Testing axioms of stochastic discrete choice using population choice probabilities

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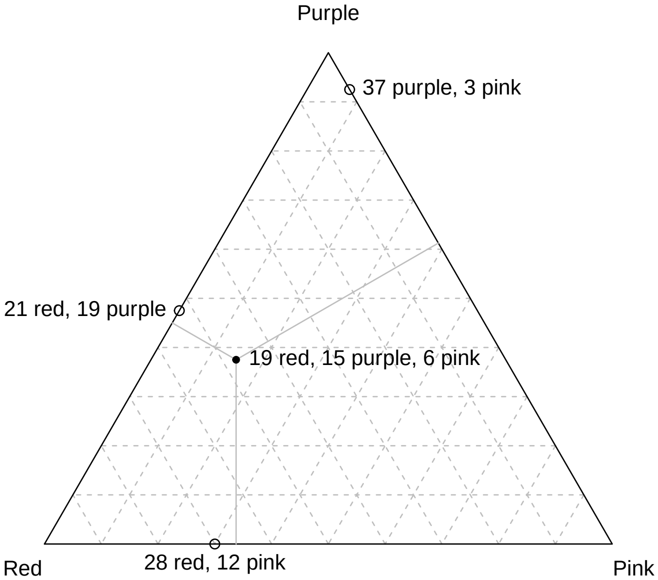
A simple discrete choice experiment

- "Which of the following colours do you like best"?

Red	Purple	Pink	Total
19	15	6	40
21	19		40
29		12	40
	37	3	40

Representing this data



Bayesian inference for choice probabilities, without theory

The unknowns: four probability spaces:

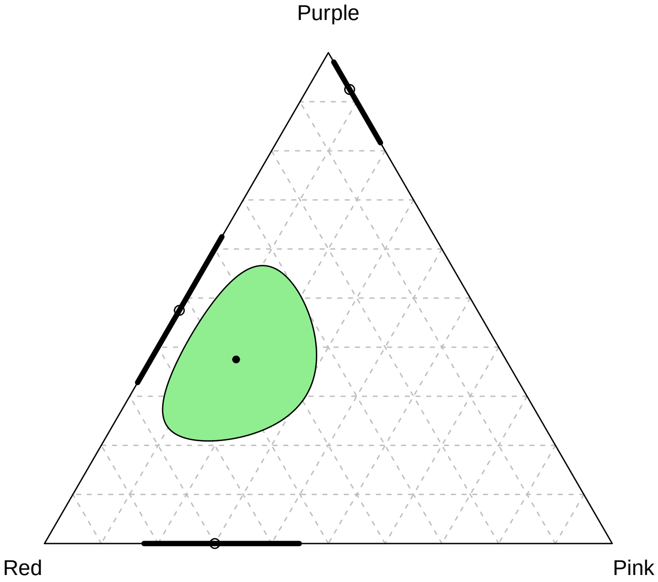
1. $P_{\{Red,Purple\}}(Red)$, $P_{\{Red,Purple\}}(Purple)$
2. $P_{\{Purple,Pink\}}(Purple)$, $P_{\{Purple,Pink\}}(Pink)$
3. $P_{\{Red,Pink\}}(Red)$, $P_{\{Red,Pink\}}(Pink)$
4. $P_{\{Red,Purple,Pink\}}(Red)$, $P_{\{Red,Purple,Pink\}}(Purple)$, $P_{\{Red,Purple,Pink\}}(Pink)$.

A prior with independent probability spaces:

- Four probability spaces are mutually independent,
- Binary probabilities are $\mathrm{Be}(\frac{\alpha}{2}, \frac{\alpha}{2})$.
- Ternary probability is $\mathrm{Di}(\frac{\alpha}{3}, \frac{\alpha}{3}, \frac{\alpha}{3})$.

We will take $\alpha = 2$ in the following examples.

High posterior density (HPD) regions with probability 0.95



Bringing theory to bear: random utility/preference

- $P_A(x)$ is the probability that an individual drawn from a population chooses item x when presented with finite choice set A.
- A *random choice structure* for a master set T specifies $P_A(x)$, all $x \in A \subseteq T$.
- Δ is the set of all random choice structures on T.
- Falmagne (1978): A random choice structure P can be induced by a random utility model iff for all $x \in A \subseteq T$,

$$\sum_{B:\ A \subseteq B \subseteq T} (-1)^{|B \setminus A|} P_B(x) \geq 0.$$

- Let Λ be the set of random choice structures satisfying random utility; we will test the hypothesis $P \in \Lambda \subset \Delta$ against $P \in \Delta$.

Two priors with the same marginals

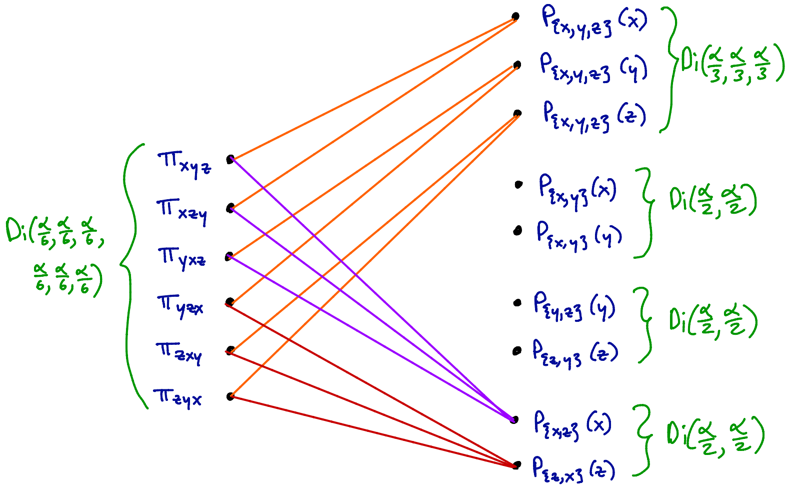
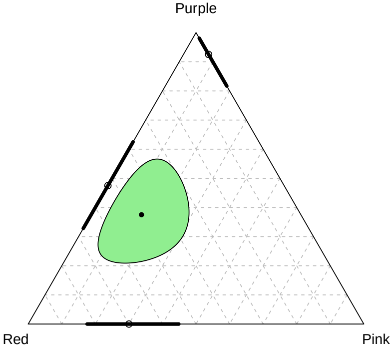


Figure 1: Two priors with the same marginals

Two posterior distributions

- Two different priors with same marginals:
  - left, $\lambda = 0$, independence across choice sets
  - right, $\lambda = 1$, support is random utility region.



Experimental design

We want to test, for population probabilities, the random utility condition, no more and no less.

We ran an experiment with these features:

1. Several different choice domains (consumer choice, taste, judgement) of five objects
    - Trying to say something general about choice.
2. Between subject design for each choice domain
    - Choices are plausibly independent (globally) and identically distributed (choice set by choice set).
3. Collect choice data for *all* subsets with at least two elements of a universe of objects.
    - Expose *all* implications of random utility (and other conditions) to possible falsification.

A consumer choice example

Coffee

You need to buy 16oz of ground coffee for a brunch with friends. Which one of the following ground coffees would you choose?

	Price ($)	Fair Trade	Name: Description
◯	18.71	Yes	Ethiopian Yirgacheffe: vibrant and intensely aromatic, fruity
◯	9.99	No	Colombian Supremo: mellow cup, complex aromas and rich flavours
◯	13.72	Yes	Colombian Organic: medium body, fragrant aroma and mild acidity
◯	13.46	No	Sumatra Mandheling: exotic, earthy, bright with low acidity

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Figure 2: Coffee

A simple taste example

Figure 3: Colours

A judgement example

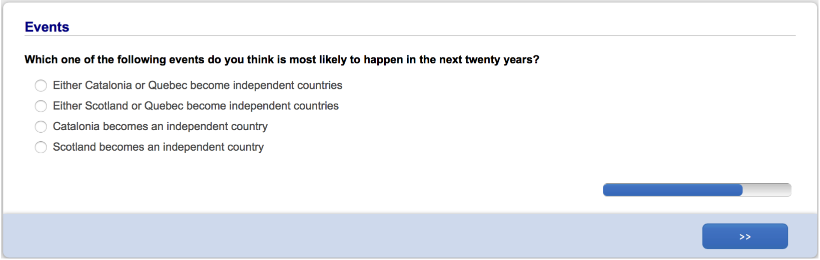

Figure 4: Events

A visual example

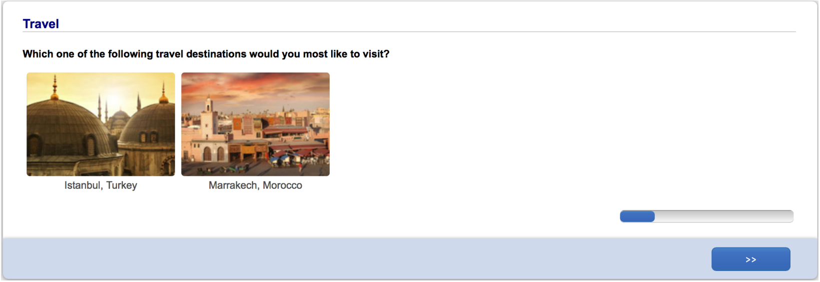

Figure 5: Travel

Testing conditions on P using Bayes factors

Definitions:

- Λ is the region where random utility (or some other condition) holds.
- Y is data, y the observed data.

The Bayes factor in favour of the restricted model against the encompassing model is

$$\mathrm{BF} \equiv \frac{\Pr[Y = y | P \in \Lambda]}{\Pr[Y = y]} = \frac{\Pr[P \in \Lambda | Y = y]}{\Pr[P \in \Lambda]}.$$

We use a hierarchical prior, with priors for

- α, governing how likely probabilities are near 0 or 1,
- λ, governing the dependence of probabilities across choice sets.

Log Bayes factors, first 16 domains

	WST	MST	SST	Reg	RU	MI
Male stars	0.4	2.2	4.2	1.8	1.5	6.3
Female stars	0.0	0.5	1.3	1.2	0.8	2.5
Films	-0.7	-0.9	-2.2	1.6	1.4	6.8
Star pairs	0.1	0.0	-0.7	1.8	1.7	3.9
Pizzas	-0.4	-1.5	-Inf	1.7	1.4	3.9
Juices	0.1	0.5	0.1	1.5	1.3	5.8
Colours	0.2	1.6	1.3	1.3	1.1	5.3
Colour Combinations	-1.1	-2.3	-3.6	1.7	1.5	5.2
Events	0.2	1.4	0.1	0.7	0.7	2.9
Radio formats	0.4	1.9	3.3	0.8	0.6	5.4
Musical artists	0.1	1.0	1.5	1.9	1.6	6.0
Aboriginal art	0.3	1.3	2.7	1.2	0.9	1.4
Impressionist art	0.3	1.5	2.4	1.5	1.2	4.9
Sentences	0.2	1.5	0.9	1.6	1.4	6.6
Travel	0.4	2.1	4.1	1.5	1.3	6.9
Marijuana	0.4	0.1	-3.6	1.5	1.4	3.6

Log Bayes factors, other 16 domains

	WST	MST	SST	Reg	RU	MI
Latitude	0.4	1.5	-Inf	0.6	0.5	-Inf
Dots	0.2	1.0	1.5	1.8	1.5	5.1
Triangles	0.0	0.9	0.8	1.2	1.0	-Inf
Population	-0.1	0.0	0.3	1.9	1.6	6.0
Surface area	0.4	1.5	4.3	1.5	1.5	5.3
Beer	-0.1	0.7	1.6	0.6	0.6	2.5
Cars	0.0	0.2	-0.2	1.1	1.0	4.4
Restaurants	0.1	0.9	0.3	0.7	0.6	3.5
Flight layovers	0.4	0.6	0.6	1.2	1.1	-Inf
Future payments	0.4	1.1	0.3	1.7	1.7	-Inf
Phone plans	-1.1	-1.9	-1.3	1.0	0.8	1.4
Hotel rooms	0.5	1.9	2.9	1.2	1.0	3.7
Two-flight itineraries	-0.5	-0.9	-1.1	1.4	1.1	2.8
Televisions	0.5	2.4	3.5	1.6	1.4	5.0
Coffee	0.3	1.9	2.8	1.6	1.4	6.7
Charity	0.2	-0.6	-Inf	0.9	0.8	1.4

Conclusions

1. For each choice domain, random utility is favoured, although never strongly.
2. Overall evidence in favour of random utility is compelling.