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CSE 122 Homework 1

Problem 1: Show $1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$

Base Case: $n = 1$

$1^2 = 1$

$1(1+1)(2(1)+1)/6 = 1$

Assume:

$1^2 + 2^2 + \ldots + k^2 = k(k+1)(2k+1)/6$

Show:

$1^2 + 2^2 + \ldots + k^2 + (k+1)^2 = (k+1)(k+2)(2k+3)/6$

Proof:

$1^2 + 2^2 + \ldots + k^2 + (k+1)^2 = [1^2 + 2^2 + \ldots + k^2] + (k+1)^2$

$= [k(k+1)(2k+1)/6] + (k+1)^2$

$= (2k^3 + 9k^2 + 13k + 6)/6$

$= (k+1)(k+2)(2k+3)/6$

$(k+1)(k+2)(2k+3)/6 = (k+1)(k+2)(2k+3)/6$

TRUE

Problem 2: Show $1^3 + 2^3 + \ldots + n^3 = [n(n+1)/2]^2$

Base Case: $n = 1$

$1^3 = 1$

$[1(1+1)/2]^2 = 1$

Assume:

$1^3 + 2^3 + \ldots + k^3 = [k(k+1)/2]^2$

Show:

$1^3 + 2^3 + \ldots + k^3 + (k+1)^3 = [(k+1)(k+2)/2]^2$

Proof:

$[1^3 + 2^3 + \ldots + k^3] + (k+1)^3 = [(k+1)(k+2)/2]^2$

$= [k(k+1)/2]^2 + (k+1)^3$

$= (k^4 + 2k^3 + k^2)/4 + (k^3 + 3k^2 + 3k + 1)$

$= (k^4 + 6k^3 + 13k^2 + 12k^4)/4$
$[(k+1)(k+2)/2]^2 = [(k+1)(k+2)/2]^2$
TRUE

Problem 3: Show $1*1! + 2*2! + \ldots + n*n! = (n+1)! - 1$
Base Case: $n = 1$
$1*1! = 1$
$(1+1)! - 1 = 1$
Assume:
$1*1! + 2*2! + \ldots + k*k! = (k+1)! - 1$
Show:
$1*1! + 2*2! + \ldots + k*k! + (k+1)*(k+1)! = (k+2)! - 1$
Proof:
$1*1! + 2*2! + \ldots + k*k! + (k+1)*(k+1)! = (k+2)! - 1$
$= [(k+1)! - 1] + (k+1)*(k+1)!$
$= (k+1)!*(1+k+1) - 1$
$= (k+1)!*(k+2) - 1$
$= (k+2)! - 1$
$(k+2)! - 1 = (k+2)! - 1$
TRUE

Problem 4: Show $2^n > n^2$ when $n > 4$
Base Case: $n = 5$
$2^5 = 32$
$5^2 = 25$
$32 > 25$
Assume:
$2^k > k^2$ when $k > 4$
Show:
$2^k + 2^{k+1} > (k+1)^2$
Proof:
$2^k + 2^{k+1} > (k+1)^2$
$2^k + 2^k * 2 > k^2 + 2k + 1$
TRUE

Problem 5: Show $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2 - 1)$
Base Case: $n = 1$
$(2(1) - 1)^3 = 1$

$1^2(2(1)^2-1)=1$

Assume:

$1^3+3^3+5^3+\ldots+(2k-1)^3=k^2(2k^2-1)$

Show:

$1^3+3^3+5^3+\ldots+(2k-1)^3+(2k+1)^3=(k+1)^2(2(k+1)^2-1)$

Proof:

$[1^3+3^3+5^3+\ldots+(2k-1)^3]+(2k+1)^3=(k+1)^2(2(k+1)^2-1)$

$=[k^2(2k^2-1)]+(2k+1)^3$

$=2k^4-k^2+(8k^3+8k^2+2k+4k^2+4k+1)$

$=2k^4+8k^3+11k^2+6k+1$

$(k+1)^2(2(k+1)^2-1)=(k+1)^2(2(k+1)^2-1)$

TRUE

Problem 6: Show

$\frac{1}{(1)(2)}+\frac{1}{(2)(3)}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Base Case: $n=1$

$\frac{1}{1(1+1)}=\frac{1}{2}$

$\frac{1}{1+1}=\frac{1}{2}$

Assume:

$\frac{1}{(1)(2)}+\frac{1}{(2)(3)}+\ldots+\frac{1}{k(k+1)}=\frac{k}{k+1}$

Show:

$\frac{1}{(1)(2)}+\frac{1}{(2)(3)}+\ldots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

Proof:

$[\frac{1}{(1)(2)}+\frac{1}{(2)(3)}+\ldots+\frac{1}{k(k+1)}]+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

$=[\frac{k}{k+1}]+\frac{1}{(k+1)(k+2)}$

$=\frac{k(k+2)}{(k+1)(k+2)}+\frac{1}{(k+1)(k+2)}$

$=\frac{k^2+2k+1}{(k+1)(k+2)}$

$=\frac{(k+1)(k+1)}{(k+1)(k+2)}$

$\frac{k+1}{k+2}=\frac{k+1}{k+2}$

TRUE

Problem 7: Show

$S=\sum_{j=0}^{n} ar^j=\frac{ar^{n+1}-a}{r-1}, r\neq 1$

Base Case: $n=0$,$r=2$, $j=0$

$a*2^0=a$

$\frac{a*2^{0+1}-a)}{2-1}=\frac{2a-a}{1}=a$

Assume:

$S = \sum\limits_{j=0}^{k} ar^j = \frac{ar^{k+1}-a}{r-1}, r \neq 1$

$ar^0 + ar^1 + ar^2 + \ldots + ar^k = \frac{ar^{k+1}-a}{r-1}$

Show:

$ar^0 + ar^1 + ar^2 + \ldots + ar^k + ar^{k+1} = \frac{ar^{k+2}-a}{r-1}$

Proof:

$[ar^0 + ar^1 + ar^2 + \ldots + ar^k] + ar^{k+1} = \frac{ar^{k+2}-a}{r-1}$

$= [\frac{ar^{k+1}-a}{r-1}] + ar^{k+1}$

$= [\frac{ar^{k+1}-a+ar^{k+2}-ar^{k+1}}{r-1}]$

$= \frac{ar^{k+2}-a}{r-1}$

$\frac{ar^{k+2}-a}{r-1} = \frac{ar^{k+2}-a}{r-1}$

TRUE

Problem 8: Show

$S = \sum\limits_{i=1}^{n+1} i * 2^i = n * 2^{n+2} + 2$, for all integers $n \geq 0$

Base Case: $n = 0$

$1 * 2^1 = 2$

$0 * 2^{0+2} + 2 = 2$

Assume:

$(1 * 2^1) + (2 * 2^2) + \ldots + (k * 2^k) + ((k+1) * 2^{k+1}) = k * 2^{k+2} + 2$

Show:

$(1 * 2^1) + (2 * 2^2) + \ldots + (k * 2^k) + ((k+1) * 2^{k+1}) + ((k+2) * 2^{k+2}) = (k+1) * 2^{k+3} + 2$

Proof:

$[(1 * 2^1) + (2 * 2^2) + \ldots + (k * 2^k) + ((k+1) * 2^{k+1})] + ((k+2) * 2^{k+2}) = (k+1) * 2^{k+3} + 2$

$= (k * 2^{k+2} + 2) + ((k+2) * 2^{k+2})$

$= k * 2^{k+2} + 2 + k * 2^{k+2} + k * 2^{k+3}$

$= 2^{k+3} * (k+1) + 2$

$(k+1) * 2^{k+3} + 2 = (k+1) * 2^{k+3} + 2$

TRUE