3.7 Orders of Magnitude

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An **order of magnitude** can be written in powers of 10. For example, the order of magnitude of 1500 is 3, since 1500 may be written as 1.5×10^3. Differences in order of magnitude can be measured on the logarithmic scale in factors of ten. Orders of magnitude are used to make approximate comparisons. If numbers differ by one order of magnitude, *x* is *about* ten times different in quantity than *y*. If values differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale: the larger value is less than ten times the smaller value.

An **order-of-magnitude estimate** of a variable whose precise value is unknown is an estimate rounded to the nearest power of ten. *For example*, an order-of-magnitude estimate for a variable between about 3 billion and 30 billion (*such as the human population of the Earth*) is 10 billion.For a number written in scientific notation, this logarithmic rounding scale requires rounding up to the next power of ten when the multiplier is greater than the square root of ten (about 3.162). For example, the nearest order of magnitude for 1.7×10^8 is 8, whereas the nearest order of magnitude for 3.7×10^8 is 9. An order-of-magnitude estimate is sometimes also called a zeroth order approximation.

Other orders of magnitude may be calculated using bases other than 10. The ancient Greeks ranked the nighttime brightness of celestial bodies by 6 levels in which each level was the fifth root of one hundred (about 2.512) as bright as the nearest weaker level of brightness, and thus the brightest level being 5 orders of magnitude brighter than the weakest indicates that it is $(100^{1/5})^5$ or a **factor** of 100 times brighter.These weren't accurate

because they used the human eye (*which yes sees everything logarithmically*) though, but it was a system used!

In words (long scale)	In words (short scale)	Prefix (Symbol)	Decimal	Power of ten	Order of magnitude
quadrillionth	septillionth	yocto- (y)	0.000000000000000000000001	10^{-24}	−24
trilliardth	sextillionth	zepto- (z)	0.000000000000000000001	10^{-21}	−21
trillionth	quintillionth	atto- (a)	0.000000000000000001	10^{-18}	−18
billiardth	quadrillionth	femto- (f)	0.000000000000001	10^{-15}	−15
billionth	trillionth	pico- (p)	0.000000000001	10^{-12}	−12
milliardth	billionth	nano- (n)	0.000000001	10^{-9}	−9
millionth	millionth	micro- (μ)	0.000001	10^{-6}	−6
thousandth	thousandth	milli- (m)	0.001	10^{-3}	−3

hundredth	hundredth	centi- (c)	0.01	10^{-2}	−2
tenth	tenth	deci- (d)	0.1	10^{-1}	−1
one	one	–	1	10^{0}	0
ten	ten	deca- (da)	10	10^{1}	1
hundred	hundred	hecto- (h)	100	10^{2}	2
thousand	thousand	kilo- (k)	1000	10^{3}	3
million	million	mega- (M)	1000000	10^{6}	6
milliard	billion	giga- (G)	1000000000	10^{9}	9
billion	trillion	tera- (T)	1000000000000	10^{12}	12
billiard	quadrillion	peta- (P)	1000000000000000	10^{15}	15
trillion	quintillion	exa- (E)	1000000000000000000	10^{18}	18
trilliard	sextillion	zetta- (Z)	10000000000000000000000	10^{21}	21

quadrillion	septillion	yotta- (Y)	1000000000000000000000000	10^{24}	24

Different decimal numeral systems of the world use a larger base to better envision the size of the number, and have created names for the powers of this larger base. The table shows what number the order of magnitude aim at for base 10 and for base 1000000.

Order of magnitude	Is $\log_{10}$ of	Is $\log_{1000000}$ of	Short scale	Long scale
1	10	1000000	million	million
2	100	1000000000000	trillion	billion
3	1000	1000000000000000000	quintillion	trillion