Homework 6

Instruction: There are 4 problems in total.
  1) Problems 1-4 – 25 points
Due November 14, 2022 (11:59 pm)

For all problems, provide both hand-written solutions and R codes (for 30 points bonus), wherever applicable.

Problem 1

Consider the following computer output for an experiment. The factor was tested over four levels.

Source	DF	SS	MS	F	P-value
Factor	?	?	330.4716	4.42	?
Error	?	?	?		
Total	31	?			

Annotations on table: $\frac{SS_T}{\alpha-1}$, $\frac{MS_T}{MS_E}$, $\alpha-1$, $\alpha(n-1)$, $\alpha n-1$, SS_T, SS_E, SS_T+SS_E

(a) How many replicates did the experimenter use?
(b) Fill in the missing information in the ANOVA table. Use bounds for the P-value if you use the tables, not a calculator nor R.
(c) What conclusions can you draw about differences in the factor-level means?

a) treatments = 2,
$n = df + 1$
$= 31 + 1$
$= 32$

$32/4 = 8$

b) From F-table: p-val = 0.0115

$4.42 = \frac{330.4716}{MS_E}$

$MS_E = 74.767$

$DF_F = 4-1$
$= 3$

$DF_E = 31 - 3$
$= 28$

$SS_{Total} = SS_T + SS_E$
$= 3084.89$

$3 = \alpha - 1$
$\alpha = 4$

$\frac{SS_T}{3} = 330.4716$

$SS_T = 991.41$

$SSE = MS_E \cdot DF_E$
$= 74.767 \cdot 28$
$= 2093.48$

c)

Using H_0: no difference in means
H_1: difference in means

We say F stat = 4.42 ⟶ p-val = 0.0115 which is < $\alpha = 0.05$

So we reject H_0 and say there are differences in the factor level means.

Problem 2

A research study described an experiment to determine the effect of C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Three flow rates are used in the experiment, and the resulting uniformity (in percent) for six replicates follows.

C_2F_6 Flow (SCCM)	Observations						AVG $(\bar{Y})$
	1	2	3	4	5	6	
125	2.7	4.6	2.6	3.0	3.2	3.8	3.32
160	4.9	4.6	5.0	4.2	3.6	4.2	4.42
200	4.6	3.4	2.9	3.5	4.1	5.1	3.93

(a) Does C_2F_6 flow rate affect etch uniformity? Perform the analysis of variance using $\alpha = 0.05$.

(b) Apply Fisher's LSD method with $\alpha = 0.01$ and determine which levels of the factor differ.

$$\bar{Y} = \frac{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3}{3}$$

$$= 3.89$$

$$SS = 6 \cdot \left[(\bar{y}_1 - \bar{y})^2 + (\bar{y}_2 - \bar{y})^2 + (\bar{y}_3 - \bar{y})^2 \right]$$

$$= 6 \cdot \left[0.327 + 0.279 + 0.002 \right]$$

$$SS = 3.65$$

$$SS_T = \sum_j^3 \sum_i^6 (Y_{ij} - \bar{Y})^2$$

$$= (2.7 - 3.89)^2 + (4.9 - 3.89)^2 + (4.6 - 3.89)^2 \dots + (3.8 - 3.89)^2 + (4.2 - 3.89)^2 + (5.1 - 3.89)^2$$

$SS_T = 11.28$

$MS_F = \frac{SS}{DF_F}$ $MS_E = \frac{SS_E}{DF_E}$ $H_0: \mu_1 = \mu_2 = \mu_3$

$SS_E = SS_T - SS$ $= 1.83$ $= 0.509$ $H_1: \mu_1 \neq \mu_2 \neq \mu_3$ or

$= 11.28 - 3.65$

$= 7.63$

$DF_T = 18 - 1$ $F_{stat} = \frac{MS_F}{MS_E}$ $F_{(2,15)(0.05)} = 3.68$

$= 17$

$DF_F = 3 - 1$ $= \frac{1.83}{0.509}$ Since F_{stat} (3.6) is less than F_{crit} (3.68)

$= 2$ $F_{stat} = 3.60$ we fail to reject and say

$DF_E = 18 - 3$ flow rates don't affect uniformity.

$= 15$

B) $LSD = t_{\frac{0.01}{2}, (18-1)} \cdot \sqrt{\frac{2 \cdot MS_E}{n}}$

$\qquad 2.898 \cdot 0.238$

$= 0.690$

$SCCM \ 125 \ and \ 160$

Problem 3

An article investigated four different methods of preparing the superconducting compound PbMo6S8. The authors contend that the presence of oxygen during the preparation process affects the material's superconducting transition temperature T_c. Preparation methods 1 and 2 use techniques that are designed to eliminate the presence of oxygen, and methods 3 and 4 allow oxygen to be present. Five observations on T_c (in °K) were made for each method, and the results are as follows:

Preparation Method	Transition Temperature T_c (°K)					Avg ($\bar{Y}$)
1	14.8	14.8	14.7	14.8	14.9	14.8
2	14.6	15.0	14.9	14.8	14.7	14.8
3	12.7	11.6	12.4	12.7	12.1	12.3
4	14.2	14.4	14.4	12.2	11.7	13.4

(a) Is there evidence to support the claim that the presence of oxygen during preparation affects the mean transition temperature? Use $\alpha = 0.05$.

(b) What is the P-value for the F-test in part (a)?

(c) Find a 95% confidence interval on mean Tc when method 1 is used to prepare the material.

(d) Apply Fisher's LSD method with $\alpha = 0.05$ and determine which levels of the factor differ.

a) $\bar{Y} = 13.83$ $SS = 5 \cdot \left[(\bar{y}_1 - \bar{y})^2 + (\bar{y}_2 - \bar{y})^2 + (\bar{y}_3 - \bar{y})^2 + (\bar{y}_4 - \bar{y})^2 \right]$

$= 22.04$

$SS_T = \sum_{j}^{4} \sum_{i}^{5} (Y_{ij} - \bar{y})^2$

$= 30.07$

$SS_E = SS_T - SS$
$= 8.03$

$DF_1 = 20 - 1$
$= 19$

$DF_E = 20 - 4$
$= 16$

$DF_F = 4 - 1$
$= 3$

$MS_F = \dfrac{SS}{DF_F}$
$= 7.35$

$MS_E = \dfrac{SS_E}{DF_E}$
$= 0.502$

$F_{stat} = \dfrac{MS_F}{MS_E}$

$= \dfrac{7.35}{0.502}$

$F_{stat} = 14.85$

$F_{crit} = F_{(3,16)(0.05)}$

$= 3.24$

Since F_{stat} is greater than F_{crit} we say there is evidence that mean is equal across all 4 prep methods

b) from R $pf(14.85, 3, 16)$

$P = 6.94 e^{-5}$

c) $\bar{y} \pm t_{\frac{\alpha}{2}, (n-1)_4} \cdot \sqrt{\dfrac{MS_E}{n}}$

$CI: 14.8 \pm 2.12 \cdot \sqrt{\dfrac{0.502}{4}}$

$CI: 13.95 \le \mu_1 \le 15.65$

d) $LSD = t_{\frac{0.01}{2}, (14)} \cdot \sqrt{\dfrac{2 \cdot MS_E}{n}}$

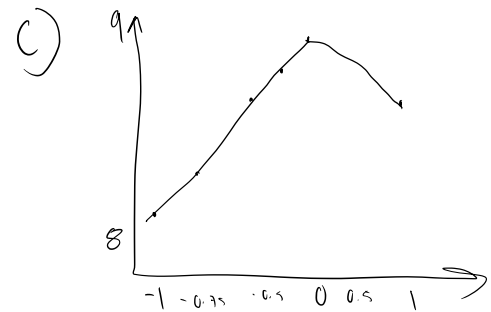$= 2.861 \cdot \sqrt{\dfrac{2 \cdot 0.502}{20}}$

$= 0.641$

Problem 4

An article reported a study on the effects of additives on final polymer properties. In this case, polyurethane additives were referred to as cross-linkers. The average domain spacing was the measurement of the polymer property. The data are as follows:

Cross-Linker Level	Domain Spacing (nm)					
−1	8.2	8	8.2	7.9	8.1	8
−0.75	8.3	8.4	8.3	8.2	8.3	8.1
−0.5	8.9	8.7	8.9	8.4	8.3	8.5
0	8.5	8.7	8.7	8.7	8.8	8.8
0.5	8.8	9.1	9.0	8.7	8.9	8.5
1	8.6	8.5	8.6	8.7	8.8	8.8

(a) Is there a difference in the cross-linker level? Draw comparative box plots and perform an analysis of variance. Use $\alpha = 0.05$.
(b) Find the P-value of the test. Estimate the variability due to random error.
(c) Plot average domain spacing against cross-linker level and interpret the results.
(d) Apply Fisher's LSD method with $\alpha = 0.05$ and determine which levels of the factor differ.

a)
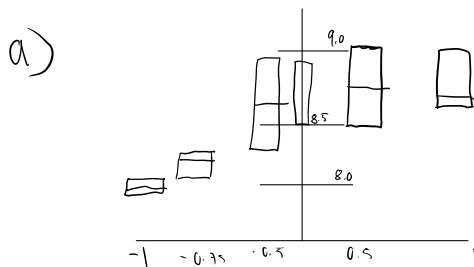
yes there is a difference
a gradual increase then slight decrease

c)

$\bar{y}_1 = 8.13 \quad \bar{y}_4 = 8.7$
$\bar{y}_2 = 8.26 \quad \bar{y}_5 = 8.83$
$\bar{y}_3 = 8.62 \quad \bar{y}_6 = 8.6$

$a = 6$
$n = 36$

$\bar{y}_1 = 8.13 \quad \bar{y}_4 = 8.7$
$\bar{y}_2 = 8.26 \quad \bar{y}_5 = 8.83$
$\bar{y}_3 = 8.62 \quad \bar{y}_6 = 8.6$
$\bar{Y} = 8.53$

$SS = 6 \cdot \left[(\bar{y}_1 - \bar{y})^2 + (\bar{y}_2 - \bar{y})^2 + (\bar{y}_3 - \bar{y})^2 + (\bar{y}_4 - \bar{y})^2 + (\bar{y}_5 - \bar{y})^2 \right]$
$= 2.189$

$SS_T = \sum_j^6 \sum_i^6 (Y_{ij} - \bar{Y})^2$
$= 3.408$

$SS_E = SS_T - SS$
$= 1.22$

$MS_F = \frac{2.189}{5}$
$= 0.438$

$MS_E = \frac{1.22}{30}$
$= 0.041$

$F_{stat} = \frac{0.438}{0.041}$
$= 10.683$

$F_{crit} = F_{(5, 30, 0.05)}$
$= 2.53$

Since $F_{stat} > F_{crit}$ we reject null hypo and say the

d) $LSD = t_{\frac{0.05}{2}, (35)} \cdot \sqrt{\frac{2 \cdot MS_E}{n}}$
$= 2.03 \cdot 0.047$
$= 0.0968$

p value $= 5.79 \cdot 10^{-6}$

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