Testing axioms of stochastic discrete choice using population choice probabilities

William McCausland[1] Tony Marley[2] Clint Davis-Stober[3]

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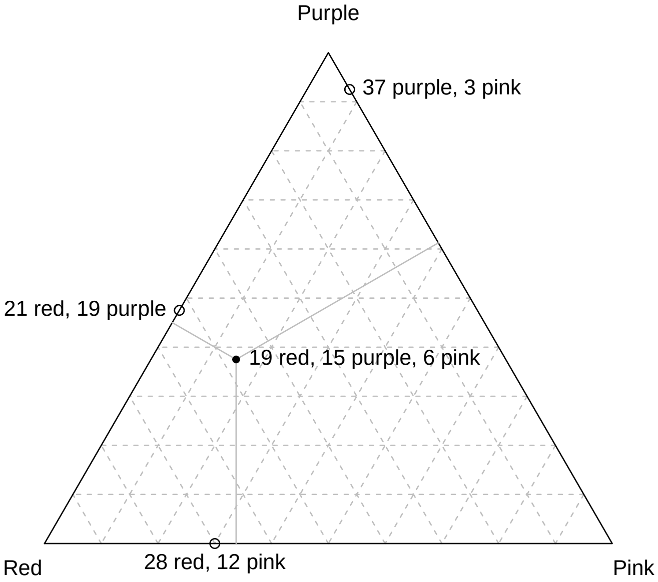
[1]Université de Montréal (Economics)
[2]University of Victoria (Psychology)
[3]University of Missouri (Psychology)

A simple discrete choice experiment

- $4 \times 40 = 160$ distinct participants
- "Which of the following colours do you like best"?

Red	Purple	Pink	Total
19	15	6	40
21	19		40
29		12	40
	37	3	40

Representing frequencies in Barycentric coordinates



Unknown choice probabilities

Let $T \equiv \{Red, Purple, Pink\}$. The unknowns are four probability vectors:

$$(P_T(Red), P_T(Purple), P_T(Pink))$$
$$(P_{\{Red,Purple\}}(Red), P_{\{Red,Purple\}}(Purple))$$
$$(P_{\{Red,Pink\}}(Red), P_{\{Red,Pink\}}(Pink))$$
$$(P_{\{Purple,Pink\}}(Purple), P_{\{Purple,Pink\}}(Pink))$$

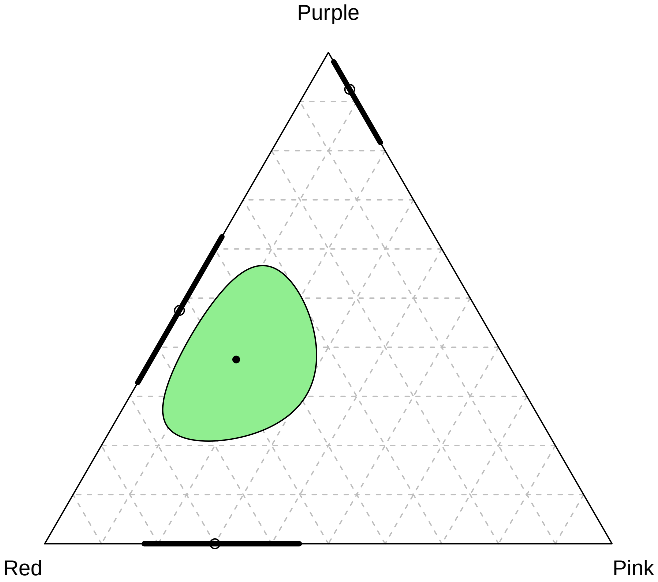
A simple conjugate prior

Here is a special case of a two-parameter (α, λ) prior we use:

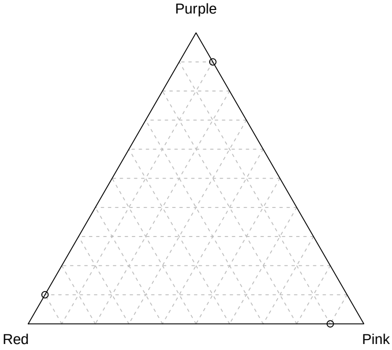
- The four probability vectors are mutually independent ($\lambda = 0$)
- Binary probabilities are all $\mathrm{Be}(\frac{\alpha}{2}, \frac{\alpha}{2})$.
- Ternary probability is $\mathrm{Di}(\frac{\alpha}{3}, \frac{\alpha}{3}, \frac{\alpha}{3})$.

We will take $\alpha = 2$ in the following examples.

Four 95% High Posterior Density (HPD) regions

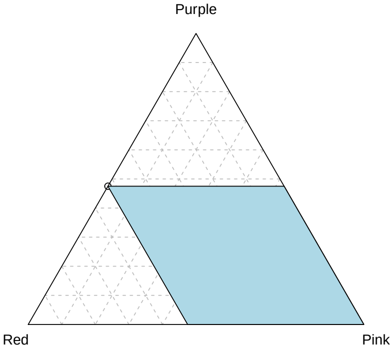


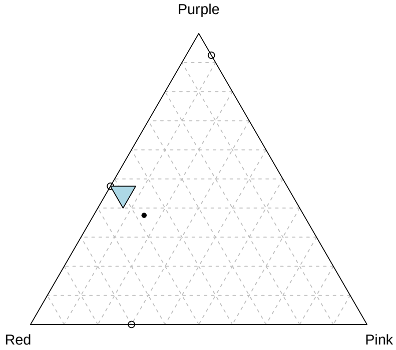
Bringing theory to bear I: Stochastic Transitivity



Bringing theory to bear II: Regularity

Regularity: $x \in A \subseteq B \Rightarrow P_B(x) \leq P_A(x)$.





Bringing theory to bear III: Random utility/preference

Let T be the universe of objects

This set of conditions (the Block-Marschak conditions) is necessary and sufficient for random utility:

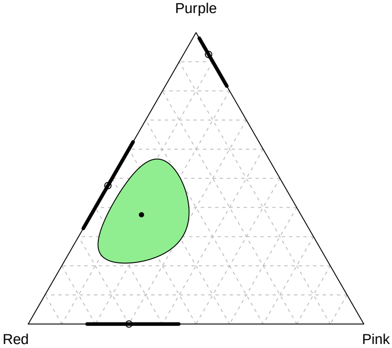
$$\forall x \in A \subseteq T, \sum_{B:\, A \subseteq B \subseteq T} (-1)^{|B \setminus A|} P_B(x) \geq 0.$$

Notes:

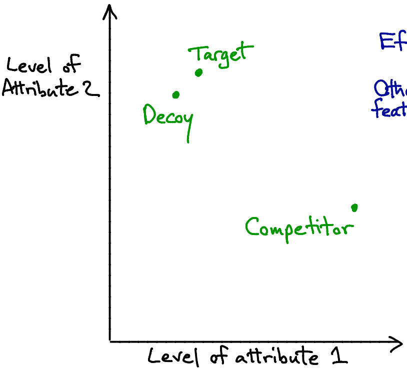
1. Each $P_A(x)$ features in multiple sums
2. Region is convex (intersection of half planes)

Two posterior distributions

- Two different priors with the same fours marginals:
  - left, $\lambda = 0$, independence across choice sets
  - right, $\lambda = 1$, support is random utility region.



The Asymmetric Dominance effect



Effect: $P_{\{T,C,D\}}(T) > P_{\{T,C\}}(T)$

Other features:

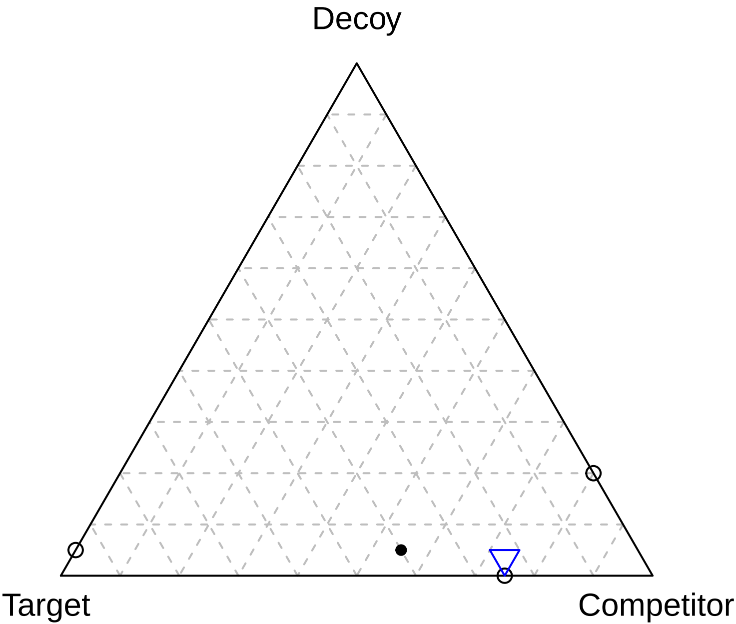
$P_{\{T,D\}}(T) \approx 1$

$P_{\{T,C\}}(T) \approx P_{\{D,C\}}(D)$

$0 \ll P_{\{T,C\}}(T) \ll 1$

Figure 1: Asymmetric Dominance Effect

Typical asymmetric dominance pattern



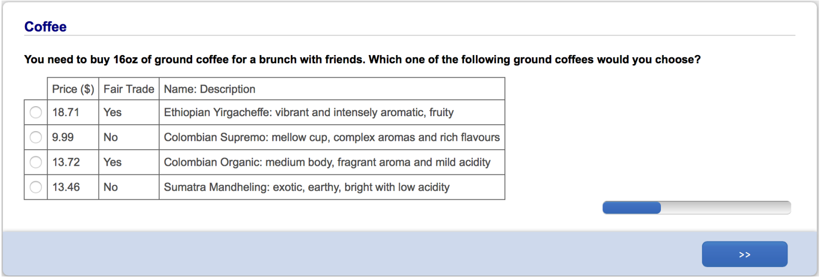
Experimental design

We want to test, for population probabilities, the random utility condition, no more and no less.

We ran an experiment with these features:

1. Several different choice domains (consumer choice, taste, judgement)
   - Trying to say something general about choice.
2. Between subject design for each choice domain
   - Choices are plausibly independent (globally) and identically distributed (choice set by choice set).
3. Collect choice data for *all* subsets with at least two elements of a universe of objects.
   - Expose *all* implications of random utility (and other conditions) to possible falsification.

A consumer choice example



Coffee

You need to buy 16oz of ground coffee for a brunch with friends. Which one of the following ground coffees would you choose?

	Price ($)	Fair Trade	Name: Description
○	18.71	Yes	Ethiopian Yirgacheffe: vibrant and intensely aromatic, fruity
○	9.99	No	Colombian Supremo: mellow cup, complex aromas and rich flavours
○	13.72	Yes	Colombian Organic: medium body, fragrant aroma and mild acidity
○	13.46	No	Sumatra Mandheling: exotic, earthy, bright with low acidity

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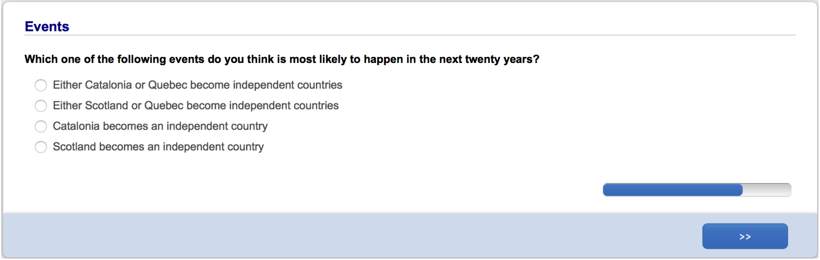
Figure 2: Coffee

A simple taste example

Figure 3: Colours

A judgement example



Events

Which one of the following events do you think is most likely to happen in the next twenty years?

- Either Catalonia or Quebec become independent countries
- Either Scotland or Quebec become independent countries
- Catalonia becomes an independent country
- Scotland becomes an independent country

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Figure 4: Events

A visual example

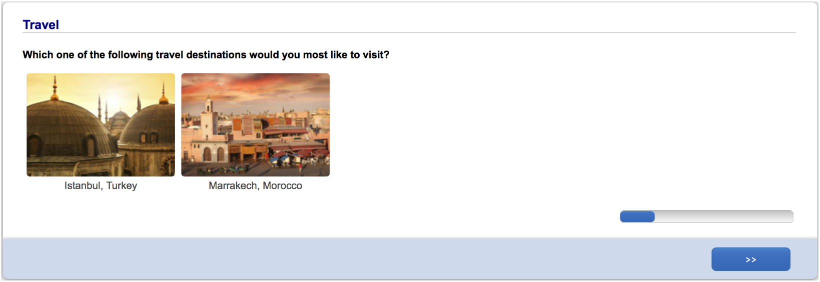

Figure 5: Travel

Assignment of subjects to choice sets

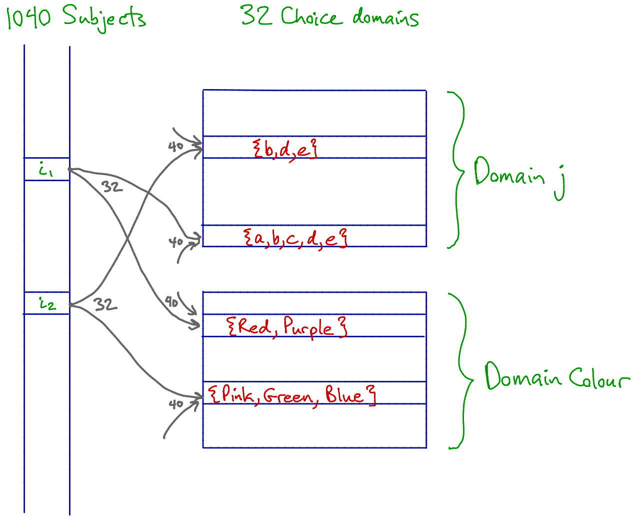

Figure 6: Assignment of subjects to choice sets

Testing conditions on P using Bayes factors

Definitions:

- Λ is the region where random utility (or some other condition) holds.
- Y is data, y the observed data.

The Bayes factor in favour of the restricted model against the encompassing model is

$$\mathrm{BF} \equiv \frac{\Pr[Y = y | P \in \Lambda]}{\Pr[Y = y]} = \frac{\Pr[P \in \Lambda | Y = y]}{\Pr[P \in \Lambda]}.$$

Log Bayes factors, first 16 domains

	WST	MST	SST	Reg	RU	MI
Male stars	0.4	2.2	4.2	1.8	1.5	6.3
Female stars	0.0	0.5	1.3	1.2	0.8	2.5
Films	-0.7	-0.9	-2.2	1.6	1.4	6.8
Star pairs	0.1	0.0	-0.7	1.8	1.7	3.9
Pizzas	-0.4	-1.5	-Inf	1.7	1.4	3.9
Juices	0.1	0.5	0.1	1.5	1.3	5.8
Colours	0.2	1.6	1.3	1.3	1.1	5.3
Colour Combinations	-1.1	-2.3	-3.6	1.7	1.5	5.2
Events	0.2	1.4	0.1	0.7	0.7	2.9
Radio formats	0.4	1.9	3.3	0.8	0.6	5.4
Musical artists	0.1	1.0	1.5	1.9	1.6	6.0
Aboriginal art	0.3	1.3	2.7	1.2	0.9	1.4
Impressionist art	0.3	1.5	2.4	1.5	1.2	4.9
Sentences	0.2	1.5	0.9	1.6	1.4	6.6
Travel	0.4	2.1	4.1	1.5	1.3	6.9
Marijuana	0.4	0.1	-3.6	1.5	1.4	3.6

Log Bayes factors, other 16 domains

	WST	MST	SST	Reg	RU	MI
Latitude	0.4	1.5	-Inf	0.6	0.5	-Inf
Dots	0.2	1.0	1.5	1.8	1.5	5.1
Triangles	0.0	0.9	0.8	1.2	1.0	-Inf
Population	-0.1	0.0	0.3	1.9	1.6	6.0
Surface area	0.4	1.5	4.3	1.5	1.5	5.3
Beer	-0.1	0.7	1.6	0.6	0.6	2.5
Cars	0.0	0.2	-0.2	1.1	1.0	4.4
Restaurants	0.1	0.9	0.3	0.7	0.6	3.5
Flight layovers	0.4	0.6	0.6	1.2	1.1	-Inf
Future payments	0.4	1.1	0.3	1.7	1.7	-Inf
Phone plans	-1.1	-1.9	-1.3	1.0	0.8	1.4
Hotel rooms	0.5	1.9	2.9	1.2	1.0	3.7
Two-flight itineraries	-0.5	-0.9	-1.1	1.4	1.1	2.8
Televisions	0.5	2.4	3.5	1.6	1.4	5.0
Coffee	0.3	1.9	2.8	1.6	1.4	6.7
Charity	0.2	-0.6	-Inf	0.9	0.8	1.4

Conclusions

Conclusions

1. For each choice domain, evidence favours random utility.
2. Overall evidence in favour of random utility is compelling.

Papers in progress

1. Analysis using data on individual choice (paper)
2. Analysis using data on population choice (this presentation)

Future work

1. Prior as model
    - Support goes beyond RU region, but in a disciplined way.
    - Discriminate within the random utility region.