Homework #1

Math 3607

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Table of Contents

Problem 1.

(a)

First, equate units of distance:

$$d = 384,400\text{km}$$

$$t = 0.05\text{mm} = 0.05 \times 10^{-6}\text{km} = 5 \times 10^{-8}\text{km}$$

The thickness of the paper doubles with each fold.

$$d = t \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = t \cdot 2^k$$

where k denotes the number of folds. Solving for k, we have

$$2^k = d/t$$

$$k = \log_2(d/t)$$

(b)

```
t = 0.00000005;
d = 384400;
k = ceil(log(d/t)/log(2));       %using change of base formula
fprintf('The number of folds is %d.', k)
```

```
The number of folds is 43.
```

(c)

```
k = 0;
while t <= d
    t = t * 2;
    k = k + 1;
end
fprintf('The number of folds is %d.', k)
```

```
The number of folds is 43.
```

Problem 2.

(a)

We are given that

$$r = \sqrt{k + \sqrt{k + \sqrt{k + \cdots}}}$$

where k is any positive integer. Note that

$$r = \sqrt{k + r}$$

Solving for r, we have

$$r^2 = k + r$$

$$r^2 - r - k = 0$$

$$r = \frac{1 \pm \sqrt{1 + 4k}}{2}$$

(b)

For $k = 2$:

```
k = 2;
r1 = (1 + sqrt(1 + 4*k))/2;
r2 = (1 - sqrt(1 + 4*k))/2;
fprintf('r1 = %f, r2 = %f', r1, r2)
```

```
r1 = 2.000000, r2 = -1.000000
```

For $k = 3$:

```
k = 3;
r1 = (1 + sqrt(1 + 4*k))/2;
r2 = (1 - sqrt(1 + 4*k))/2;
fprintf('r1 = %f, r2 = %f', r1, r2)
```

```
r1 = 2.302776, r2 = -1.302776
```

For $k = 4$:

```
k = 4;
r1 = (1 + sqrt(1 + 4*k))/2;
r2 = (1 - sqrt(1 + 4*k))/2;
fprintf('r1 = %f \nr2 = %f', r1, r2)
```

```
r1 = 2.561553
r2 = -1.561553
```

Problem 3.

(a)

By Euler's formula,

$$1 = e^{i0\pi} = e^{i2\pi} = e^{i4\pi} = \cdots$$

$$1 = \cos(2n\pi) + i\sin(2n\pi)$$

for any integer $n \geq 0$. Also,

$$-1 = e^{i1\pi} = e^{i3\pi} = e^{i5\pi} = \cdots$$

$$-1 = \cos(2n\pi + \pi) + i\sin(2n\pi + \pi)$$

for any integer $n \geq 0$. Therefore,

$$x = \sqrt[5]{-1} = e^{\frac{(2n+1)\pi i}{5}}$$

for any integer $n \geq 0$. Using the method from the textbook to find five distinct solutions,

```
format short
x1 = exp(((2*0+1)*pi+1i)/5);
x2 = exp(((2*1+1)*pi+1i)/5);
x3 = exp(((2*2+1)*pi+1i)/5);
x4 = exp(((2*3+1)*pi+1i)/5);
x5 = exp(((2*4+1)*pi+1i)/5);
fprintf('x1 = %f \nx2 = %f \nx3 = %f \nx4 = %f \nx5 = %f', x1, x2, x3, x4, x5)
```

```
x1 = 1.837092
x2 = 6.454779
x3 = 22.679419
x4 = 79.686082
x5 = 279.983873
```

(b)

Problem 4.

(a)

Temperature in Fahrenheit, F, converted to Celsius, C, Kelvin, K, and Rankine, R:

$$C = \frac{5}{9}(F - 32)$$

$$K = \frac{5}{9}(F - 32) + 273.15$$

$$R = F + 459.67$$

(b)

```
[celsius, kelvin, rankine] = FahrenheitConverter(32)
```

```
celsius = 0
kelvin = 273.1500
rankine = 491.6700
```

```
[celsius, kelvin, rankine] = FahrenheitConverter(134)
```

```
celsius = 56.6667
kelvin = 329.8167
rankine = 593.6700
```

```
[celsius, kelvin, rankine] = FahrenheitConverter(212)
```

```
celsius = 100
kelvin = 373.1500
rankine = 671.6700
```

Problem 5.

```
format long g
[surfaceAreaOfEarth, approximation] = OblateSpheroidSurfaceArea(6378.137, 6356.752)
```

```
surfaceAreaOfEarth =
          510065604.944206
approximation =
          509495321.639745
```

Functions Used

```
function [c, k, r] = FahrenheitConverter(f)
    c = 5/9*(f - 32);
    k = 5/9*(f - 32) + 273.15;
    r = f + 459.67;
end

function[surfaceArea, approx] = OblateSpheroidSurfaceArea(r1, r2)
    surfaceArea = 2*pi*(r1^2 + (r2^2)/sin(acos(r2/r1)) ...
        * log(cos(acos(r2/r1))/(1-sin(acos(r2/r1)))));
    approx = 4*pi*((r1+r2)/2)^2;
end
```