Homework #2

Math 3607

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Problem 1.

```
y1 = 2020;
isLeapYear(y1)
```

```
ans = logical
   1
```

```
y2 = 1900;
isLeapYear(y2)
```

```
ans = logical
   0
```

```
y3 = 2000;
isLeapYear(y3)
```

```
ans = logical
   1
```

```
leapYears = [];
for i = 1900:2023
    if isLeapYear(i)
        leapYears = [leapYears, [i]];
    end
end
disp(reshape(leapYears, 6, 5)')
```

```
        1904        1908        1912        1916        1920        1924
        1928        1932        1936        1940        1944        1948
        1952        1956        1960        1964        1968        1972
        1976        1980        1984        1988        1992        1996
        2000        2004        2008        2012        2016        2020
```

Problem 2.

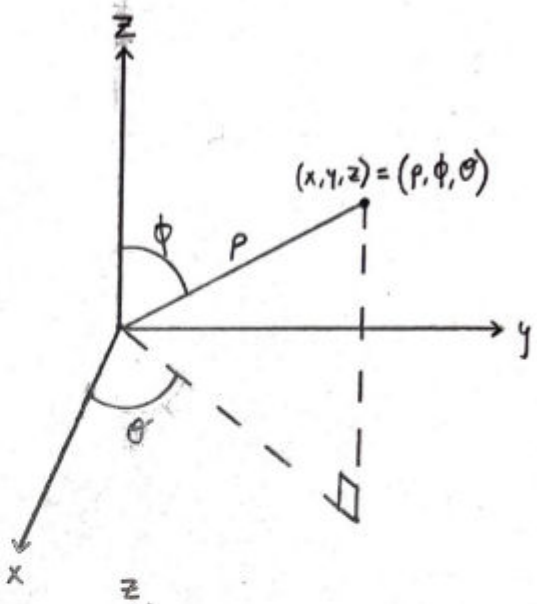
Problem 2

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

where $\phi \in [0, \pi]$, $\theta \in [0, 2\pi)$, $\rho \geq 0$



- ρ is the distance from the origin to (x,y,z). So,

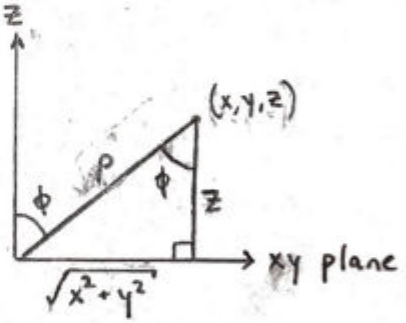
$$\boxed{\rho = \sqrt{x^2+y^2+z^2}}$$

- ϕ is therefore found by

$$z = \sqrt{x^2+y^2+z^2}\cos(\phi)$$

$$\phi = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\boxed{\phi = \arctan\left(\frac{\sqrt{x^2+y^2}}{z}\right)}$$



- θ can be found in two ways:

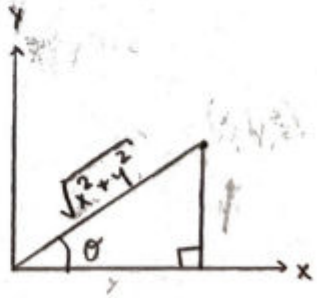
① $x = \sqrt{x^2+y^2+z^2}\sin\phi\cos\theta$

$$x = \sqrt{x^2+y^2+z^2}\cdot\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\cdot\cos\theta$$

$$x = \sqrt{x^2+y^2}\cos\theta$$

$$\theta = \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$

$$\boxed{\theta = \arctan\left(\frac{y}{x}\right)}$$



② $y = \sqrt{x^2+y^2+z^2}\sin\phi\sin\theta$

$$y = \sqrt{x^2+y^2+z^2}\cdot\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\cdot\sin\theta$$

$$y = \sqrt{x^2+y^2}\sin\theta$$

$$\theta = \arcsin\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$\boxed{\theta = \arctan\left(\frac{y}{x}\right)}$$

```
[rho, phi, theta] = toSpherical(1, 2, -2)
```

```
rho = 3
phi = 2.3005
theta = 1.1071
```

```
[rho, phi, theta] = toSpherical(0, -3, 4)
```

```
rho = 5
phi = 0.6435
theta = 4.7124
```

```
[rho, phi, theta] = toSpherical(-5, -12, 1)
```

```
rho = 13.0384
phi = 1.4940
theta = 1.1760
```

```
[rho, phi, theta] = toSpherical(0, 0, -7)
```

```
rho = 7
phi = 0
theta = 0
```

Problem 3.

```
format shortE
points = [-30 -3 3 30];
myexp(points)
```

```
ans = 1×4
   9.3576e-14   4.9787e-02   2.0086e+01   1.0686e+13
```

Problem 4.

```
format default
wins = 0;
trials = 0;

while(trials < 1000000)
    trials = trials + 1;
    stick1 = rand();
    stick2 = rand();
    stick3 = rand();

    if(stick1 < stick2 + stick3 && stick2 < stick1 + stick3 && stick3 < stick1 + stick2)
        wins = wins + 1;
    end
end

winprob = wins/trials;
fprintf('Estimated win probability: %.2f%%', winprob * 100)
```

```
Estimated win probability: 49.97%
```

Problem 5.

```
p_heads = 1/3;
p_tails = 2/3;

for i = 1:10
    countHeads = 0;
    countTails = 0;
    while (abs(countTails - countHeads) < 10)
        toss = rand();
        if(toss < 2/3)
            countTails = countTails + 1;
        else
            countHeads = countHeads + 1;
        end
    end
    trials = countHeads + countTails;
    fprintf('Simulation %2.15g done in %2.15g tosses.\n', i, trials)
end
```

```
Simulation  1 done in 32 tosses.
Simulation  2 done in 36 tosses.
Simulation  3 done in 50 tosses.
Simulation  4 done in 14 tosses.
Simulation  5 done in 30 tosses.
Simulation  6 done in 20 tosses.
Simulation  7 done in 24 tosses.
Simulation  8 done in 22 tosses.
Simulation  9 done in 34 tosses.
Simulation 10 done in 14 tosses.
```

Problem 6.

```
n = 0;
a_n = 0;
while abs(a_n - pi) > 10^-6
    a_n = 0;
    for k = 0:n
        a_n = a_n + (6/sqrt(3))*((-1)^k)/(3^k*(2*k+1));
    end
    fprintf('a_%d = %4.4f\n', n, a_n)
    n = n + 1;
end
```

```
a_0 = 3.4641
a_1 = 3.0792
a_2 = 3.1562
a_3 = 3.1379
a_4 = 3.1426
a_5 = 3.1413
a_6 = 3.1417
a_7 = 3.1416
a_8 = 3.1416
```

```
a_9 = 3.1416
a_10 = 3.1416
```

```
n = 0;
b_n = 0;
while abs(b_n - pi) > 10^-6
    b_n = 0;
    for k = 0:n
        b_n = b_n + 16*((-1)^k)/(5^(2*k+1)*(2*k+1)) - 4*((-1)^k)/(239^(2*k+1)*(2*k+1));
    end
    fprintf('b_%d = %4.4f\n', n, b_n)
    n = n + 1;
end
```

```
b_0 = 3.1833
b_1 = 3.1406
b_2 = 3.1416
b_3 = 3.1416
```

Functions Used

```
function [result] = isLeapYear(year)
    if mod(year, 100) == 0 && mod(year, 400) == 0
        result = true;
    elseif mod(year, 100) ~= 0 && mod(year, 4) == 0
        result = true;
    else
        result = false;
    end
end
```

NOTE: $\arctan(x) : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\phi \in [0, \pi]$, $\theta \in [0, 2\pi)$.

```
function[rho, phi, theta] = toSpherical(x, y, z)
    rho = sqrt(x^2 + y^2 + z^2);
    phi = atan(sqrt(x^2+y^2)/z);
    theta = atan(y/x);

    if phi < 0
        phi = phi + pi;
    end
    if theta < 0
        theta = theta + 2*pi;
    end
    if x == 0 && y == 0
        theta = 0;
    end
end
```

```
function[results] = myexp(points)
    polynomial = zeros(1, 16);
    results = zeros(1, length(points));
    dividedBy2 = zeros(1, length(points));

    % calculating 16 Taylor Series coefficients
    for i = 1:16
        polynomial(i) = 1/factorial(i-1);
    end

    % dividing each input by 2 until absolute value < 0.5
    for j = 1:length(points)
        while abs(points(j)) >= 0.5
            points(j) = points(j) / 2;
            dividedBy2(j) = dividedBy2(j) + 1;
        end
    end

    % evaluating Taylor Series at each input, squaring repeatedly
    polynomial = flip(polynomial);
    for k = 1:length(results)
        results(k) = polyval(polynomial, points(k));
        for x = 1:dividedBy2(k)
            results(k) = results(k)^2;
        end
    end
end
```