IE 335: Operations Research - Optimization   Fall 2022

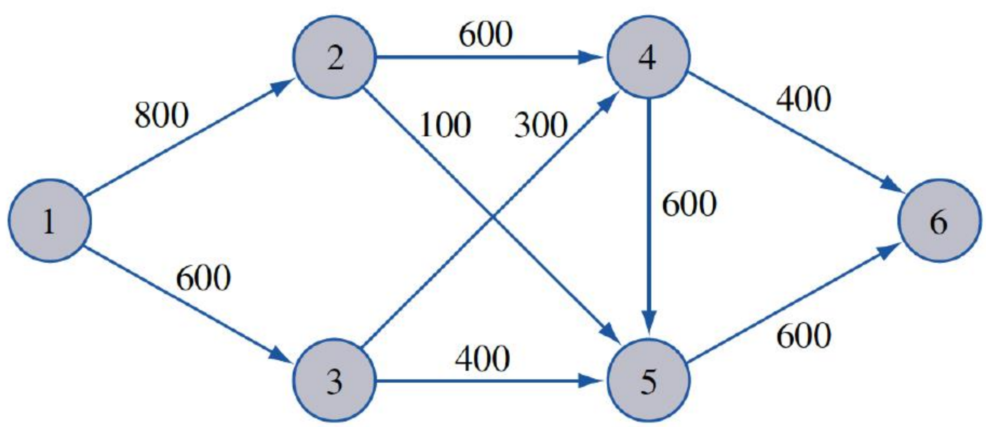
HOMEWORK 5
Due: November 15, 11:59 PM

Homework assignments must be prepared individually and submitted online through Gradescope. See the syllabus for more information on Homework requirements and expectations. Any necessary modifications to this assignment will be posted to Brightspace as an announcement.
NOTE: Please show your final solution clearly and include your all Excel settings such as model display and solver settings along with the final solution in your Gradescope submission. Do the same for Gurobi solutions.

(25 pts) Question 1: Each hour, an average of 900 cars enter the network in the figure below at node 1 and seek to travel to node 6. The time it takes a car to traverse each arc is shown in the following table. In the figure, the number above each arc is the maximum number of cars that can pass by any point on the arc during a one-hour period.

a) Formulate this problem as a minimum cost flow problem that minimizes the total time required for all cars to travel from node 1 to node 6. Clearly define in words what your decision variables represent. In the LP, label each constraint clearly for what the constraint corresponds to.

b) Solve the Question using Excel Solver. Submit a screenshot of the result in Excel spreadsheet with Solver setting and Excel spreadsheet display.



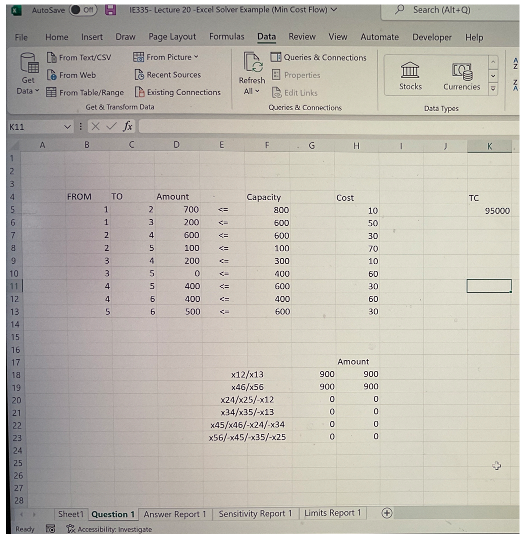
Arc	Time (Minutes)
(1, 2)	10
(1, 3)	50
(2, 5)	70
(2, 4)	30
(5, 6)	30
(4, 5)	30
(4, 6)	60
(3, 5)	60
(3, 4)	10

a) minimize : $Z = 10x_{12} + 50x_{13} + 70x_{25} + 30x_{24} + 30x_{56} + 30x_{45} + 60x_{46} + 60x_{35} + 10x_{34}$

st:

$x_{12} + x_{13} = 900$ — node 1
$x_{24} + x_{25} - x_{12} = 0$ — node 2
$x_{34} + x_{35} - x_{13} = 0$ — node 3
$x_{45} + x_{46} - x_{24} - x_{34} = 0$ — node 4
$x_{56} - x_{45} - x_{35} - x_{25} = 0$ — node 5
$x_{46} + x_{56} = 900$ — node 6
$x_{12} \le 800$ — arc (1-2)
$x_{13} \le 600$ — arc (1-3)
$x_{24} \le 600$ — arc (2-4)
$x_{25} \le 100$ — arc (2-5)
$x_{34} \le 300$ — arc (3-4)
$x_{35} \le 400$ — arc (3-5)
$x_{45} \le 600$ — arc (4-5)
$x_{46} \le 400$ — arc (4-6)
$x_{56} \le 600$ — arc (5-6)

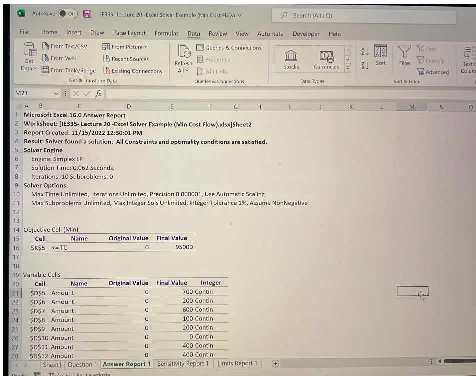
All variables ≥ 0 (non negative)


IE335- Lecture 20 -Excel Solver Example (Min Cost Flow)

FROM	TO	Amount		Capacity	Cost	TC
1	2	700	<=	800	10	95000
1	3	200	<=	600	50	
2	4	600	<=	600	30	
2	5	100	<=	100	70	
3	4	200	<=	300	10	
3	5	0	<=	400	60	
4	5	400	<=	600	30	
4	6	400	<=	400	60	
5	6	500	<=	600	30	

		Amount
x12/x13	900	900
x46/x56	900	900
x24/x25/-x12	0	0
x34/x35/-x13	0	0
x45/x46/-x24/-x34	0	0
x56/-x45/-x35/-x25	0	0

Sheet1 | Question 1 | Answer Report 1 | Sensitivity Report 1 | Limits Report 1

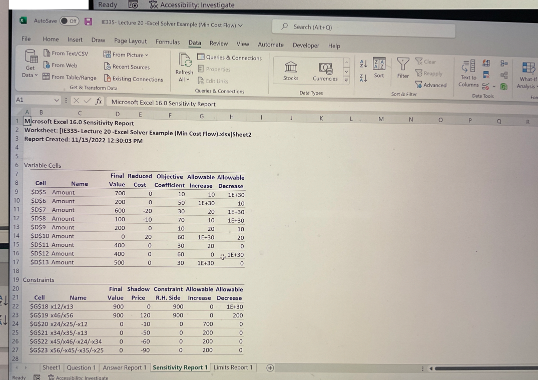

Microsoft Excel 16.0 Answer Report
Worksheet: [IE335- Lecture 20 -Excel Solver Example (Min Cost Flow).xlsx]Sheet2
Report Created: 11/15/2022 12:30:01 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.062 Seconds.
Iterations: 10 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
K5	<= TC	0	95000

Variable Cells

Cell	Name	Original Value	Final Value	Integer
D5	Amount	0	700	Contin
D6	Amount	0	200	Contin
D7	Amount	0	600	Contin
D8	Amount	0	100	Contin
D9	Amount	0	200	Contin
D10	Amount	0	0	Contin
D11	Amount	0	400	Contin
D12	Amount	0	400	Contin

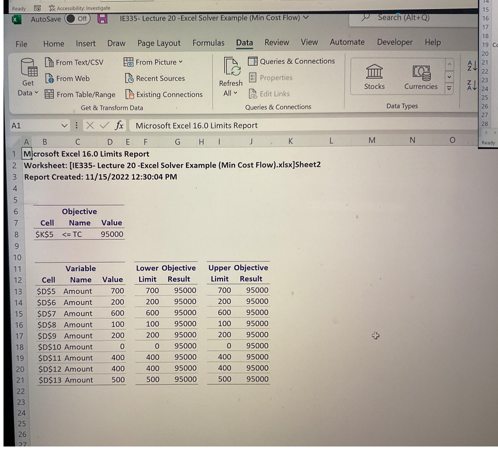

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [IE335- Lecture 20 -Excel Solver Example (Min Cost Flow).xlsx]Sheet2
Report Created: 11/15/2022 12:30:03 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D5	Amount	700	0	10	10	1E+30
D6	Amount	200	0	50	1E+30	10
D7	Amount	600	-20	30	20	1E+30
D8	Amount	100	-10	70	10	1E+30
D9	Amount	200	0	10	20	10
D10	Amount	0	20	60	1E+30	20
D11	Amount	400	0	30	20	0
D12	Amount	400	0	60	0	1E+30
D13	Amount	500	0	30	1E+30	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G18	x12/x13	900	0	900	0	1E+30
G19	x46/x56	900	120	900	0	200
G20	x24/x25/-x12	0	-10	0	700	0
G21	x34/x35/-x13	0	-50	0	200	0
G22	x45/x46/-x24/-x34	0	-60	0	200	0
G23	x56/-x45/-x35/-x25	0	-90	0	200	0

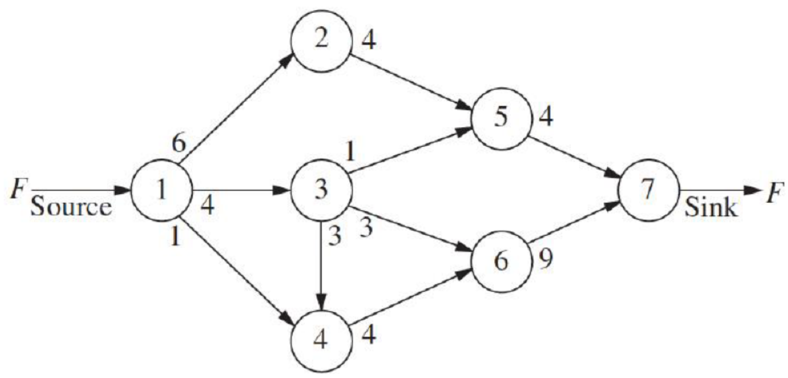

Microsoft Excel 16.0 Limits Report
Worksheet: [IE335- Lecture 20 -Excel Solver Example (Min Cost Flow).xlsx]Sheet2
Report Created: 11/15/2022 12:30:04 PM

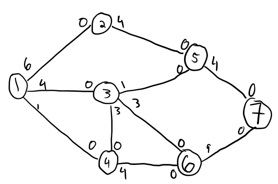
Objective

Cell	Name	Value
K5	<= TC	95000

Variable			Lower	Objective	Upper	Objective
Cell	Name	Value	Limit	Result	Limit	Result
D5	Amount	700	700	95000	700	95000
D6	Amount	200	200	95000	200	95000
D7	Amount	600	600	95000	600	95000
D8	Amount	100	100	95000	100	95000
D9	Amount	200	200	95000	200	95000
D10	Amount	0	0	95000	0	95000
D11	Amount	400	400	95000	400	95000
D12	Amount	400	400	95000	400	95000
D13	Amount	500	500	95000	500	95000

(20pts) Question 2: For the network shown below, use the augmenting path algorithm to find the flow pattern giving the maximum flow from the source to the sink, given that the arc capacity from node i to node j is the number nearest node i along the arc between these nodes. Show your work.

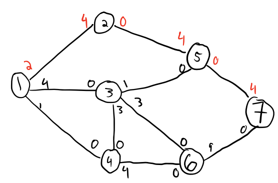




Round 1:

path {1, 2, 5, 7}

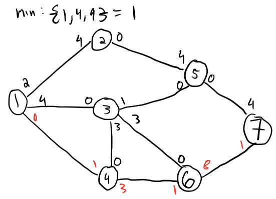
min {6, 4, 4} = 4



Round 2:

path: {1, 4, 6, 7}

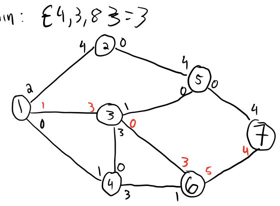
min: {1, 4, 9} = 1



Round 3:

Path: {1, 3, 6, 7}

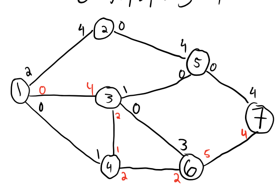
min: {4, 3, 8} = 3



Round 4:

Path: {1, 3, 4, 6, 7}

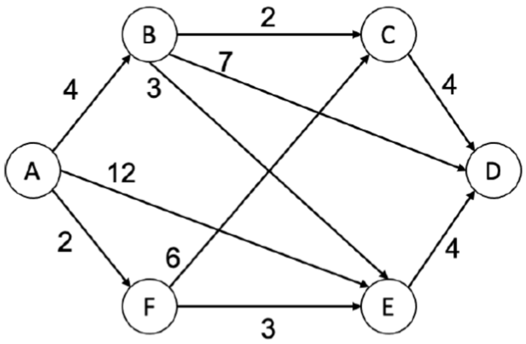
min: {1, 3, 3, 5} = 1



Arc	Flow
1-2	4
1-3	4
1-4	1
2-5	4
3-4	1
3-5	0
3-6	3
4-6	2
5-7	4
6-7	5

Max Flow = 9

(10pts) Question 3: For the graph below, write the LP formulation to find the maximum flow from node A to node D. (Define/show any required variable, node, arc, etc.)



Maximize: $Z = X_{CD} + X_{ED} + X_{BD}$

S.t.: $X_{AB} - X_{BC} - X_{BD} - X_{BE} = 0$ — Node B

$X_{AF} - X_{FC} - X_{FE} = 0$ — Node F

$X_{BC} + X_{FC} - X_{CD} = 0$ — Node C

$X_{FE} + X_{AE} + X_{BE} - X_{ED} = 0$ — Node E

$X_{AB} \le 4$ — arc (A-B)

$X_{AF} \le 2$ — arc (A-F)

$X_{AE} \le 12$ — arc (A-E)

$X_{BC} \le 2$ — arc (B-C)

$X_{BD} \le 7$ — arc (B-D)

$X_{BE} \le 3$ — arc (B-E)

$X_{CD} \le 4$ — arc (C-D)

$X_{ED} \le 4$ — arc (E-D)

$X_{FC} \le 6$ — arc (F-C)

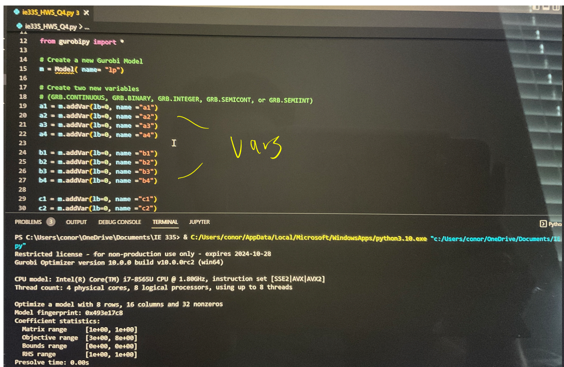
$X_{FE} \le 3$ — arc (F-E)

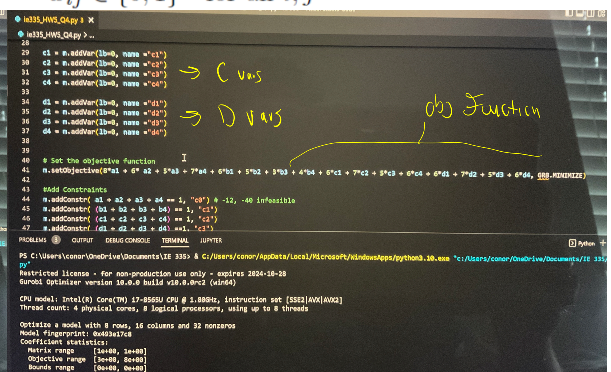
All arcs non negative (≥ 0)

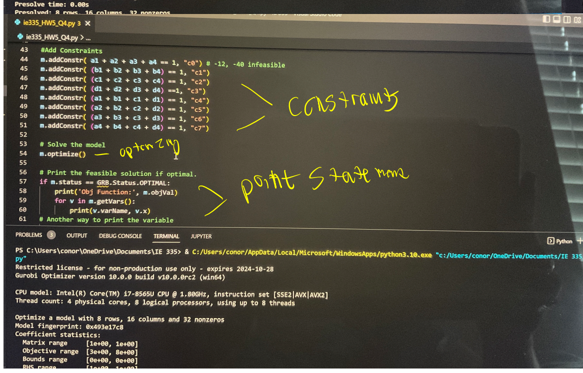
(15pts) Question 4: Considering the following parameter table and LP formulation, solve this problem using **Gurobi in Python**. Submit a screenshot of your code and a screenshot of your results including the final objective value and non-zero decision variables.

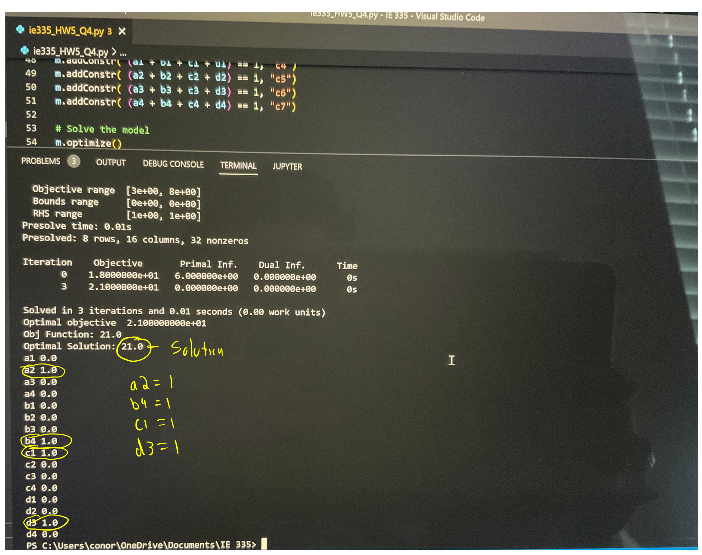
	1	2	3	4	Supply
A	8	6	5	7	1
B	6	5	3	4	1
C	6	7	5	6	1
D	6	7	5	6	1
Demand	1	1	1	1	

$$
\begin{aligned}
\underset{x_{ij}}{\text{minimize}} \quad & \sum_i \sum_j c_{ij} x_{ij} \\
\text{subject to} \quad & \sum_j x_{ij} = 1 \qquad \text{for } i = A, B, C, D, \\
& \sum_i x_{ij} = 1 \qquad \text{for } j = 1, 2, 3, 4, \\
& x_{ij} \in \{0, 1\} \quad \text{for all } i, j
\end{aligned}
$$







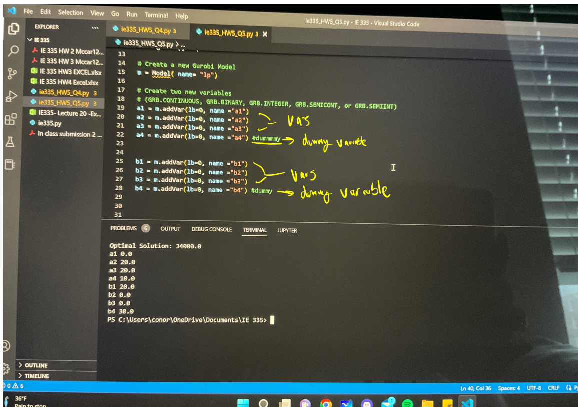


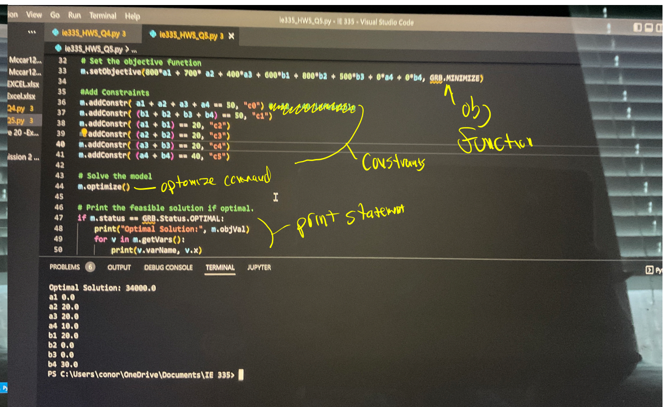
(15pts) Question 5: Considering the following parameter table and LP formulation, solve this problem using **Gurobi in Python**. Submit a screenshot of your code and a screenshot of your results including the final objective value and non-zero decision variables.

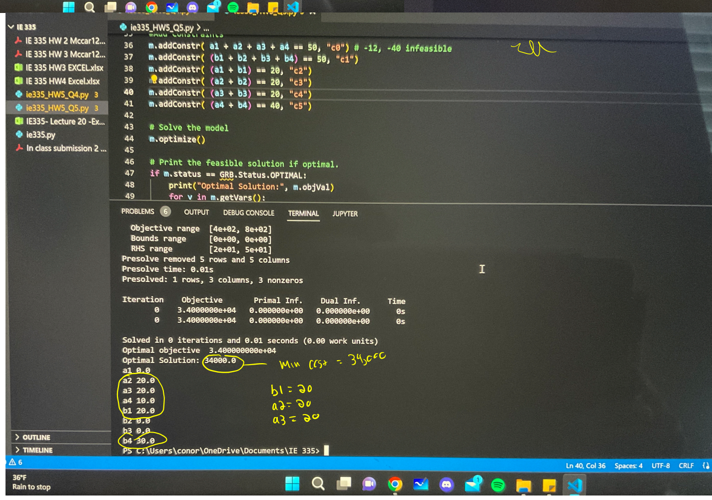
	1	2	3	4(dummy)	Supply
A	\$800	\$700	\$400	0	50
B	\$600	\$800	\$500	0	50
Demand	20	20	20	40	

$$\begin{aligned} \underset{x_{ij}}{\text{minimize}} \quad & \sum_i \sum_j c_{ij} x_{ij} \\ \text{subject to} \quad & \sum_j x_{ij} = S_i \quad \text{for } i = A, B, \\ & \sum_i x_{ij} = D_j \quad \text{for } j = 1, 2, 3, 4, \\ & x_{ij} \geq 0 \quad \text{for all } i, j \end{aligned}$$

where c_{ij} are the shipping cost for each truck from i to j, S_i is the maximum supply of trucks from plant i, and D_j is the exact demand of trucks shipped to distribution center j.







(15pts) **Question 6**: Solve Problem 4.4-6 in the textbook using **Gurobi in Python**. Submit a screenshot of your code and a screenshot of your results including the final objective value and non-zero decision variables. Only use the Problem formulation from the Problem.