right click to open a **new tab** for *certain words* in red to learn something new!

3.1: Mathematical Foundations

Physics is a physical science, meaning it explains what we know about the physical world through principles that govern behaviors; that we have learned through both observational and experimentation. For this reason, the study of physics history, and a sort of **metacognition** while explaining physics is important. So, it is important to us because it leads us through the scientific methodology that lead the original discoverers to the principles that we have today. For this reason we will try to incorporate both physics and history.

The second thing, apart from the intertwining of *physics and the history of physics* is the intertwining of *physics and mathematics*; and ultimately *physics and mathematics history.* One should not fool themselves into thinking that every science is disconnected, because it is exactly the interconnections that describe the whole of physical processes.

I like to think of it like.

Mathematical Base + Physics Base + i(Philosophy Base) = Fundamental World View

Pulling from mathematics and physics fundamentals, we have some important names that we should keep in mind and remember.

Euclid	Newton	Archimedes	Gauss	Euler	Lambert
Riemann	Leibniz	Lagrange	Hilbert	Fermat	Legendre
Descartes	Jacobi	Cauchy	Pythagoras	Pascal	Fibonacci
Hamilton	Laplace	Hipparchus	Kepler	Bernoulli	Einstein
Aristotle	Minkowski				

Now a lot of the basic mathematics principles comes from the Greeks (*and was then lost to us in the Dark Ages and revived during the Renaissance).* Using generalities, which typically includes Greek driven variables, letters, and notation, is the best to use when trying to learn. **Think in generalities**, because often enough exact numbers leads to mistakes and confusion over the course of a long problem. It is also important to use symbols rather than numerical values when one is doing calculations.

Typically generalized variables follow the pattern as follows.

end of alphabet	X, Y, Z	→ unknown varibles	$f(x, y, z)$
letters	a, b, c	→ constant quantities	$f(x) = ax + by$
Greek letters	α, θ, κ	→ angular varibles higher order unknowns	$\alpha = \frac{a_1}{r}$ $\theta = \theta_0 + \omega t$
Sub/superscripts	X^n_μ, X_n, X^n	→ additional information for variable	$Y', t_1, X_2,$

We also have commonly used symbols.

$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a \simeq b$	a is approximately equal to b
$a \propto b$	a is proportional to b
$a \sim b$	a is with an order of magnitude of b
$a > b$	a is greater than b
$a < b$	a is less than b
$a \gg b$	a is much much greater than b
$a \ll b$	a is much much smaller than b

And an example of it's useage.

example:

A metal ball is 3cm in radius heated to a temperature of 500°C in a very hot furnace. If it's emissivity is .5, what rate does it radiate energy?

Given: Radiation Energy (P)

$P = \varepsilon \sigma A T^4$ $\qquad$ $A = 4\pi r^2$

$T(C^\circ \to K) = (\text{temp in } ^\circ C) + 273K$

Best Way to solve:

$A = 4\pi r^2$

$= 4\pi(.03m)^2$

$= .0113112\ m^2$

$T = 500\cancel{C} + 273K$

$= 773K$

Given:
$r = 3cm \to .03m$
$T = 500^\circ C$
$\varepsilon = .5$
$\sigma = 5.67\times10^{-8}$

$P = \varepsilon \sigma A T^4$

$P = (.5)(5.67\times10^{-8})(.0113112)(773)^4$

$P = 114.37W$

(no units used can use as a check)

Alternate Way

$P = \varepsilon \sigma A T^4$

$P = \varepsilon \sigma (4\pi r^2) T^4$

$P = 4\pi\ \varepsilon \sigma\ r^2\ T^4$

$P = (4)\cdot(\pi)\cdot(.5)\cdot(5.67\times10^{-8})(.03)^2(773K)^4$

$P = 114.37W$

Now what we have been using are variables, equations, and constants. We use these to describe physical phenomenon **quantitatively** and talk about it **qualitatively**. A number that is used to describe this quantitatively is called a **physical quantity**. It is defined two ways, one is a procedure for measuring the quantity and the second is a way to describe how to calculate the quantity.

For example, we want to know how fast something is traveling. So we figure that we know that the velocity of an object can be calculated by taking the distance the object travels (say in meters) in how long that takes (say in seconds). We determine that our physical quantity for velocity has to be in **units** of meters per second (*per* is a fun word that is Latin via Old French meaning 'through, by means of', so a **rate**). This leads to the topic of *standardizing units*.

It is important to have a standardized set of units so we can talk to each other in the *language of science*. We can describe a pig moving with a speed of 73 gugglehauts per quintives; but what the heck is a gugglehaut or a quintive? We take to inherently know the definition of the unit to describe properties correctly. This is why we have standardized units, typically we use a set of units described by the International System or SI units (*it is SI because in French it is the Système Internationale d'Unités*).

Typically we think of the basics so; second as a unit of time, meter as a unit of length, kilogram as a unit of mass. We like to use these units, and also know how to convert from one unit to another unit.

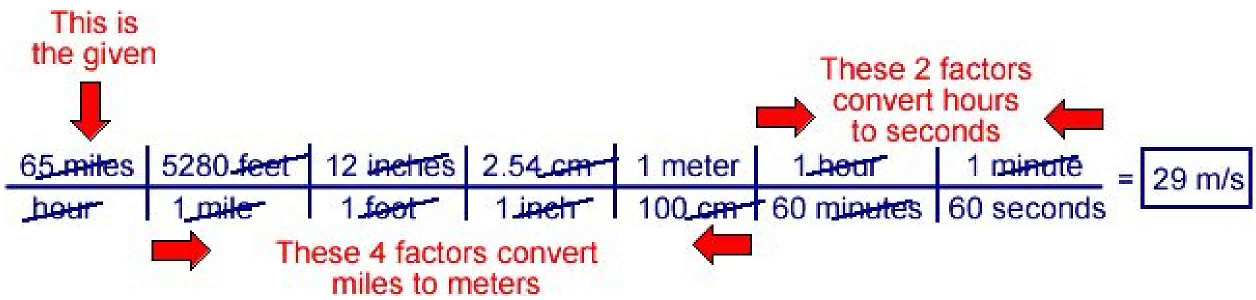
Base quantity	SI base unit: Name	SI base unit: Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	m/s
acceleration	meter per second squared	m/s^2
wave number	reciprocal meter	m^{-1}
mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m^3/kg
current density	ampere per square meter	A/m^2
magnetic field strength	ampere per meter	A/m
amount-of-substance concentration	mole per cubic meter	mol/m^3
luminance	candela per square meter	cd/m^2
mass fraction	kilogram per kilogram, which may be represented by the number 1	kg/kg = 1

Converting from one unit to another unit is where most people get messed up when doing a problem, or have an error called **perturbation error** (*where one mistake leads to more and more and more)*. When we convert from one unit to another it is called **dimensional analysis**, because we are taking the "dimension" of one unit and analyzing it to make it a different unit. We will show the long way to do this first, then show it using **Wolfram Alpha** and **Google** .

When we do dimensional analysis we want to convert from one unit to another, if we do that by hand we want every quantity on the top and bottom to “cancel out” until we are left with the correct unit that we want. So when we convert from a simple example, take inches to meters to kilometers. Our original quantity is in inches, our conversion is x inches to y meters, and our next conversion is from y meters to z kilometers. So inches on top cancels with inches on bottom, meters on top cancels with meters on bottom, and we are left with kilometers on top.

In a more complicated example say we are given a unit in miles per hour and want in in meters per second.



Let’s show this with the following example as well.

- We are given that the length of an object is 5´2 feet. We want to know this distance in inches, meters, kilometers, lightyears and parces. If we are in Earth's reference frame, which would be the best to use? 1 foot = 12 inches

Known: object is $\underbrace{5´2}$. – 5 feet 2 inches

↳ Though the phrase 5´2 feet is bad wording we (assume) it means 5 feet 2 inches from the English colloquisms we know like the phrase. "How tall are you?" "I'm 5´2".

↳ We need 5´2 to be a whole quanity in inches so.

→ unit feet (num) & foot cancel, so we are left in the unit of inches WHAT WE WANT

$$\frac{5\ \cancel{\text{feet}} \mid 12\text{ inches}}{\mid 1\ \cancel{\text{foot}}} = 60\text{ inches}$$

In Inches: Total height in inches = $\underbrace{60\text{ inches}}_{5\text{ feet}} + \underbrace{2\text{ inches}}_{2\text{ inches}} = \boxed{62\text{ inches}}$

Now we can easily convert to our other units.

$$\frac{62\text{ inches} \mid .0254\text{m}}{\mid 1\text{ inches}} = \boxed{1.5748\text{m}}\ \frac{\mid 1\text{km}}{\mid 1000\text{m}} = \boxed{.0015748\text{km}}$$

$$\frac{.0015748\ \cancel{\text{km}} \mid 1.057\times10^{-13}\text{ly}}{\mid 1\ \cancel{\text{km}}} = \boxed{1.664\times10^{-16}\text{ly}}$$

$$\frac{1.66\times10^{-16}\text{ly} \mid .306601\text{ pc}}{\mid 1\text{ly}} = \boxed{5.101847\times10^{-17}\text{pc}}$$

↳ In Earth's reference frame we use relative units, so unit's that are on the scale of humans (inches, meters, kilometers). Whereas lightyears and parces we use for the scale of the Cosmos (Universe). Meters is the best choice here

- We are given that the radius of the Sun is 1 R☉, what is this in meters? Kilometers? AU?

The radius of the Sun is approximated as it is not a perfect sphere & has gravitational effects that "swish" it's mass towards the Sun's equator. This approximation is designated as 1 Solar Radius (1 R☉) where ☉ is the Greek symbol (for Earth it is ⊕) and these are standard units for our solar system so we don't have to say ~ 10^{13} meters for everything)

$$1\,R_\odot = 432,687\text{ miles}$$

$$1\text{ mile} = 1609.34\text{ meters}$$

$$\frac{432,687\ \cancel{\text{miles}} \mid 1609.34\text{ meters}}{\mid 1\ \cancel{\text{mile}}}$$

$= 696342227.3\text{ m}$

7 decimal place → 10^7

↳ $\boxed{6.9\times10^7\text{m}}$ approximate

$$\frac{6.9\times10^7\text{m} \mid 1\text{km}}{\mid 1000\text{m}}$$

$= 69000$

4 decimal place

$= \boxed{6.9\times10^4\text{km}}$ approx

or order of magnitude

$6.9\ \frac{10^7}{10^3} \rightarrow 10^{7-3} = 10^4$

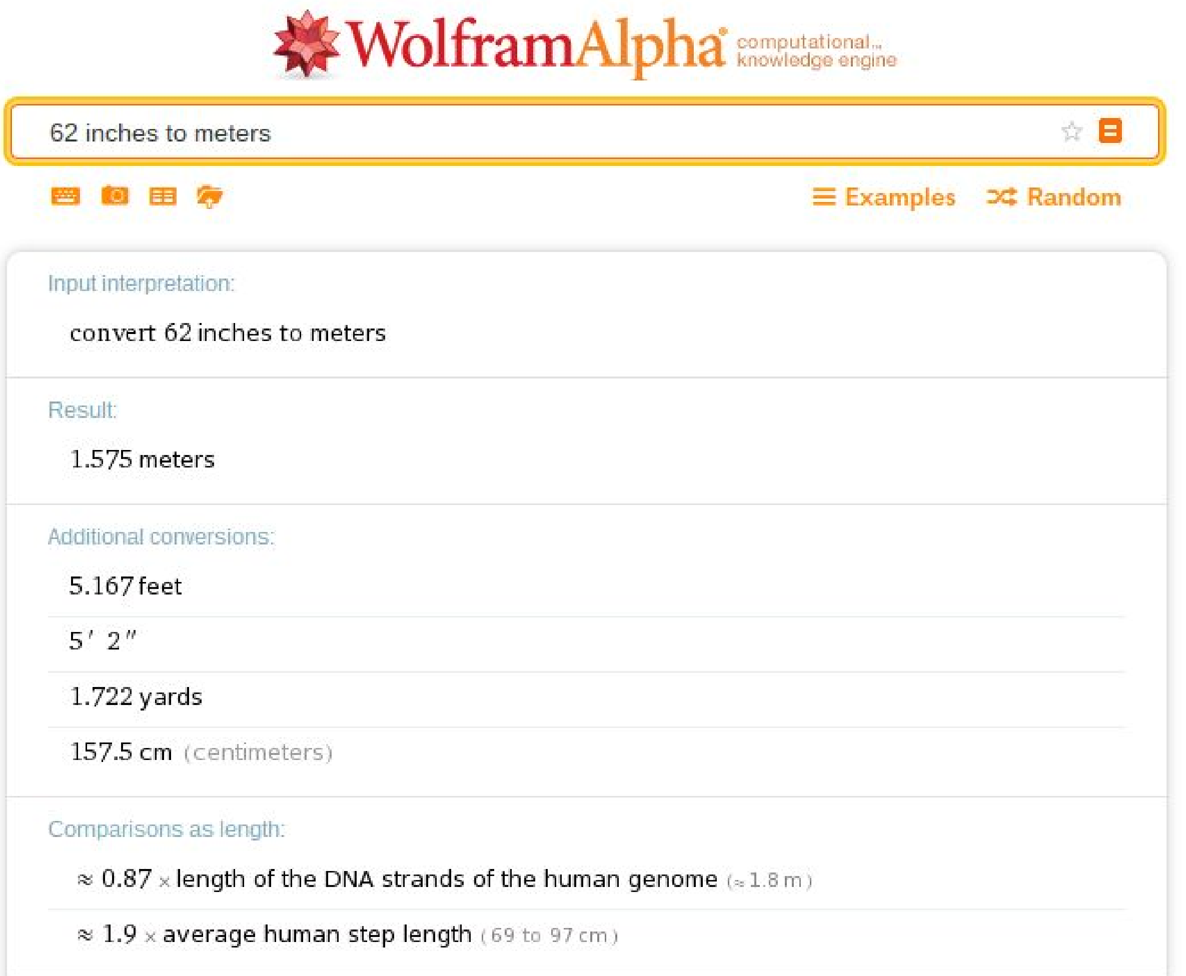
↳ $\sim 6.9\times10^4$

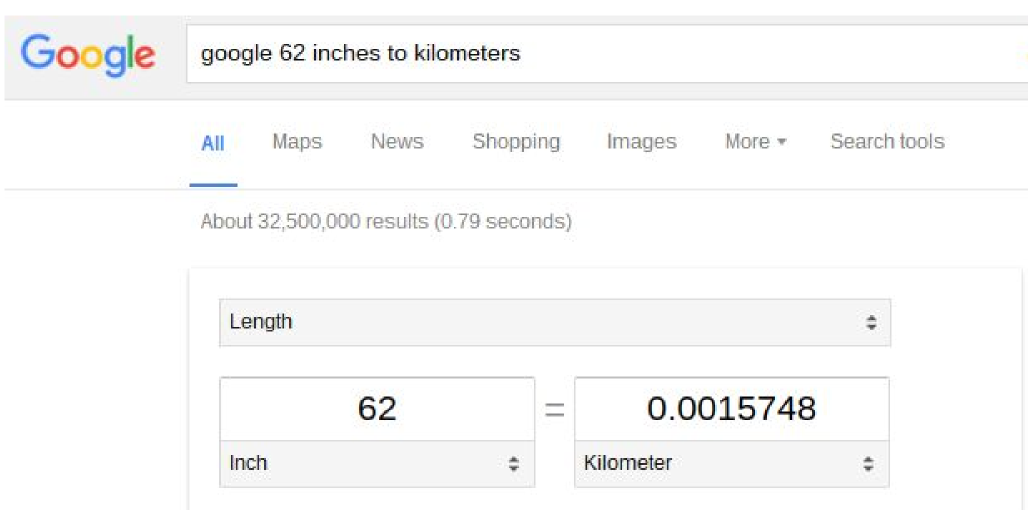
AU? $1\text{AU} = 1.49\times10^{11}\text{m}$

$$\frac{1\text{AU} \mid 6.9\times10^7\text{m}}{1.49\times10^{11}\text{m} \mid}$$

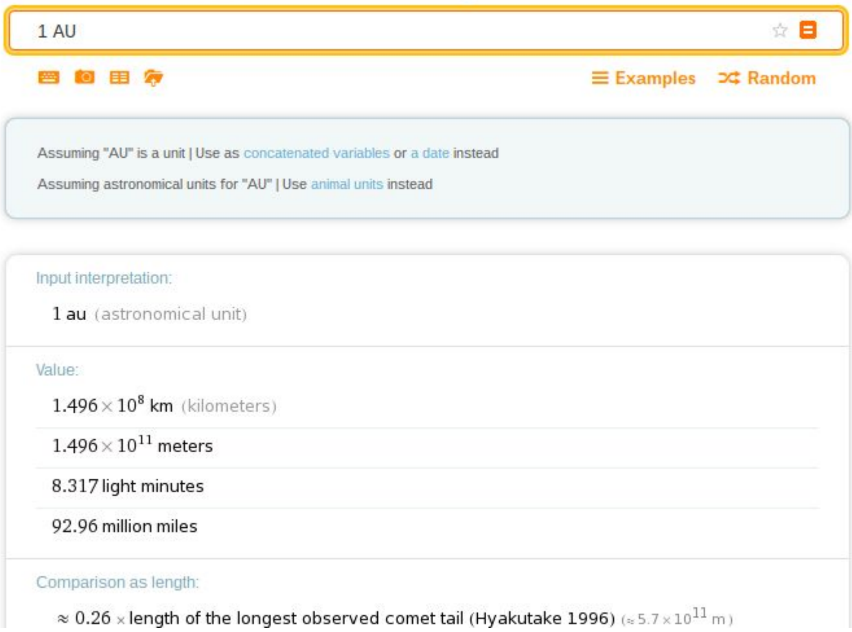
$= \boxed{.000461\text{ AU}}$

There is a simple function in both Wolfram Alpha and Google that does this for us, and is able to keep track of digits and significant figures better.





We can also use Wolfram Alpha to gather more details about units, constant, and everything really. Say we want to look up the definition for an Astronomical Unit, 1AU. Wolfram will automatically generate the conversion for an AU in useful quantities such as a km, meter, light minute (how long it takes for light to travel between two points), and miles.



Once we know how to use units we have to check units, this is used abstractly in order to make sure that a problem is correct. Let's take the follow examples in SI units.

v/d (velocity/distance) --> (m/s)/(m) --> 1/s --> Hz (measure of sound) --> YES
v*t (velocity*time) --> (m/s)*(s) --> m (measure of length) --> YES
||v|| +d --> (speed) + (distance) --> (m/s) + (m) --> NOPE
N/m --> (Force)/(Distance) --> (kg*m/s^2)/ (m) --> kg/s^2 (is a rate) --> YES

Input interpretation:

force (physical quantity) | mass (physical quantity) | acceleration (physical quantity)

Common symbols:

force	F
mass	m
acceleration	a

Basic dimensions:

force	$[\text{mass}]\,[\text{length}]\,[\text{time}]^{-2}$
mass	[mass]
acceleration	$[\text{length}]\,[\text{time}]^{-2}$

Standard units:

force	N (newton)
mass	kg (kilogram)
acceleration	m/s^2 (meter per second squared)

Dimensionless combination:

$$\frac{[\text{acceleration}]\,[\text{mass}]}{[\text{force}]}$$ (Newton's second law)