IE 33000: Probability and Statistics in Engineering II (Fall 2022)
School of Industrial Engineering, Purdue University

Homework 8

Instruction: There are 2 problems in total.
1) Problems 1 and 2 – 50 points
Due December 5, 2022 (11:59 pm)

For all problems, provide both hand-written solutions and R codes (for 30 points bonus), wherever applicable.

Problem 1

The level of cholesterol (in mg/dL) is an important index for human health. The sample size is $n = 5$. The following summary statistics are obtained from cholesterol measurements:

$$\sum_{i=1}^{30} \bar{x}_i = 140.03 \qquad \sum_{i=1}^{30} r_i = 13.63 \qquad \sum_{i=1}^{30} s_i = 5.10$$

(a) Find control limits for $\bar{X}$ and R charts.
(b) Repeat part (a) for $\bar{X}$ and S charts.

$\bar{\bar{X}} = 140.03/30 = 4.67$ $\quad \bar{R} = 13.63/30 = 0.45$ $\quad \bar{S} = 5.1/30 = 0.17$

X chart
$CL = 4.67 \quad A_3\bar{r} = (0.577 \cdot 0.45)$
$UCL = CL + A_3\bar{r} \rightarrow 4.93$
$LCL = CL - A_3\bar{r} \rightarrow 4.41$

R Chart
$CL = 0.45$
$UCL = D_4\bar{r} = 2.11 \cdot 0.45 \rightarrow 0.95$
$LCL = D_3\bar{r} = 0 \cdot 0.45 \Rightarrow 0$

$\bar{X}$ chart
$CL = 4.67$
$UCL = CL + \frac{3 \cdot \bar{S}}{C_4 \cdot \sqrt{n}}$
$= 4.67 + \frac{3 \cdot 0.17}{0.94 \cdot \sqrt{5}}$
$= 4.91$
$LCL = CL - \frac{3 \cdot \bar{S}}{C_4 \cdot \sqrt{n}}$
$= 4.43$

$CL = 0.17$
$UCL = \bar{S} + \frac{3\bar{S} \cdot \sqrt{1 - C_4^2}}{C_4}$
$= 0.17 + \frac{3 \cdot 0.17 \cdot \sqrt{1 - 0.94^2}}{0.94}$
$= 0.355$
$LCL = \bar{S} - \frac{3\bar{S} \cdot \sqrt{1 - C_4^2}}{C_4}$
$= -0.02$
$= 0$

Problem 2

In the following data, the syringe length is measured during a pharmaceutical manufacturing process. The following table provides data (in inches) for 20 samples each of size 5.

Sample	x_1	x_2	x_3	x_4	x_5
1	4.960	4.946	4.950	4.956	4.958
2	4.958	4.927	4.935	4.940	4.950
3	4.971	4.929	4.965	4.952	4.938
4	4.940	4.982	4.970	4.953	4.960
5	4.964	4.950	4.953	4.962	4.956
6	4.969	4.951	4.955	4.966	4.954
7	4.960	4.944	4.957	4.948	4.951
8	4.969	4.949	4.963	4.952	4.962
9	4.984	4.928	4.960	4.943	4.955
10	4.970	4.934	4.961	4.940	4.965
11	4.975	4.959	4.962	4.971	4.968
12	4.945	4.977	4.950	4.969	4.954
13	4.976	4.964	4.970	4.968	4.972
14	4.970	4.954	4.964	4.959	4.968
15	4.982	4.962	4.968	4.975	4.963
16	4.961	4.943	4.950	4.949	4.957
17	4.980	4.970	4.975	4.978	4.977
18	4.975	4.968	4.971	4.969	4.972
19	4.977	4.966	4.969	4.973	4.970
20	4.975	4.967	4.969	4.972	4.972

$A_2 = 0.577$ $C_4 = 0.94$ $D_4 = 2.115$ $D_3 = 0$ $m = 20$ $n = 5$

(a) Using all the data, find control limits for $\bar{X}$ and R charts, construct the chart, and plot the data. Is this process in statistical control?

(b) Use the control limits from part (a) to identify out-of-control points.

(c) Repeat part (a) for $\bar{X}$ and S charts.

$\bar{\bar{X}} = \frac{\sum_{i=1}^{20} \bar{X}_i}{20} = 4.96$ $\bar{r} = \frac{\sum_{i=1}^{20} r_i}{25} = 0.022$ $\bar{s} = \frac{\sum_{i=1}^{20} s_i}{20} = 0.009$

$\bar{X}$ chart

$CL = 4.96$

$UCL = 4.96 + 0.577 \cdot 0.022 = 4.973$

$LCL = 4.96 - 0.577 \cdot 0.022 = 4.948$

R chart

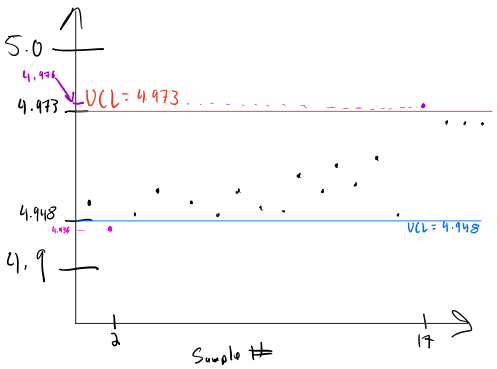
$CL = 0.022$

$UCL = D_4 \cdot \bar{r} = 0.047$

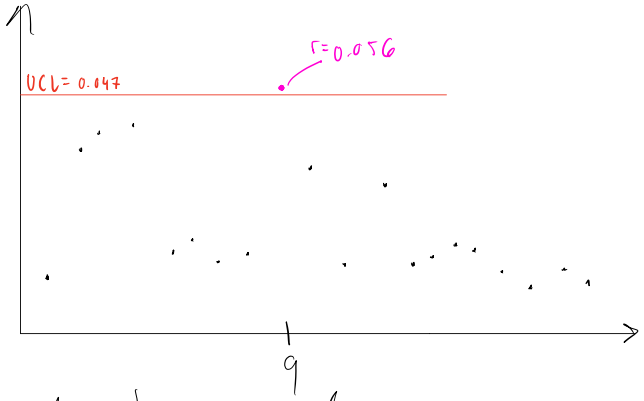
$LCL = D_3 \cdot \bar{r} = 0$

I would say this process isn't in control due to points out of the limits ~ based on next pose charts

X Chart



R Chart



b) Samples 2, 9, and 17 should be removed

c) $\bar{x}$ bar

$CL = 4.96$

$UCL = CL + \frac{3\bar{s}}{c_4\sqrt{n}}$

$= 4.973$

$LCL = CL - \frac{3\bar{s}}{c_4\sqrt{n}}$

$= 4.948$

$\bar{s}$ bar

$\bar{s} = 0.009$

$UCL = \bar{s} + \frac{3 \cdot \bar{s} \cdot \sqrt{1 - c_4^2}}{c_4}$

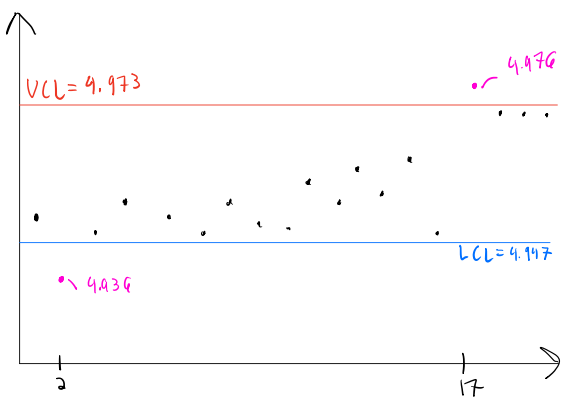
$= 0.019$

$LCL = \bar{s} - \frac{3 \cdot \bar{s} \cdot \sqrt{1 - c_4^2}}{c_4}$

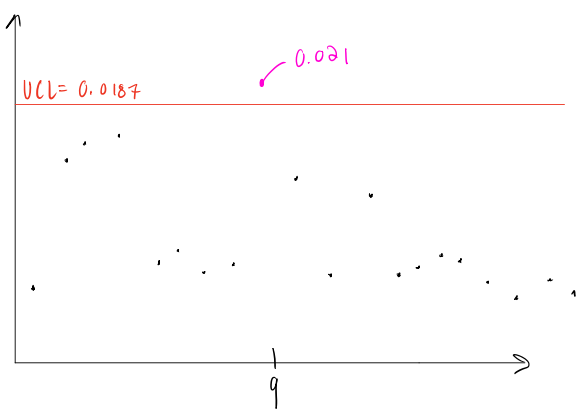
$= -0.001$

$= 0$

$\bar{x}$ bar



$\bar{s}$ bar



```
### Problem 3 ###
q1_xbar = 140.03
q1_r = 13.63
q1_s = 5.1
m = 30
n = 5
q1_xdbar = q1_xdbar / m
q1_rbar = q1_r / m
q1_sbar = q1_s / m

## X and R Charts Limits ##
# X Chart Limits #
q1_xr_x_CL = q1_xdbar
q1_xr_x_UCL = q1_xr_x_CL + 0.577 * q1_sbar
q1_xr_x_LCL = q1_xr_x_CL - 0.577 * q1_sbar

# R Chart Limits #
q1_xr_r_CL = q1_rbar
q1_xr_r_UCL = 2.11 * q1_rbar
q1_xr_r_LCL = 0 * q1_rbar

## X and S Charts Limits ##
# X Chart Limits #
q1_xs_x_CL = q1_xdbar
q1_xs_x_UCL = q1_xs_x_CL + (3 * q1_sbar) / (0.94 * sqrt(n))
q1_xs_x_LCL = q1_xs_x_CL - (3 * q1_sbar) / (0.94 * sqrt(n))

# S Chart Limits #
q1_xs_s_CL = q1_sbar
q1_xs_s_UCL = q1_sbar + (3 * q1_sbar * sqrt(1-0.94^2))/0.94
q1_xs_s_LCL = q1_sbar - (3 * q1_sbar * sqrt(1-0.94^2))/0.94
q1_xs_s_LCL = 0 #the previous row will give you a negative


### Problem 2 ###
library(qcc)
data <- matrix(data = c(4.960, 4.946, 4.950, 4.956, 4.958,
                4.958, 4.927, 4.935, 4.940, 4.920,
                4.971, 4.929, 4.965, 4.952, 4.938,
                4.940, 4.982, 4.970, 4.953, 4.960,
                4.964, 4.951, 4.953, 4.962, 4.956,
                4.969, 4.951, 4.955, 4.966, 4.954,
                4.960, 4.944, 4.957, 4.948, 4.951,
                4.969, 4.949, 4.963, 4.952, 4.962,
                4.984, 4.928, 4.960, 4.943, 4.955,
                4.970, 4.934, 4.961, 4.940, 4.965,
                4.975, 4.959, 4.962, 4.971, 4.968,
                4.945, 4.977, 4.950, 4.969, 4.954,
                4.976, 4.964, 4.970, 4.968, 4.972,
                4.970, 4.954, 4.964, 4.959, 4.968,
                4.982, 4.962, 4.968, 4.975, 4.963,
                4.961, 4.943, 4.950, 4.949, 4.957,
                4.980, 4.970, 4.975, 4.978, 4.977,
                4.975, 4.968, 4.971, 4.969, 4.972,
                4.977, 4.966, 4.969, 4.973, 4.970,
                4.975, 4.967, 4.969, 4.972, 4.972), nrow = 20, ncol = 5)
```

```
XbarChart = qcc(data, type = "xbar", nsigmas = 5)
RChart = qcc(data, type = "R", nsigmas = 5)
SChart = qcc(data, type = "S", nsigmas = 5)
```