3.9 Vectors

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In mathematics, physics, and engineering, a **Euclidean vector** (*sometimes called a **geometric** or **spatial vector**, or—as here—simply a **vector***) is a geometric object that has magnitude (*length*) and direction and can be added to other vectors according to vector algebra. A Euclidean vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an *initial point A* with a *terminal point B*,and denoted by $\overrightarrow{AB}$.

A vector is what is needed to "carry" the point *A* to the point *B*; the Latin word *vector* means "carrier". It was first used by 18th century astronomers investigating planet rotation around the Sun. The magnitude of the vector is the distance between the two points and the direction refers to the direction of displacement from *A* to *B*. Many *algebraic operations* on real numbers such as addition, subtraction, multiplication, and negation have close analogues for vectors, operations which obey the familiar algebraic laws of *commutativity, associativity*, and *distributivity*. These operations and associated laws qualify Euclidean vectors as an example of the more generalized concept of vectors defined simply as elements of a vector space (a space that vectors can be in).

Vectors play an important role in physics: velocity and acceleration of a moving object and forces acting on it are all described by vectors. Many other physical quantities can be usefully thought of as vectors. Although most of them do not represent distances (like position or displacement), their magnitude and direction can be still represented by the length and direction of an arrow. The mathematical representation of a physical vector depends on the coordinate system used to describe it. Other vector-like objects that describe physical quantities and transform in a similar way under changes of the coordinate system include pseudovectors and tensors.

A vector is formally defined as an element of a vector space. In the commonly encountered vector space $\mathbb{R}^n$ (a Euclidean *n*-space), a vector is given by n coordinates and can be specified as $(A_1, A_2, \ldots, A_n)$.

Vectors are sometimes referred to by the number of coordinates they have, so a 2-dimensional vector (x_1, x_2) is often called a two-vector, an n-dimensional vector is often called an *n*-vector, and so on.

Vectors can be added together (vector addition), subtracted (vector subtraction) and multiplied by scalars (scalar multiplication). Vector multiplication is not uniquely defined, but a number of different types of products, such as the dot product, cross product, and tensor direct product can be defined for pairs of vectors.

A vector from a point A to a point B is denoted $\overrightarrow{AB}$, and a vector v may be denoted $\vec{v}$, or more commonly, $\mathbf{v}$. The point A is often called the "tail" of the vector, and B is called the vector's "head." A vector with unit length is called a unit vector and is denoted using a hat, $\hat{\mathbf{v}}$.

When written out componentwise, the notation $\mathbf{x}$ generally refers to $\mathbf{x} = (x_1, x_2, \ldots)$. On the other hand, when written with a subscript, the notation $\mathbf{x}_1$ generally refers to $\mathbf{x}_1 = (x_1, y_1, z_1, \ldots)$.

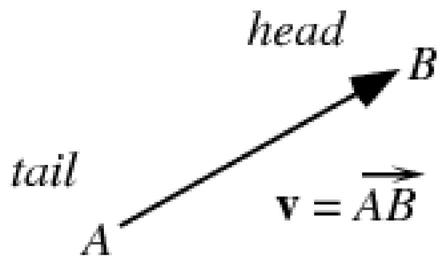
An arbitrary vector may be converted to a unit vector by dividing by its norm (which is the vectors length or magnitude). The bottom is the unit vector, the top is the magnitude, and the unit vector is the vector over the magnitude of the vector.

$$|\mathbf{v}| \equiv \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2},$$

$$\hat{\mathbf{v}} \equiv \frac{\mathbf{v}}{|\mathbf{v}|}.$$

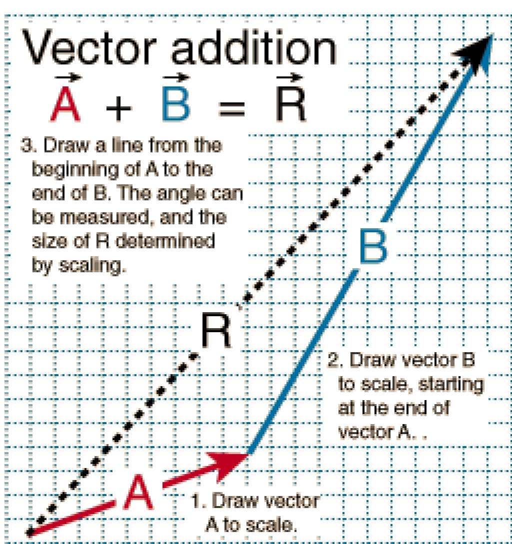
A zero vector, is a vector of length 0, and thus has all components equal to zero.

A Vector



The most basic vector will be a line with an arrow, the vector will have a length (magnitude) and a direction (where it is pointing). The start of it is called a tail and the end is called a head. We add component vectors from head to tail. Look below and see this. We can see that the tail of A is the starting point, the head of A touches the tail of B, and the head of B touches the head of R where this vector touches its tail to the tail of A and it's head to the head of B, this is how we add and subtract vectors.

Graphical Vector Addition

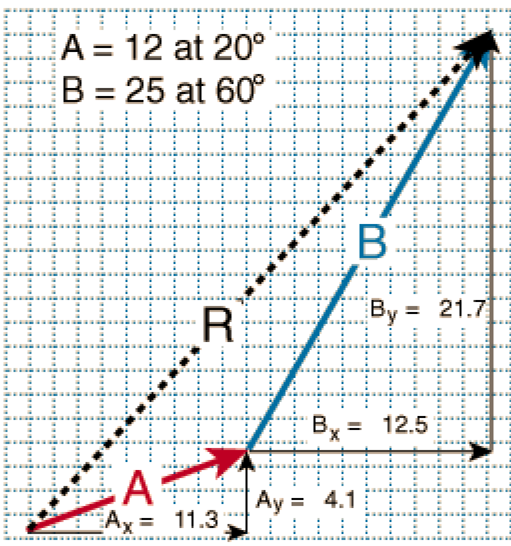


Adding two vectors A and B graphically can be visualized like two successive walks, with the vector sum being the vector distance from the beginning to the end point.

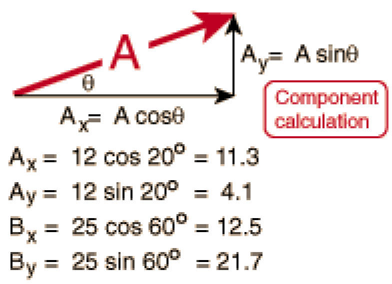
Representing the vectors by arrows drawn to scale, the beginning of vector B is placed at the end of vector A. The vector sum R can be drawn as the vector from the beginning to the end point.

The process can be done mathematically by finding the components of A and B, combining to form the components of R, and then converting to polar form.

Example of Vector Components



Finding the components of vectors for vector addition involves forming a right triangle from each vector and using the standard triangle trigonometry.



The vector sum can be found by combining these components and converting to polar form.

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