3.8 Precision, Accuracy, Significant Figures

Precision and Accuracy

Even if the theory is absolutely correct, a lot of things can go wrong in the experiment: perhaps, when we measure the height, we measure off 51.2 cm, instead of 50 cm that we were supposed to. Or, more likely, we press the "stop" botton on the stopwatch a little bit after the marble falls. We have to say "roughly" because we cannot expect any data set to have an *exact* $D \propto T^2$ (*or directly proportional*) relationship.

This can happen because of our reaction time (*time between when we hear the marble fall and when we press the button*) and this results in an error in **accuracy**. There is another type of error, called **precision**. When the stopwatch records 1.52 seconds, we do not know whether it's not really 1.5187 seconds, because our equipment, assuming it works perfectly, measures reliably only up to 0.01 seconds. The difference between "accuracy" and "precision" can be described in analogy with target-shooting in this way: "high precision" means each shot hits relatively closer to each other; the spread of points is smaller. "High accuracy" means that all the shots are close to the mark. Given a set of data, it can have "high precision" but "low accuracy", for example if we got a better stopwatch that records to four digits past decimal but did not solve the problem with the reaction time, all our measurements would be relatively close to each other, but still their average will be 0.1 second away from what it should be. We can also have "high accuracy" but "low precision", if we solved the reaction time problem by concocting an arrangement where the stopwatch will be pressed automatically when the marble fell, but the stopwatch itself measures reliably only up to 1/10th of a second.

It is a common goal in any experiment to achieve "high precision", "high accuracy" measurements. Generally, reducing the error in accuracy is regarded as part of a good experimental technique, but there is less we can do about the errors in precision. While a better equipment would help, in various situations, there are several types of fundamental noises that limit even the state-of-the-art instruments. In fact, it is a fundamental fact of physics, partly due to quantum mechanics, that *it is impossible to make an absolutely exact measurement*.

So, this is what we mean by "roughly". For each measurement, one defines error boundaries which indicate how much one estimates the measurement can be off by (*using statistics, concepts like "confidence level" and standard deviation is often used to make this meaning more precise*).

Significant Figures

Because all experimentally measured quantities have a certain uncertainty, or error, associated with them, we should also state how precise we expect the measurement to be. While this can be a somewhat complicated matter when conducting a thorough experiment, for us, a convention called significant figures, allows us to indicate the error in a concise manner.

Simply put, **significant figures** indicates how many meaningful digits are present in a measurement, with the error taken as ± 1 on the last digit. For example, when we say a stick measures 1.25 m, this measurement has three significant figures: We are pretty sure about the first two digits, 1 and 2, but because we don't know what comes after the last digit, we have ± 0.01 uncertainty (because, for example, if the actual number was 1.256, after rounding off, it would be 1.26 m). On the other hand, if we said that it measured 1.250 m, then we have four significant figures.

The convention is, "the number of significant figures is equal to the number of digits written, including the zeros after the decimal". However, this convention does not say how many significant figures "5000 m" has. It could mean that it is accurate up to the meter or it could also be a rough estimate accurate only up to a 1000 m---because there is no other way to write 5000 m, we simply can't know. One way to resolve this ambiguity is to use **scientific notation**, and in scientific notation, $5000\ \mathrm{m} = 5.00 \times 10^3\ \mathrm{m}$, if it had three significant figures.

The number of significant figures matters to us when we calculate another quantity from measured quantities. Suppose, we have a rectangular piece of foil and we want to know its area. We measure the sides to be 12.1 cm and 27.2 cm. And, as we know from geometry, the area of a rectangle is given by, $A = wl$ where w and l represent the width and length, so we multiply the numbers together to obtain, $A = 329.12cm^2$. However, the precision of this result is misleading, and $A = 329cm^2$, which has the same number of significant figures as what we started with, represents the value of the area and our confidence in the value better.

The general rules is, "when we take a product or quotient of two numbers, the number of significant figures of the final result is equal to that of the number with less significant figures". So, when we multiply 12 cm and 27.2 cm together, we should state our answer is $A = 330\ \mathrm{cm}^2$, or in scientific notation, $A = 3.3 \times 10^2\ \mathrm{cm}^2$.

When we add or subtract two numbers, the rule is slightly different, instead of trying to keep the number of significant figures the same, we try to keep the decimal place the same, taking after that of the less-precisely known number. For example, $12.3 + 421.5234 = 433.8$.

In applying these rules, one thing to keep in mind is common sense. These rules are meant only to be rough guidelines, and as such, we have to apply these rules as it makes sense.

Also, when we make a series of calculations, it is often preferable to wait until the very end before rounding off to the correct number of significant figures. Otherwise, you will get what is known as a "*round-off error*".