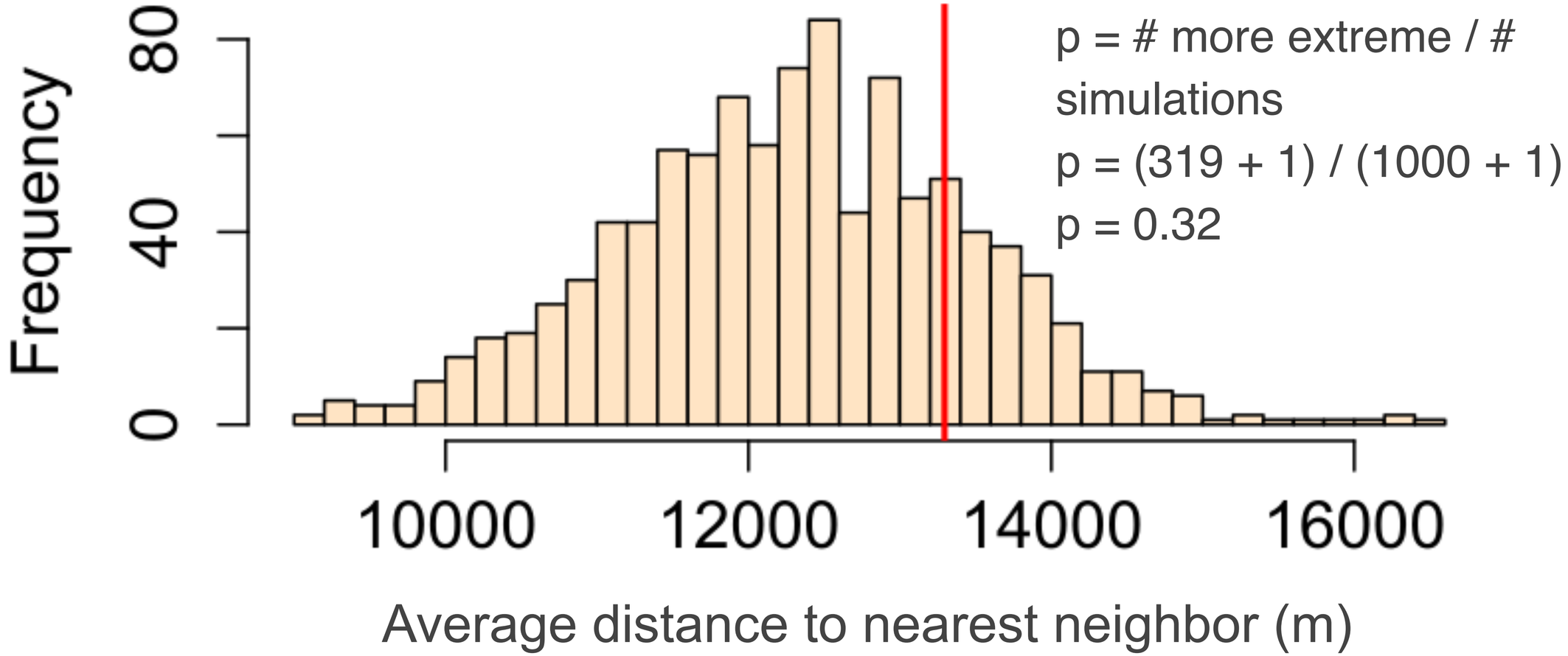
(Distance-based Methods - how the points are distributed relative to one another)




| From | To | Distance | From | To | Distance |
|---|---|---|---|---|---|
| 1 | 9 | 2.32 | 11 | 20 | 2.55 |
| 2 | 10 | 2.43 | 12 | 20 | 2.39 |
| 3 | 8 | 0.81 | 13 | 4 | 1.85 |
| 4 | 19 | 1.56 | 14 | 13 | 2.67 |
| 5 | 6 | 1.05 | 15 | 12 | 3.58 |
| 6 | 7 | 0.3 | 16 | 18 | 0.29 |
| 7 | 6 | 0.3 | 17 | 9 | 0.37 |
| 8 | 3 | 0.81 | 18 | 16 | 0.29 |
| 9 | 17 | 0.37 | 19 | 4 | 1.56 |
| 10 | 2 | 2.43 | 20 | 12 | 2.39 |

ANN = 1.52 units

https://mgimond.github.io/Spatial/introGIS.html

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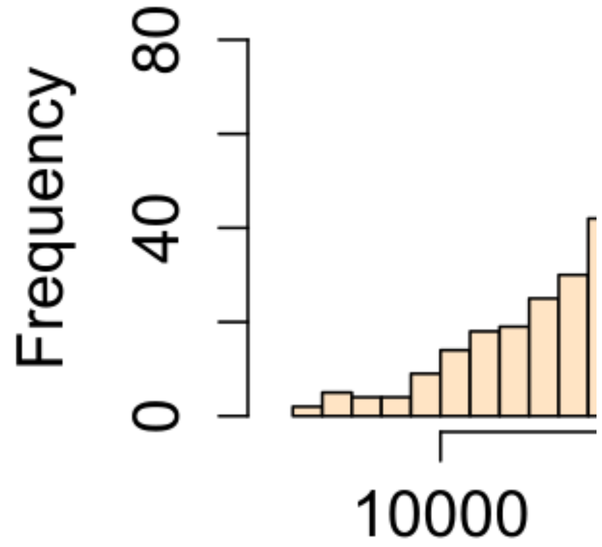



# What does the histogr

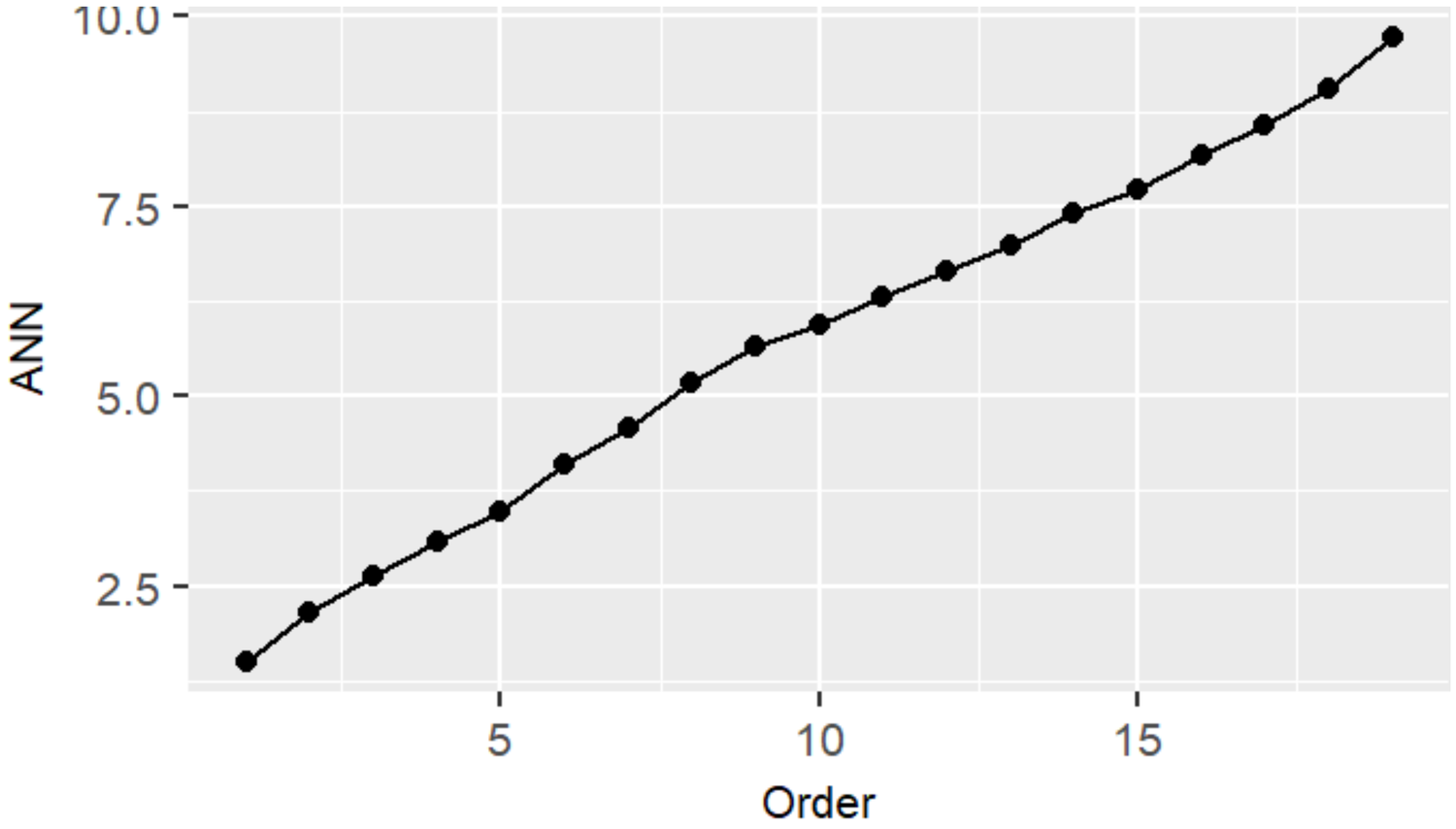
A
Alternative
Hypothesis

B
Null
Hypothesis

https://forms.gle/5a2vacFEz3qLjU7K9

# We aren’t just limited to “nearest neighbor” and calculating a statistic

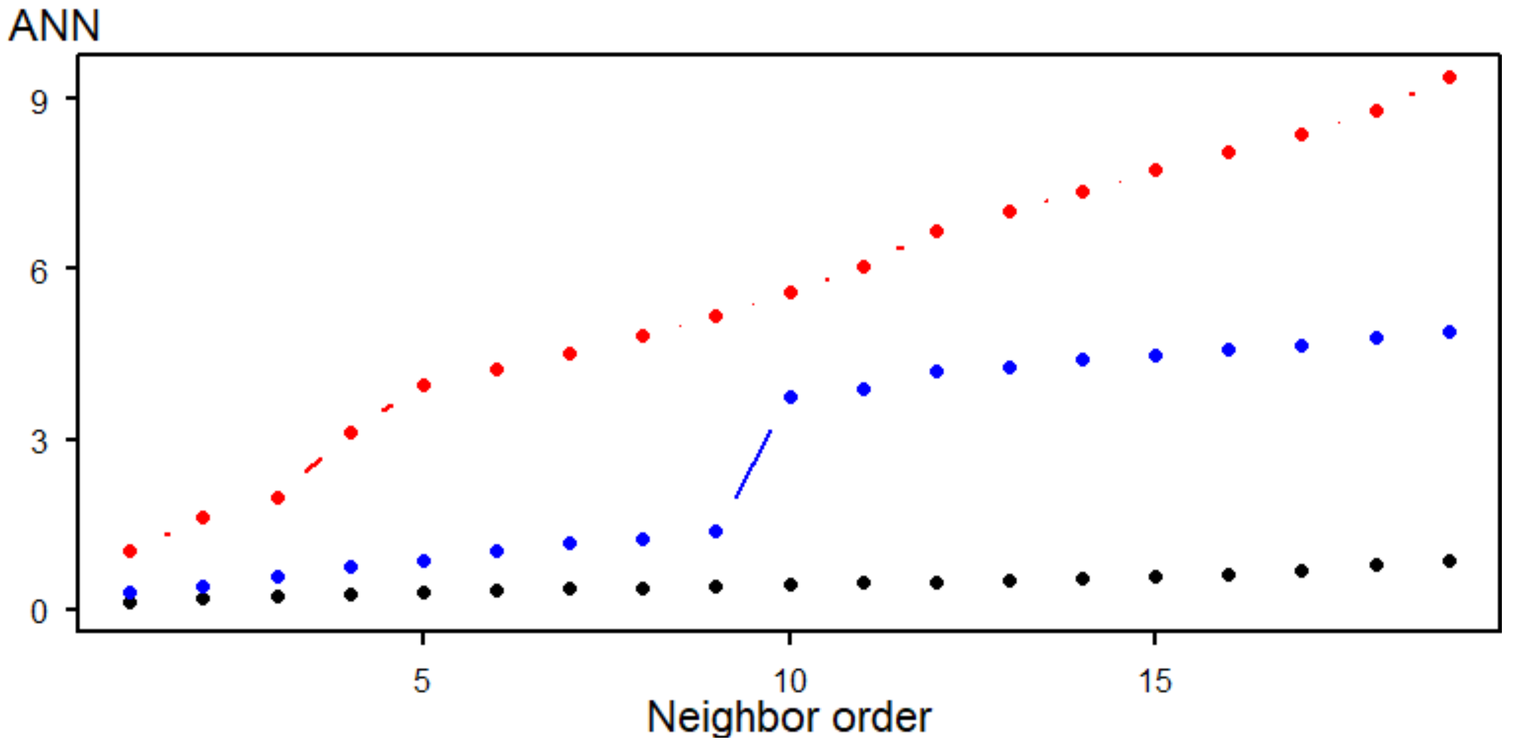
plot the ANN values for different order neighbors, that is for the first closest point, then the second closest point, and so forth.

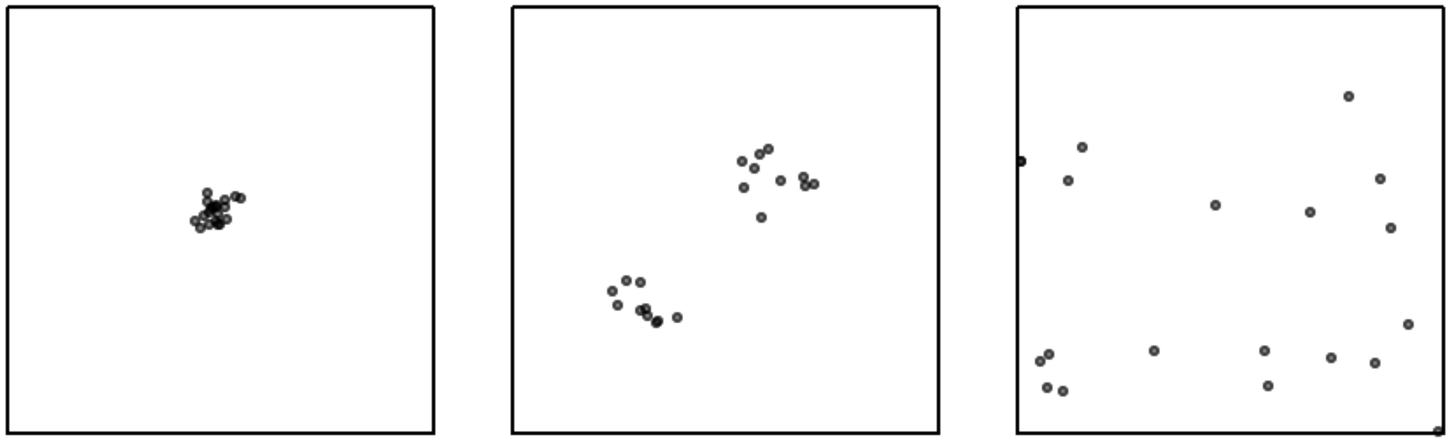



https://mgimond.github.io/Spatial/introGIS.html

ANN vs neighbor order offers insight into underlying spatial relationship

Note: study space definition affects this measure



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