Testing random utility (and other axioms of stochastic discrete choice)

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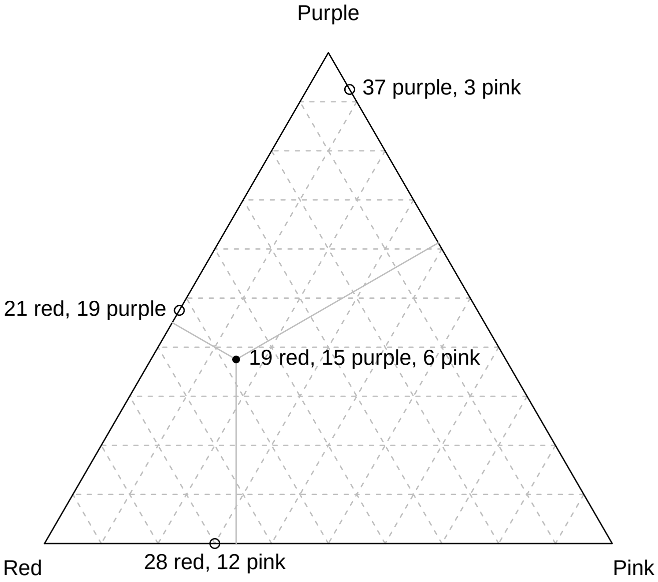
A simple choice experiment

- $4 \times 40 = 160$ distinct participants
- "Which of the following colours do you like best"?

Red	Purple	Pink	Total
19	15	6	40
21	19		40
29		12	40
	37	3	40

Representing frequencies in Barycentric coordinates



An encompassing model

- T is a finite universe of choice objects.
- $P_A(x)$ is the probability of choosing x when presented with choice set A.
- A *random choice structure* for T specifies $P_A(x)$ for all $x \in A \subseteq T$.
- For example, a random choice structure for $T \equiv \{Red, Purple, Pink\}$ gives four probability vectors:

$$(P_T(Red), P_T(Purple), P_T(Pink))$$
$$(P_{\{Red,Purple\}}(Red), P_{\{Red,Purple\}}(Purple))$$
$$(P_{\{Red,Pink\}}(Red), P_{\{Red,Pink\}}(Pink))$$
$$(P_{\{Purple,Pink\}}(Purple), P_{\{Purple,Pink\}}(Pink))$$

- The encompassing model: choices are mutually independent, governed by a static P.

Random utility

A Random Utility Model (RUM) for T is a probability space $(\Omega, \mathcal{F}, \mu)$ and a function $u\colon T \times \Omega \to \mathbb{R}$ such that

$$x, y \in T,\, x \neq y \Rightarrow \mu(\{u(x,\omega) = u(y,\omega)\}) = 0.$$

A RUM *induces* an RCS through the construction

$$P_A(x) = \mu(\{u(x,\omega) = \max_{y \in A} u(y,\omega)\}).$$

We say an RCS satisfies random utility if it can be induced by a RUM.

The content of RU is a kind of context invariance: P does not depend on the choice set A: $P_{\{x,y,z\}}(x) > P_{\{x,y\}}(x)$ is impossible for a P satisfying RU.

Block-Marschak conditions

A random choice structure P can be induced by a random utility model iff

$$\forall x \in A \subseteq T, \sum_{B:\, A \subseteq B \subseteq T} (-1)^{|B \setminus A|} P_B(x) \geq 0.$$

Notes:

1. Each $P_A(x)$ features in multiple sums.
2. Region is convex (intersection of half planes).
3. Block and Marschak show necessity; Falmagne (1978), sufficiency.

Random utility and context dependence

- EBA models (Tversky, 1972) describe choice from A as a stochastic procedure of sequential elimination, and yet they satisfy EBA.
- Random Regret Model (Chorus, 2010) satisfies random utility for some values of the parameters.
- So-called context effects not all inconsistent with random utility
  - Similarity and compromise effects are *consistent* with random utility.
  - Asymmetric dominance effect is inconsistent with random utility.

Main objective

- Testing random utility, no more and no less
  - Minimal additional assumptions.
  - Collect data for all subsets of a universe with at least two elements, to expose all implications of random utility to possible falsification.

Other conditions we test

1. Weak, Moderate and Strong Stochastic Transitivity
2. Triangle Inequality
   - Necessary for regularity
3. Regularity
   - Necessary for random utility
4. Multiplicative Inequality
   - Necessary for Elimination By Aspects (EBA, Tversky 1972)
   - Necessary for independent random utility

A testing ground for axioms of stochastic discrete choice

Definitions:

- Δ is the space of random choice structures.
- $\Lambda \subseteq \Delta$ is the region where random utility (or some other condition) holds.
- Y is chioce data, y the observed (realized) data.

Testing ground consists of

1. (McCausland and Marley, 2013) A two-parameter family of prior distrubutions
   - α (marginals)
   - λ (degree of dependence across choice sets)
2. (McCausland and Marley, 2014) Simulation methods to compute approximations of
   - $\Pr[P \in \Lambda]$,
   - $\Pr[P \in \Lambda | Y = y]$,
   - standard errors.

A test using Bayes factors

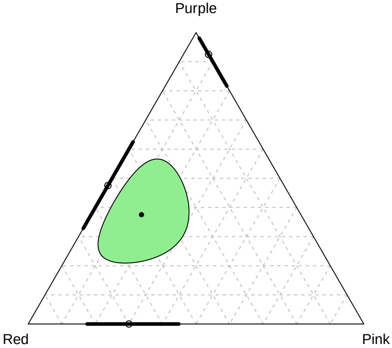
The Bayes factor in favour of a restricted model in which the axiom holds, against the encompassing model, is

$$\mathrm{BF} \equiv \frac{\Pr[Y = y | P \in \Lambda]}{\Pr[Y = y]} = \frac{\Pr[P \in \Lambda | Y = y]}{\Pr[P \in \Lambda]}.$$

If the two models are *a-priori* equiprobable, BF is posterior odds ratio (ratio of posterior probabilities).

Two posterior distributions

- Two different priors with the same four marginals:
  - left, $\lambda = 0$, independence across choice sets
  - right, $\lambda = 1$, support is random utility region.
- Each panel shows four 95% High Probability Density regions



Two experiments

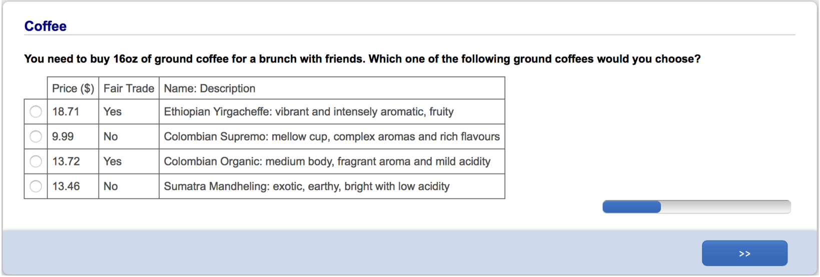
1. McCausland et al. “Testing the Random Utility Hypothesis Directly” (accepted, EJ)
   - Repeated individual choice
   - 141 subjects
   - Universe is five simple lotteries, similar to Tversky (1969), “Intransitivity of Preferences”
   - 6 trials each of 26 doubleton and larger subsets of five lotteries
   - Strong support for hypothesis that most, but not all subjects
2. McCausland et al. “Testing axioms of stochastic discrete choice using population choice probabilities” (incomplete and preliminary).
   - Population choice
   - 1040 subjects
   - 32 choice domains, each a universe of five choice objects

Experimental design, population choice experiment

We ran an experiment with these features:

1. 32 different choice domains (consumer choice, taste, judgement)
   - Trying to say something general about choice.
2. Strictly between-subject design for each choice domain
   - No subject sees the same domain twice.
   - Choices are plausibly independent (globally) and identically distributed (choice set by choice set).
3. Collect choice data for *all* subsets with at least two elements of a universe of objects.
   - Each choice set of each domain presented to 40 different subjects.
   - Expose *all* implications of random utility (and other conditions) to possible falsification.

A consumer choice example



Coffee

You need to buy 16oz of ground coffee for a brunch with friends. Which one of the following ground coffees would you choose?

	Price ($)	Fair Trade	Name: Description
○	18.71	Yes	Ethiopian Yirgacheffe: vibrant and intensely aromatic, fruity
○	9.99	No	Colombian Supremo: mellow cup, complex aromas and rich flavours
○	13.72	Yes	Colombian Organic: medium body, fragrant aroma and mild acidity
○	13.46	No	Sumatra Mandheling: exotic, earthy, bright with low acidity

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Figure 1: Coffee

A simple taste example

Figure 2: Colours

A judgement example

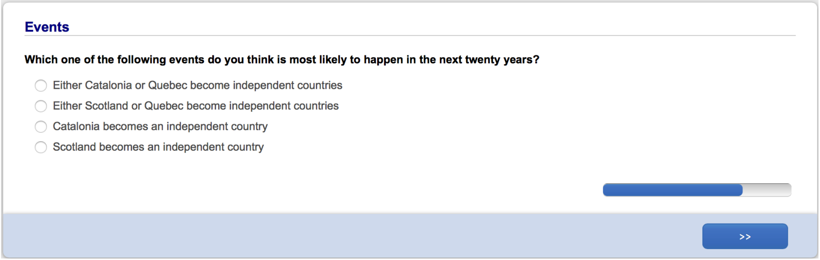

Figure 3: Events

A visual example

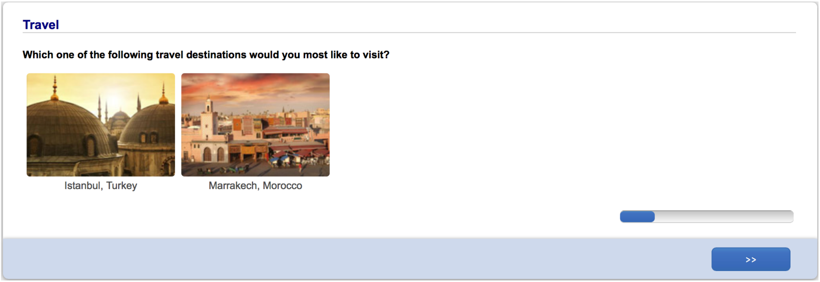

Figure 4: Travel

Assignment of subjects to choice sets

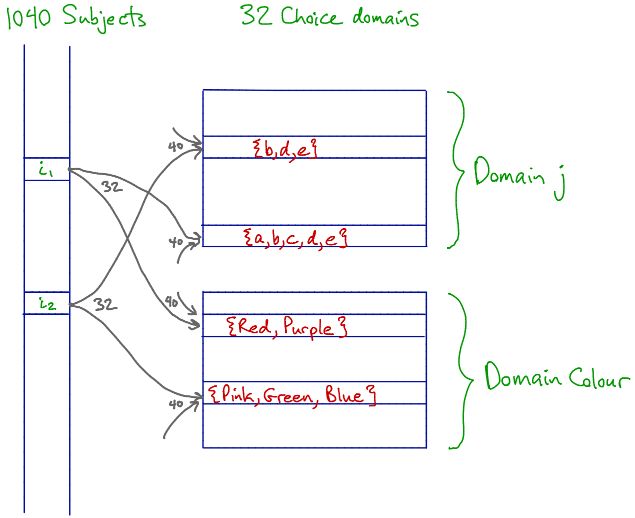

Figure 5: Assignment of subjects to choice sets

Log Bayes factors, first 16 domains

	WST	MST	SST	Reg	RU	MI
Male stars	0.4	2.2	4.2	1.8	1.5	6.3
Female stars	0.0	0.5	1.3	1.2	0.8	2.5
Films	-0.7	-0.9	-2.2	1.6	1.4	6.8
Star pairs	0.1	0.0	-0.7	1.8	1.7	3.9
Pizzas	-0.4	-1.5	-Inf	1.7	1.4	3.9
Juices	0.1	0.5	0.1	1.5	1.3	5.8
Colours	0.2	1.6	1.3	1.3	1.1	5.3
Colour Combinations	-1.1	-2.3	-3.6	1.7	1.5	5.2
Events	0.2	1.4	0.1	0.7	0.7	2.9
Radio formats	0.4	1.9	3.3	0.8	0.6	5.4
Musical artists	0.1	1.0	1.5	1.9	1.6	6.0
Aboriginal art	0.3	1.3	2.7	1.2	0.9	1.4
Impressionist art	0.3	1.5	2.4	1.5	1.2	4.9
Sentences	0.2	1.5	0.9	1.6	1.4	6.6
Travel	0.4	2.1	4.1	1.5	1.3	6.9
Marijuana	0.4	0.1	-3.6	1.5	1.4	3.6

Log Bayes factors, other 16 domains

	WST	MST	SST	Reg	RU	MI
Latitude	0.4	1.5	-Inf	0.6	0.5	-Inf
Dots	0.2	1.0	1.5	1.8	1.5	5.1
Triangles	0.0	0.9	0.8	1.2	1.0	-Inf
Population	-0.1	0.0	0.3	1.9	1.6	6.0
Surface area	0.4	1.5	4.3	1.5	1.5	5.3
Beer	-0.1	0.7	1.6	0.6	0.6	2.5
Cars	0.0	0.2	-0.2	1.1	1.0	4.4
Restaurants	0.1	0.9	0.3	0.7	0.6	3.5
Flight layovers	0.4	0.6	0.6	1.2	1.1	-Inf
Future payments	0.4	1.1	0.3	1.7	1.7	-Inf
Phone plans	-1.1	-1.9	-1.3	1.0	0.8	1.4
Hotel rooms	0.5	1.9	2.9	1.2	1.0	3.7
Two-flight itineraries	-0.5	-0.9	-1.1	1.4	1.1	2.8
Televisions	0.5	2.4	3.5	1.6	1.4	5.0
Coffee	0.3	1.9	2.8	1.6	1.4	6.7
Charity	0.2	-0.6	-Inf	0.9	0.8	1.4

Conclusions

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1. For each choice domain, evidence favours random utility.
2. Overall evidence in favour of random utility is compelling.

Future work

1. Prior as model
   - Support goes beyond RU region, but in a disciplined way.
   - Discriminate within the random utility region.