Math 3607: Homework 1

6PM, Friday, January 20, 2023

- Problems marked with are to be done by hand; those marked with are to be solved using a computer. In addition, see below [1] for reference keys and notations.
- **Important note.** This is a course on numerical computations, not on symbolic ones[2]. Though MATLAB is capable of carrying out symbolic calculations using *Symbolic Math Toolbox*, we will never use it. Any work done using `sym` or `syms` will receive NO credit.

1. (Folding paper) How many times must a sheet of paper with thickness $t = 0.05$mm (that is sufficiently large) be folded to reach the Moon (distance from Earth $d = 384,400$km)?
   (a) Write down an analytical formula using elementary mathematical functions. Justify all your steps.
   (b) Evaluate the formula found in part (a) in MATLAB.
   (c) Now solve the problem without using the formula from part (a). *Hint.* Use a while-loop.

2. (**LM** 2.1–15(a): Continued square roots) Let r be defined by

$$r = \sqrt{k + \sqrt{k + \sqrt{k + \sqrt{k + \sqrt{k + \cdots}}}}}$$

   where k is any positive integer.
   (a) Calculate r by hand and write down the result as an analytical formula (in terms of k).
   (b) Using the formula found in (a), calculate the value of r numerically for $k = 2, 3,$ and 4.

3. (**LM** 2.1–39(b): Roots of unity)
   (a) Use the approach explained in the problem (the one using Euler's formula) to find the five distinct solutions of $x^5 = -1$ in MATLAB.

[1] References:

- **FNC**: *Fundamentals of Numerical Computation* (Driscoll and Braun)
- **LM**: *Learning MATLAB, Problem Solving, and Numerical Analysis Through Examples* (Overman)

The notation **LM** 2.2–5 indicates Problem 5 at the end of section 2.2 of the textbook by Overman.

[2] To learn about the difference between numerical and symbolic computations, please read the prologue of **FNC**.

(b) (Optional) If you want more challenge, by factoring $x^5 + 1$ and solving quadratic equations, find the analytical expressions for the solutions not in terms of trigonometric functions, but in terms of radicals. Then use MATLAB to confirm your results (against the ones obtained using Euler's formula).

4. (Temperature conversion; adapted from **LM** 2.2–5.)

   (a) Write down formulas which convert a temperature in Fahrenheit to the corresponding temperature in degrees Celsius, kelvins, and degree Rankine[3]. Be sure to cite any resources used.

   (b) Write a function which takes a temperature in Fahrenheit as an input and converts it into temperature in degrees Celsius, kelvins, and degree Rankine. (Instead of writing an external function m-file, write the function at the end of your live script.) Then run your function with temperatures 32°F, 134°F, and 212°F.

5. (Oblate spheroid) An *oblate spheroid* such as the Earth is obtained by revolving an ellipse about its minor axis as shown in the figure.

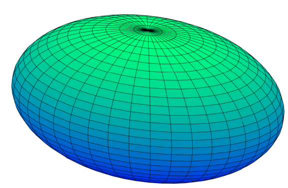

Figure 1: An oblate spheroid. Image created using MATLAB.

The Earth's equatorial radius is about 20km longer than its polar radius. The surface area of an oblate spheroid is given by

$$A(r_1, r_2) = 2\pi \left(r_1^2 + \frac{r_2^2}{\sin(\gamma)} \log \left(\frac{\cos(\gamma)}{1 - \sin(\gamma)} \right) \right),$$

where r_1 is the equatorial radius, r_2 is the polar radius, and

$$\gamma = \arccos \left(\frac{r_2}{r_1} \right).$$

We assume $r_2 < r_1$. Write a function (at the end of your live script) that inputs the equatorial and polar radii and displays both $A(r_1, r_2)$ and the approximation $4\pi((r_1 + r_2)/2)^2$. Run the

[3] William Rankine was a Scottish engineer and physicist who proposed this scale in 1859, 11 years after the Kelvin scale. It also is zero at absolute zero, an increase of 1°Ra is exactly the same as an increase of 1°F.

function to the Earth data $(r_1, r_2) = (6378.137, 6356.752)$. Use `format long g` to display enough digits.