IE 33000: Probability and Statistics in Engineering II (Fall 2022)
School of Industrial Engineering, Purdue University

Homework 2

Instruction: There are 10 problems in total.

1) Problems 1 to 10 – 10 points

Due October 3, 2022 (11:59 pm)

For all problems, provide both hand-written solutions and R codes, wherever applicable.

Problem 1

State whether each of the following situations is a correctly stated hypothesis testing problem and why.

a. H_0: μ = 25, H_1: μ ≠ 25
b. H_0: σ > 10, H_1: σ = 10
c. H_0: $\bar{x}$ = 50, H_1: $\bar{x}$ ≠ 50
d. H_0: s = 30, H_1: s > 30

Problem 2

A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed with standard deviation 0.25 volt, and the manufacturer wishes to test H_0: μ = 5 volts against H_1: μ ≠ 5 volts, using n = 8 units.
a. The acceptance region is 4.85 ≤ x ≤ 5.15. Find the value of α.
b. Find the power of the test for detecting a true mean output voltage of 5.1 volts.

Problem 1

State whether each of the following situations is a correctly stated hypothesis testing problem and why.

a. H_0: μ = 25, H_1: μ ≠ 25 → Correct : pop is parameter → null has "=" opp doesn't have "="
b. H_0: σ > 10, H_1: σ = 10 → Wrong : pop is parameter → null hypo doesn't "=" while opp does
c. H_0: $\bar{x}$ = 50, H_1: $\bar{x}$ ≠ 50 → Wrong : x isn't a pop parameter
d. H_0: s = 30, H_1: s > 30 → s is a sample stat not population parameter

Problem 2

A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed with standard deviation 0.25 volt, and the manufacturer wishes to test H_0: μ = 5 volts against H_1: μ ≠ 5 volts, using n = 8 units.

a. The acceptance region is 4.85 ≤ x ≤ 5.15. Find the value of α.
b. Find the power of the test for detecting a true mean output voltage of 5.1 volts.

null = 5 opp ≠ 5
$\sigma = 0.25$ $n=8$
Accpt Region: $4.85 \leq \bar{x} \leq 5.15$
Rejection Region $4.85 > \bar{x}$ & $\bar{x} > 5.15$

a) $\alpha = P(\bar{x} < 4.85 \text{ or } \bar{x} > 5.15)$
$= P\left[z < \frac{4.85-5}{\frac{0.25}{\sqrt{8}}}\right] + P\left(z > \frac{5.15-5}{\frac{0.25}{\sqrt{8}}}\right)$
$= P(z < -1.7) + P(z > 1.7)$
$= 0.0446 + (1 - 0.9554)$
$\alpha = 0.0892$

b) $\beta = P(\text{Type II Error})$
$= P(4.85 < \bar{x} < 5.15)$
$= P\left(\frac{4.85-5.1}{\frac{0.25}{\sqrt{8}}} < z < \frac{5.15-5.1}{\frac{0.25}{\sqrt{8}}}\right)$
$= P(-2.83 < z < 0.57)$
$= P(z < 0.57) - P(z < -2.83)$
$= 0.7157 - 0.0023$
$\beta = 0.7134$

Pwr $= 1 - 0.7134$
$= 0.2866$

Problem 3

The life in hours of a battery is known to be approximately normally distributed with standard deviation σ = 1.25 hours. A random sample of 10 batteries has a mean life of $\bar{x}$ = 40.5 hours.
a. Is there evidence to support the claim that battery life exceeds 40 hours? Use α = 0.5.
b. What is the P-value for the test in part (a)?
c. What is the β-error for the test in part (a) if the true mean life is 42 hours?

null hypo = $\mu \le 40$
opp hypo = $\mu > 40$
$\alpha = 0.05$

a) $Z = \left(\frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}\right)$

$= \left(\frac{40.5 - 40}{\frac{1.25}{\sqrt{10}}}\right)$ $\quad Z(0.95) = 1.65$

$= 1.26$

since $1.26 < 1.65$
null hypo Fails - Can't exceed 40 hrs

b) $P(Z > Z_0) = 1 - Z^{-1}(1.26)$
$= 1 - 0.8962$
$\boxed{P = 0.1038}$

c) $\delta = \mu - \mu_0$
$= 42 - 40$
$= 2$

$\beta = Z\left(Z[1-\alpha] - \left(\frac{\delta\sqrt{n}}{\sigma}\right)\right)$
$Z\left(1.65 - \frac{2\sqrt{10}}{1.25}\right)$
$= Z(-3.41)$
$\boxed{\beta = 0.000325}$

Problem 4

The bacterial strain Acinetobacter has been tested for its adhesion properties. A sample of five measurements gave readings of 2.69, 5.76, 2.67, 1.62 and 4.12 dyne-cm^2. Assume that the standard deviation is known to be 0.66 dyne-cm^2 and that the scientists are interested in high adhesion (at least 2.5 dyne-cm^2).

a. Should the alternative hypothesis be one-sided or two-sided?
b. Test the hypothesis that the mean adhesion is 2.5 dyne-cm^2.
c. What is the P-value of the test statistic?

a) $H_0: \mu \le 2.50$ | $H_{opp}: \mu > 2.5$
The opp should be one sided

b) $\bar{x} = \frac{2.69 + 5.76 + 2.67 + 1.62 + 4.12}{5}$
$= 3.372$

$Z_1 = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow \frac{3.372 - 2.5}{\frac{0.66}{\sqrt{5}}} \Rightarrow 2.95$

$\Rightarrow Z_1[x] = 0.95$
$x_2 = 1.65$

since $Z > Z_1$
$2.\ldots > 1.65$
we say yes it will be greater than 2.5

c) $P(2.95) = ?$
$\boxed{P = 0.021}$

Problem 5

An article in the ASCE Journal of Energy Engineering (1999, Vol. 125, pp. 59–75) describes a study of the thermal inertia properties of autoclaved aerated concrete used as a building material. Five samples of the material were tested in a structure, and the average interior temperatures (°C) reported were as follows: 23.01, 22.22, 22.04, 22.62, and 22.59.

a. Test the hypotheses H_0: $\mu = 22.5$ versus H_1: $\mu \neq 22.5$, using $\alpha = 0.05$. Find the P-value.
b. Explain how the question in part (a) could be answered by constructing a two-sided confidence interval on the mean interior temperature.

a) $n = 5$, mean $= 22.496$, $\sigma = 0.38$

$$t = \frac{22.496 - 22.5}{\frac{0.38}{\sqrt{5}}} \Rightarrow -0.024$$

$P(t[= -0.024, \alpha = 0.05, DF = 4])$

$p = 0.982$

b) $MoE = t_{\underset{4}{n-1}, \underset{0.025}{\frac{\alpha}{2}}} \cdot \frac{s}{\sqrt{n}}$

$= 2.776 \cdot \frac{0.38}{\sqrt{5}}$

$= 0.472$

$22.496 \pm .472$

95% CI = (22.024, 22.968)

22.5 falls in this range

we fail to reject

Problem 6

Cloud seeding has been studied for many decades as a weather modification procedure (for an interesting study of this subject, see the article in Technometrics, "A Bayesian Analysis of a Multiplicative Treatment Effect in Weather Modification," 1975, Vol. 17, pp. 161–166). The rainfall in acre-feet from 20 clouds that were selected at random and seeded with silver nitrate follows: 18.0, 30.7, 19.8, 27.1, 22.3, 18.8, 31.8, 23.4, 21.2, 27.9, 31.9, 27.1, 25.0, 24.7, 26.9, 21.8, 29.2, 34.8, 26.7, and 31.6.

a. Can you support a claim that mean rainfall from seeded clouds exceeds 25 acre-feet? Use $\alpha = 0.01$. Find the P-value.
b. Explain how the question in part (a) could be answered by constructing a one-sided confidence bound on the mean diameter.

a) $H_0 = 25$

$$t = \frac{26.03 - 25}{\left(\frac{4.78}{\sqrt{20}}\right)}$$

$t = 0.9637$

$t_{crit} = 2.5315$

$P = 0.1737$

Since $P > \alpha$ and $t < t_{crit}$ we accept H_0

b) $M.E = 2.5395 \cdot \frac{4.78}{\sqrt{20}}$

$= 2.7143$

$1 - 0.01 = P(26.03 - 2.7143 < \mu < 26.03 + 2.7143)$

$0.99 = P(23.32 < \mu < 28.74)$

Problem 7

An article in Medicine and Science in Sports and Exercise ["Electrostimulation Training Effects on the Physical Performance of Ice Hockey Players" (2005, Vol. 37, pp. 455–460)] considered the use of electromyostimulation (EMS) as a method to train healthy skeletal muscle. EMS sessions consisted of 30 contractions (4-second duration, 85 Hz) and were carried out three times per week for 3 weeks on 17 ice hockey players. The 10-meter skating performance test showed a standard deviation of 0.09 seconds.

a. Is there strong evidence to conclude that the standard deviation of performance time exceeds the historical value of 0.75 seconds? Use $\alpha = 0.05$. Find the P-value for this test.

b. Discuss how part (a) could be answered by constructing a 95% one-sided confidence interval for σ.

$n = 5$, $\sigma^{n} = 0.09$, $\sigma = 0.75$

$\alpha = 0.05 \rightarrow 0.025$

$df = 16$

CV: $(\chi^2_{0.025,16}, \chi^2_{0.975,16})$

$= (6.91, 28.85)$

test Stat: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} \Rightarrow \frac{14 \cdot 0.09^2}{0.75^2} \Rightarrow 0.202$

$P = 1.0$ (right tail)

We fail to reject null hypo since P value high

b) CI@95% $= \left(\sqrt{\frac{16 \cdot 0.09^2}{28.85}} \le \sqrt{\frac{16 \cdot 0.09^2}{6.91}}\right)$

$= 0.067 \le \sigma \le 0.14$

this shows 0.09 inside confidence interval this its ok

Problem 8

The percentage of titanium in an alloy used in aerospace castings is measured in 51 randomly selected parts. The sample standard deviation is s = 0.37.

a. Test the hypothesis H_0: σ = 0.25 versus H_1: σ ≠ 0.25 using α = 0.05. State any necessary assumptions about the underlying distribution of the data. Find the P-value.
b. Explain how you could answer the question in part (a) by constructing a 95% two-sided confidence interval for σ.

a) $n = 51$, $s = 0.37$

$H_0 = .25$; test Stat $= \chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$

$\alpha = .05$

$= \frac{50 \cdot .37^2}{0.25^2}$

$= 109.52$

$P = \chi^2(109.52, 50)$

$= 0.\,224$

if $P > \alpha$ we accept H_0
thus we accept

b) $1 - \alpha = P\left(\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2, df}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2, df}}}\right)$

$= 0.31 < \sigma < 0.46$

Problem 9

Ten samples were taken from a plating bath used in an electronics manufacturing process, and the pH of the bath was determined. The sample pH values are 7.91, 7.85, 6.82, 8.01, 7.46, 6.95, 7.05, 7.35, 7.25, and 7.42. Manufacturing engineering believes that pH has a median value of 7.0.

Do the sample data indicate that this statement is correct? Use the sign test with α = 0.05 to investigate this hypothesis. Find the P-value for this test.

$H_0 = 7.0$

$R^+ = $ # of diff > 0

$R^+ = 8$

$n/2 = 5$

$P = 2 \cdot \sum_{r=8}^{10} \binom{10}{r} (0.5)^8 \cdot (0.5)^2$

$= 0.11$

Since $P = .11$ is $> \alpha = 0.05$ we don't reject hypo

Problem 10

The impurity level (in ppm) is routinely measured in an intermediate chemical product. The following data were observed in a recent test: 2.4, 2.5, 1.7, 1.6, 1.9, 2.6, 1.3, 1.9, 2.0, 2.5, 2.6, 2.3, 2.0, 1.8, 1.3, 1.7, 2.0, 1.9, 2.3, 1.9, 2.4, 1.6.

Can you claim that the median impurity level is less than 2.5 ppm? State and test the appropriate hypothesis using the sign test with α = 0.05. What is the P-value for this test?

$H_0: M \geq 2.5$

$\alpha = 0.05$

$N = 22$

$R^+ = 2$

$R^- = 18$

$p' = \left(\frac{1}{2}\right)^n \cdot \sum^{k} \binom{n}{i}$

$= \left(\frac{1}{2}\right)^{20} \cdot \sum_i^{18} \binom{20}{i}$

$= 0.99$

Since $p' > \alpha$ we fail to reject H_0

The median impurity level is not less than 2.5 ppm

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#QUESTION 2
#A manufacturer is interested in the output voltage of a power supply used in
a PC.
#Output voltage is assumed to be normally distributed with standard deviation
0.25
#volt, and the manufacturer wishes to test H0: μ = 5 volts against H1: μ ≠ 5
volts,
#using n = 8 units.
#a. The acceptance region is 4.85 ≤ x ≤ 5.15. Find the value of α.
library(ltm)
data1a <- as.data.frame((rnorm(8,5,0.25)))
cronbach.alpha(data1a)
#b. Find the power of the test for detecting a true mean output voltage of 5.1
volts.
oneb_m <- 5
oneb_s <- 0.25
oneb_n <-8
oneb_error <- qnorm(0.975)*oneb_s/sqrt(oneb_n)
oneb_left <- oneb_m + oneb_error
oneb_right <- oneb_m + oneb_error
oneb_assumed <- oneb_m + 0.1
oneb_Zleft <- (oneb_left-oneb_assumed)/(oneb_s/sqrt(oneb_n))
oneb_Zright <-(oneb_right-oneb_assumed)/(oneb_s/sqrt(oneb_n))
oneb_p <- pnorm(oneb_Zright)-pnorm(oneb_Zleft)
oneb_beta <- 1 - oneb_p

#Problem 3
#The life in hours of a battery is known to be approximately normally
distributed with
#standard deviation σ = 1.25 hours. A random sample of 10 batteries has a mean
life
#of x̄ = 40.5 hours.
#a. Is there evidence to support the claim that battery life exceeds 40 hours?
Use α = 0.5.
threea_z <- (40.5 - 40) / (1.25 / sqrt(10))
threea_evidence <- threea_z > qnorm(0.95)
#b. What is the P-value for the test in part (a)?
threeb_p <- 2*pnorm(-abs(threea_z))
#c. What is the β-error for the test in part (a) if the true mean life is 42
hours?
threec_b <- pnorm(1.65 - (2 * sqrt(10))/1.25)

#Problem 4
#The bacterial strain Acinetobacter has been tested for its adhesion
properties. A
#sample of five measurements gave readings of 2.69, 5.76, 2.67, 1.62 and 4.12
dyne-
#  cm2. Assume that the standard deviation is known to be 0.66 dyne-cm2 and
that the
#scientists are interested in high adhesion (at least 2.5 dyne-cm2).
#a. Should the alternative hypothesis be one-sided or two-sided?
#b. Test the hypothesis that the mean adhesion is 2.5 dyne-cm2.
foura_z <- (3.372 - 2.5) / (0.66 / sqrt(5))
foura_evidence <- threea_z > qnorm(0.95)
#c. What is the P-value of the test statistic?
fourb_p <- 2*pnorm(-abs(foura_z))
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#Problem 5
#An article in the ASCE Journal of Energy Engineering (1999, Vol. 125, pp. 59–
75)
#describes a study of the thermal inertia properties of autoclaved aerated
concrete
#used as a building material. Five samples of the material were tested in a
structure,
#and the average interior temperatures (∘C) reported were as follows: 23.01,
22.22,
#22.04, 22.62, and 22.59.
#a. Test the hypotheses H0: μ = 22.5 versus H1: μ ≠ 22.5, using α = 0.05. Find
the P-
#value.
fivea_n <- 5
fivea_s <-0.38
fivea_mean<- 22.496
fivea_t <- ((22.496 - 22.5) / (0.38 / sqrt(5)))
fivea_p <- 2*pt(-abs(fivea_t),df=fivea_n-1)
#b. Explain how the question in part (a) could be answered by constructing a
two-
#sided confidence interval on the mean interior temperature.
fiveb_MoE <- 2.776 * fivea_s / sqrt(fivea_n)
fiveb_CI <- c(fivea_mean - fiveb_MoE, fivea_mean + fiveb_MoE)

#Problem 6
#Cloud seeding has been studied for many decades as a weather modification
#procedure (for an interesting study of this subject, see the article in
Technometrics,
#"A Bayesian Analysis of a Multiplicative Treatment Effect in Weather
#Modification," 1975, Vol. 17, pp. 161–166). The rainfall in acre-feet from 20
clouds
#that were selected at random and seeded with silver nitrate follows: 18.0,
30.7, 19.8,
#27.1, 22.3, 18.8, 31.8, 23.4, 21.2, 27.9, 31.9, 27.1, 25.0, 24.7, 26.9, 21.8,
29.2, 34.8,
#26.7, and 31.6.
#a. Can you support a claim that mean rainfall from seeded clouds exceeds 25
acre-
#feet? Use α = 0.01. Find the P-value.
sixa_t <- ((26.03 - 25) / (4.78 / sqrt(20)))
sixa_p <- 2*pt(-abs(sixa_t),df=19)
#b. Explain how the question in part (a) could be answered by constructing a
one-
#sided confidence bound on the mean diameter.
sixb_MoE <- 2.776 * 4.78 / sqrt(20)

#Problem 7
#An article in Medicine and Science in Sports and Exercise
["Electrostimulation
#Training Effects on the Physical Performance of Ice Hockey Players" (2005,
Vol.
#37, pp. 455–460)] considered the use of electromyostimulation (EMS) as a
method
#to train healthy skeletal muscle. EMS sessions consisted of 30 contractions
(4-
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#second duration, 85 Hz) and were carried out three times per week for 3 weeks
on
#17 ice hockey players. The 10-meter skating performance test showed a
standard
#deviation of 0.09 seconds.
#a. Is there strong evidence to conclude that the standard deviation of
performance
#time exceeds the historical value of 0.75 seconds? Use α = 0.05. Find the P-
value for
#this test.
seven_n <- 17
seven_ss<- 0.09
seven_s <- 0.75
seven_alpha <- 0.05
seven_CV <- c(qchisq(0.025, 16, lower.tail=TRUE), qchisq(0.975, 16,
lower.tail=TRUE))
seven_test<- (seven_n - 1)*seven_ss^2 / seven_s
#b. Discuss how part (a) could be answered by constructing a 95% one-sided
#confidence interval for σ.
sevenb_CI <-sqrt(((seven_n - 1)*seven_ss^2) / seven_CV)

#Problem 8
#The percentage of titanium in an alloy used in aerospace castings is measured
in 51
#randomly selected parts. The sample standard deviation is s = 0.37.
#a. Test the hypothesis H0: σ = 0.25 versus H1: σ ≠ 0.25 using α = 0.05. State
any
#necessary assumptions about the underlying distribution of the data. Find the
P-
#  value.
eighta_n <- 51
eighta_s <- 0.37
eight_Test <- ((eighta_n - 1) * eighta_s^2) / 0.25^2
eight_Chi <- qchisq(eight_Test, eighta_n, lower.tail=TRUE)
#b. Explain how you could answer the question in part (a) by constructing a
95% two-
#  sided confidence interval for σ.

#Problem 9
#Ten samples were taken from a plating bath used in an electronics
manufacturing
#process, and the pH of the bath was determined. The sample pH values are
7.91,
#7.85, 6.82, 8.01, 7.46, 6.95, 7.05, 7.35, 7.25, and 7.42. Manufacturing
engineering
#believes that pH has a median value of 7.0.
#Do the sample data indicate that this statement is correct? Use the sign test
with α =
#  0.05 to investigate this hypothesis. Find the P-value for this test

nine_p <- (8:20, sum(choose(10,8)*0.5))*2
nine_eval <- nine_p > 0.05
#Problem 10
#The impurity level (in ppm) is routinely measured in an intermediate chemical
#product. The following data were observed in a recent test: 2.4, 2.5, 1.7,
1.6, 1.9,
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#2.6, 1.3, 1.9, 2.0, 2.5, 2.6, 2.3, 2.0, 1.8, 1.3, 1.7, 2.0, 1.9, 2.3, 1.9,
2.4, 1.6.
#Can you claim that the median impurity level is less than 2.5 ppm? State and
test the
#appropriate hypothesis using the sign test with α = 0.05. What is the P-value
for this
#test?
ten_p <- (1/20)^20 * (1:18, sum(choose(20,i)))
```