IE335

HW2

mccar122

4.1-6. Describe graphically what the simplex method does step by step to solve the following problem.

$$\text{Maximize} \qquad Z = 2x_1 + 3x_2,$$

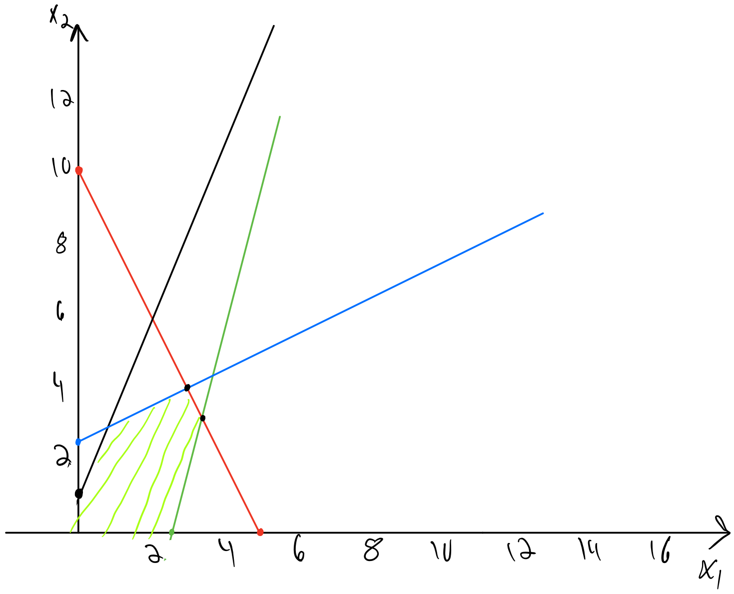
subject to

$$\begin{aligned} -3x_1 + x_2 &\leq 1 \\ 4x_1 + 2x_2 &\leq 20 \\ 4x_1 - x_2 &\leq 10 \\ -x_1 + 2x_2 &\leq 5 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

4.1.6)



$-3x_1 + x_2 \leq 1$ $\longrightarrow$ (0, 1) and (-1/3, 0)

$4x_1 + 2x_2 \leq 20$ $\longrightarrow$ (0, 10) and (5, 0)

$4x_1 - x_2 \leq 10$ $\longrightarrow$ (0, -10) and (2.5, 0)

$-x_1 + 2x_2 \leq 5$ $\longrightarrow$ (0, 2.5) (-5, 0)

Corner Point	Output	Next Step
(3, 4)	18	check extremes (0,10) and (5,0)
(0,10)	30	
(5,0)	10	

4.4-6. Consider the following problem.

Maximize $\quad Z = 3x_1 + 5x_2 + 6x_3,$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &\leq 4 \\ x_1 + 2x_2 + x_3 &\leq 4 \\ x_1 + x_2 + 2x_3 &\leq 4 \\ x_1 + x_2 + x_3 &\leq 3 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

D,I **(a)** Work through the simplex method step by step in algebraic form.

D,I **(b)** Work through the simplex method in tabular form.

C **(c)** Use a computer package based on the simplex method to solve the problem.

B)

$$\begin{aligned} 2x_1 + x_2 + x_3 &\le 4 \\ x_1 + 2x_2 + x_3 &\le 4 \\ x_1 + x_2 + 2x_3 &\le 4 \\ x_1 + x_2 + x_3 &\le 3 \end{aligned}$$

Maximize $Z = 3x_1 + 5x_2 + 6x_3$,

$0 = Z - 3x_1 - 5x_2 - 6x_3$

$2x_1 + x_2 + x_3 + s_1 = 4$

$x_1 + 2x_2 + x_3 + s_2 = 4$

$x_1 + x_2 + 2x_3 + s_3 = 4$

$x_1 + x_2 + x_3 + s_4 = 3$

$x_{123} \ge 0 \quad s_{123} \ge 0$

	x_1	x_2	x_3	s_1	s_2	s_3	s_4	RHS
Z	-3	-5	-6	0	0	0	0	0
s_1	2	1	1	1	0	0	0	4 → 4/1
s_2	1	2	1	0	1	0	0	4 → 4/1
s_3	1	1	2	0	0	1	0	4 → 4/2 = 2
s_4	1	1	1	0	0	0	1	3 → 3/1
Z	0	-2	0	0	0	3	0	
s_1	3/2	1/2	0	1	0	-1/2	0	2 → 4
s_2	1/2	3/2	0	0	1	-1/2	0	2 → 4/3
x_3	1/2	1/2	1	0	0	1/2	0	2 → 4
s_4	1/2	1/2	0	0	0	-1/2	1	1 → 2
Z	2/3	0	0	0	4/3	7/3	0	
s_1	4/3	0	0	1	-1/3	-1/3	0	4/3
x_2	1/3	1	0	0	2/3	-1/3	0	4/3
x_3	1/3	0	1	0	-1/3	2/3	0	4/3
s_4	1/3	0	0	0	-1/3	-1/3	1	1/3
Z								0

Operations:

$s_3[\ ] \to s_3/2$

$s_1[\ ] \to s_1 - s_3[\ ]$

$s_2[\ ] \to s_2 - s_3[\ ]$

$s_4[\ ] \to s_4 - s_3[\ ]$

$Z_1[\ ] \to Z_0 + 6 \cdot x_3[\ ]$

Initial $Z[x_3]$ 0 is out

$x_2[\ ] \to s_2 \cdot 2/3$

$s_1[\ ] \to s_1 - x_2[\ ]$

$x_3[\ ] \to$

$s_4[\ ] \to$

$x_1 = 0$

$x_2 = 4/3$

$x_3 = 4/3$

$Z = 0 + \frac{20}{3} + \frac{24}{3}$

$6\frac{2}{3}$ $\quad$ 8

$\boxed{Z = 14\,2/3}$

D,I **4.5-4.** Consider the following problem.

Maximize $\quad Z = 5x_1 + x_2 + 3x_3 + 4x_4,$

subject to

$$\begin{aligned} x_1 - 2x_2 + 4x_3 + 3x_4 &\leq 20 \\ -4x_1 + 6x_2 + 5x_3 - 4x_4 &\leq 40 \\ 2x_1 - 3x_2 + 3x_3 + 8x_4 &\leq 50 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0, \qquad x_4 \geq 0.$$

Work through the simplex method step by step to demonstrate that Z is unbounded.

3) Maximize $Z = 5x_1 + x_2 + 3x_3 + 4x_4,$

$$x_1 - 2x_2 + 4x_3 + 3x_4 \le 20$$
$$-4x_1 + 6x_2 + 5x_3 - 4x_4 \le 40$$
$$2x_1 - 3x_2 + 3x_3 + 8x_4 \le 50$$

$$Z - 5x_1 - x_2 - 3x_3 - 4x_4$$
$$x_1 - 2x_2 + 4x_3 + 3x_4 + s_1 = 20$$
$$-4x_1 + 6x_2 + 5x_3 - 4x_4 + s_2 = 40$$
$$2x_1 - 3x_2 + 3x_3 + 8x_4 + s_3 = 50$$

$x_1, x_2, x_3, x_4 \ge 0 \quad s_1, s_2, s_3 \ge 0$

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
Z	-5	-1	-3	-4	0	0	0		
s_1	1	-2	4	3	1	0	0	20	20/1
s_2	-4	6	5	-4	0	1	0	40	x
s_3	2	-3	3	8	0	0	1	50	50/2 = 25
Z	0	-11	17	11	5	0	0		
x_1	1	-2	4	3	1	0	0	20	x
s_2	0	-2	21	8	4	1	0	120	x
s_3	0	1	-5	2	-2	0	1	10	10/1
Z	0	0	-38	33	17	0	-11		
x_1	1	0	-6	7	-3	0	2	40	x
s_2	0	0	11	12	0	1	2	140	140/11
x_2	0	1	-5	2	-2	0	1	10	x
Z	0	0	0	819/11	-17	38/11	197/11		
x_1	1	0	0	149/11	-3	6/11	34/11	1280/11	x
x_3	0	0	1	12/11	0	1/11	2/11	140/11	x
x_2	0	1	0	82/11	-2	5/11	21/11	810/11	x

$x_1[1] =$

$S_2[1] = 0 \quad S_2[\,] = S_2[\,] + 4 \cdot x_1[\,]$

$S_3[1] = 0 \quad S_3[\,] = S_3[\,] + -2 \cdot x_1[\,]$

$x_1[\,] = x_1[\,] + 2 \cdot x_2[\,]$

$S_2[\,] = S_2[\,] + 2 \cdot x_2[\,]$

$x_3[\,] = x_3 / 11$

$x_1[\,] = x_1[\,] + 6 \cdot x_3[\,]$

$x_2[\,] = x_2[\,] + 5 \cdot x_3[\,]$

s_1 enters but there are all negative or zero. Thus solution is unbounded!!

4.6-4. Consider the following problem.

$$\text{Minimize} \qquad Z = 2x_1 + x_2 + 3x_3,$$

subject to

$$5x_1 + 2x_2 + 7x_3 = 420$$
$$3x_1 + 2x_2 + 5x_3 \geq 280$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

Introduce artificial variables to reformulate this problem as a convenient artificial problem for preparing to apply the simplex method.

4) Minimize $Z = 2x_1 + x_2 + 3x_3,$

$$5x_1 + 2x_2 + 7x_3 = 420$$
$$3x_1 + 2x_2 + 5x_3 \geq 280$$

$$Z = A_1 + A_2$$
$$5x_1 + 2x_2 + 7x_3 + A_1 = 420$$
$$3x_1 + 2x_2 + 5x_3 - S_1 + A_2 = 280$$

$$x_1, x_2, x_3, A_1, A_2, S_1 \geq 0$$

4.6-9. Consider the following problem.

$$\text{Maximize} \qquad Z = x_1 + 4x_2 + 2x_3,$$

subject to

$$4x_1 + x_2 + 2x_3 \le 5$$
$$-x_1 + x_2 + 2x_3 \le 10$$

and

$$x_2 \ge 0, \qquad x_3 \ge 0$$

(no nonnegativity constraint for x_1).

(a) Reformulate this problem so all variables have nonnegativity constraints.

D,I **(b)** Work through the simplex method step by step to solve the problem.

C **(c)** Use a software package based on the simplex method to solve the problem.

5) a and b

Maximize $Z = x_1 + 4x_2 + 2x_3,$

$$4x_1 + x_2 + 2x_3 \leq 5$$
$$-x_1 + x_2 + 2x_3 \leq 10$$

a) Since x_1 has no non negativity $\rightarrow$ $x_1 = y_1 - y_2$

$y_1 \geq 0 \quad y_2 \geq 0$

thus new eqns:

$$Z = y_1 - y_2 + 4x_2 + 2x_3$$

b) $$4y_1 - 4y_2 + x_2 + 2x_3 = 5$$
$$-y_1 + y_2 + x_2 + 2x_3 = 10$$

$y_1, y_2, x_2, x_3 \geq 0$ | $y_1 = x^+$, $y_2 = x^-$

$$Z - y_1 + y_2 - 4x_2 - 2x_3 = 0$$
$$4y_1 - 4y_2 + x_2 + 2x_3 + s_1 = 5$$
$$-y_1 + y_2 + x_2 + 2x_3 + s_2 = 10$$

$y_1, y_2, x_2, x_3, s_1, s_2 \geq 0$

	y_1	y_2	x_2	x_3	s_1	s_2	RHS	
Z	-1	1	-4	-2	0	0		
$\leftarrow s_1$	4	-4	1	2	1	0	5	5/1
s_2	-1	1	1	2	0	1	10	10/1
Z	15	-15	0	6	-4	0		
x_1	1	-1	1/4	1/2	1/4	0	5/4	
s_2	0	0	5/4	5/2	1/4	1	45/4	
Z								
~~x_1~~								
x_2								

$y_1 = 0$

$y_2 = -1$

$x_2 = 9$

$x_3 = 0$

$$\boxed{Z = 35}$$

I **4.8-1.** Consider the following problem.

$$\text{Maximize} \quad Z = 4x_1 + 5x_2 + 3x_3,$$

subject to

$$\begin{aligned} x_1 + x_2 + 2x_3 &\geq 20 \\ 15x_1 + 6x_2 - 5x_3 &\leq 50 \\ x_1 + 3x_2 + 5x_3 &\leq 30 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

After reformulating the problem appropriately, work through phase 1 of the two-phase method step by step to demonstrate that this problem does not possess any feasible solutions.

e)

$$Z = 4x_1 + 5x_2 + 3x_3,$$

$$x_1 + x_2 + 2x_3 \geq 20$$

$$15x_1 + 6x_2 - 5x_3 \leq 50$$

$$x_1 + 3x_2 + 5x_3 \leq 30$$

$$Z - 4x_1 - 5x_2 - 3x_3 + MA_1 = 0$$

$$x_1 + x_2 + 2x_3 - s_1 + A_1 = 20$$

$$15x_1 + 6x_2 + 5x_3 + s_2 = 50$$

$$x_1 + 3x_2 + 5x_3 + s_3 = 30$$

$$x_1, x_2, x_3, s_1, s_2, s_3, A_1 \geq 0$$

	x_1	x_2	x_3	s_1	s_2	s_3	A_1	RHS	
Z	$-4-M$	$-5-M$	$-3-2M$	M	0	0	$M-M$		
s_1	1	1	2	-1	0	0	1	20	$\to 20/2$
s_2	15	6	5	0	1	0	0	50	$50/5$
s_3	1	3	5	0	0	1	0	30	$30/5$
Z	$\frac{3(1-M)}{5}$	$\frac{M+9}{5}$	3	M	0	$\frac{2M+3}{5}$	$-M$		
A_1	$3/5$	$-1/5$	0	-1	0	$-2/5$	1	$40/3$	
s_2	14	3	0	0	1	-1	0	$10/7$	
x	$1/5$	$3/5$	1	0	0	$1/5$	0	6	
Z									
A_1	0	-.33	0	-1	-.04	-.36	1	7.14	
x_1	1	.21	0	0	0.07	-.07	0	1.43	
x_2	0	.56	1	0	0.01	.21	0	5.71	

$A_1[\,] = s_1 \cdot -2\, x[\,]$

$x_1 = 1.43$

$x_2 = 5.71$

$Z = 22.86$