

### Data for choice domain 3, films

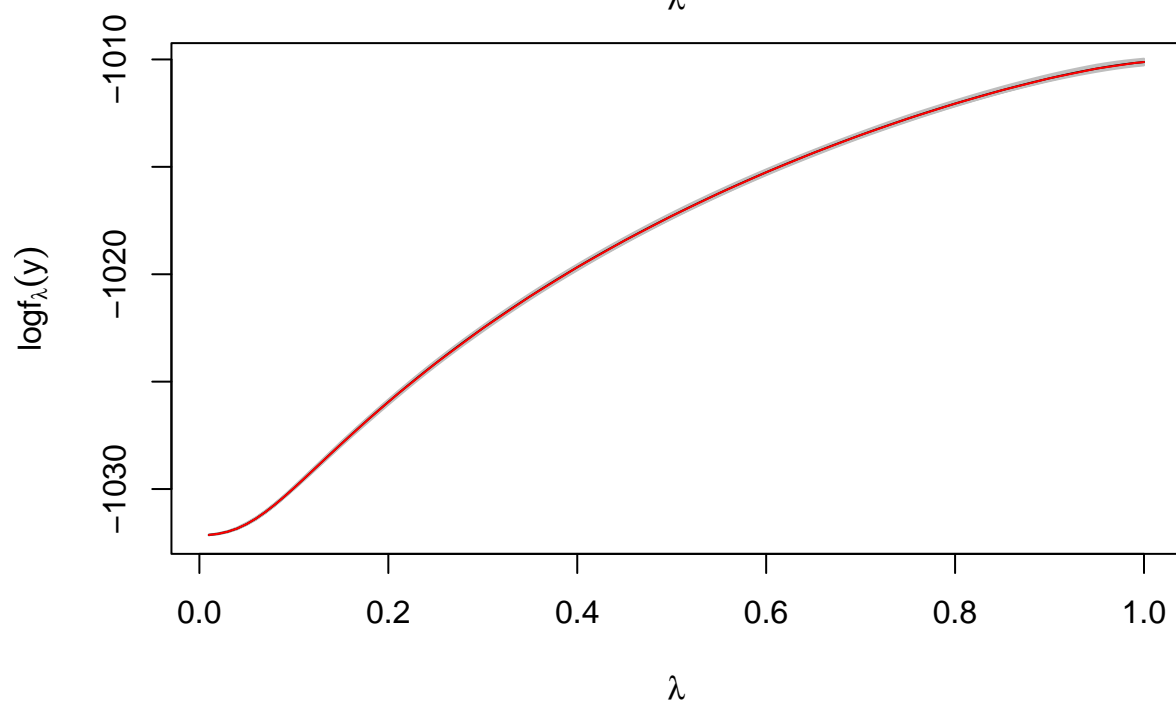
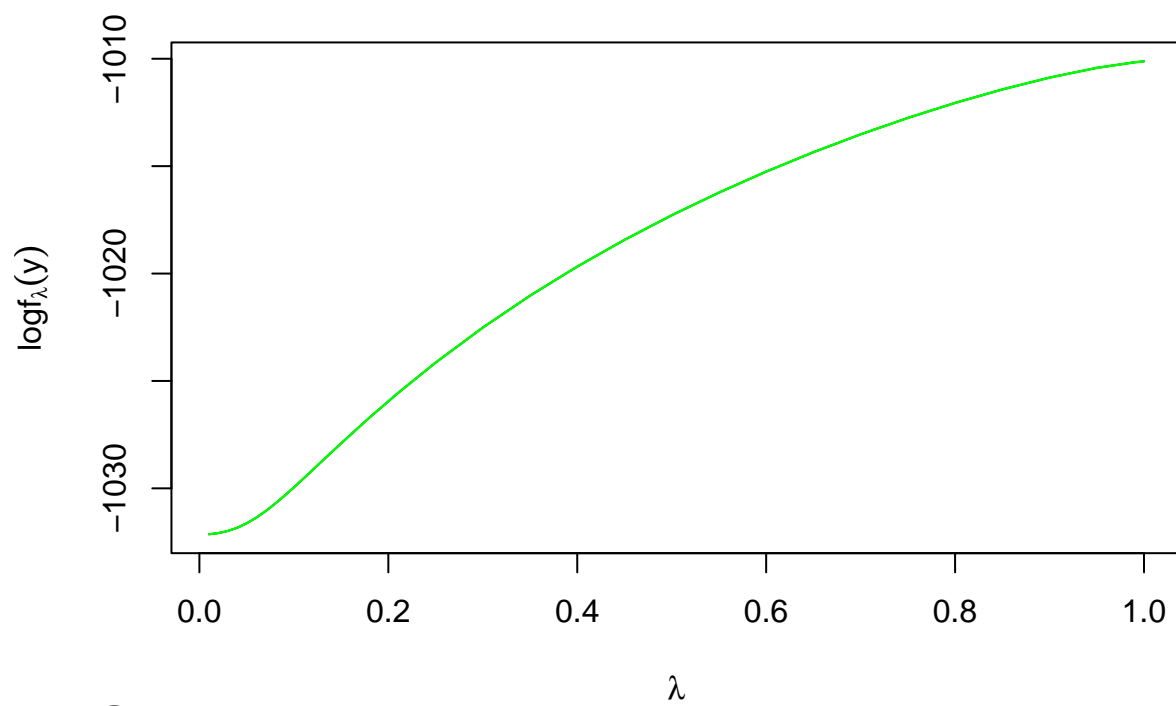
Subset	Object				
	a	b	c	d	e
a	0	-	-	-	-
b	-	0	-	-	-
ab	17	23	-	-	-
c	-	-	0	-	-
ac	27	-	13	-	-
bc	-	23	17	-	-
abc	17	17	6	-	-
d	-	-	-	0	-
ad	27	-	-	13	-
bd	-	29	-	11	-
abd	11	27	-	2	-
cd	-	-	21	19	-
acd	20	-	13	7	-
bcd	-	15	15	10	-
abcd	15	14	8	3	-
e	-	-	-	-	0
ae	21	-	-	-	19
be	-	19	-	-	21
abe	12	13	-	-	15
ce	-	-	27	-	13
ace	15	-	10	-	15
bce	-	16	10	-	14
abce	7	14	12	-	7
de	-	-	-	5	36
ade	16	-	-	4	21
bde	-	21	-	5	14
abde	8	17	-	4	11
cde	-	-	15	6	19
acde	12	-	9	5	14
bcde	-	12	8	5	15
abcde	7	11	5	3	14

### Results of SMC for choice domain 3, films

#### Marginal likelihood of hybrid model as a function of $\lambda$

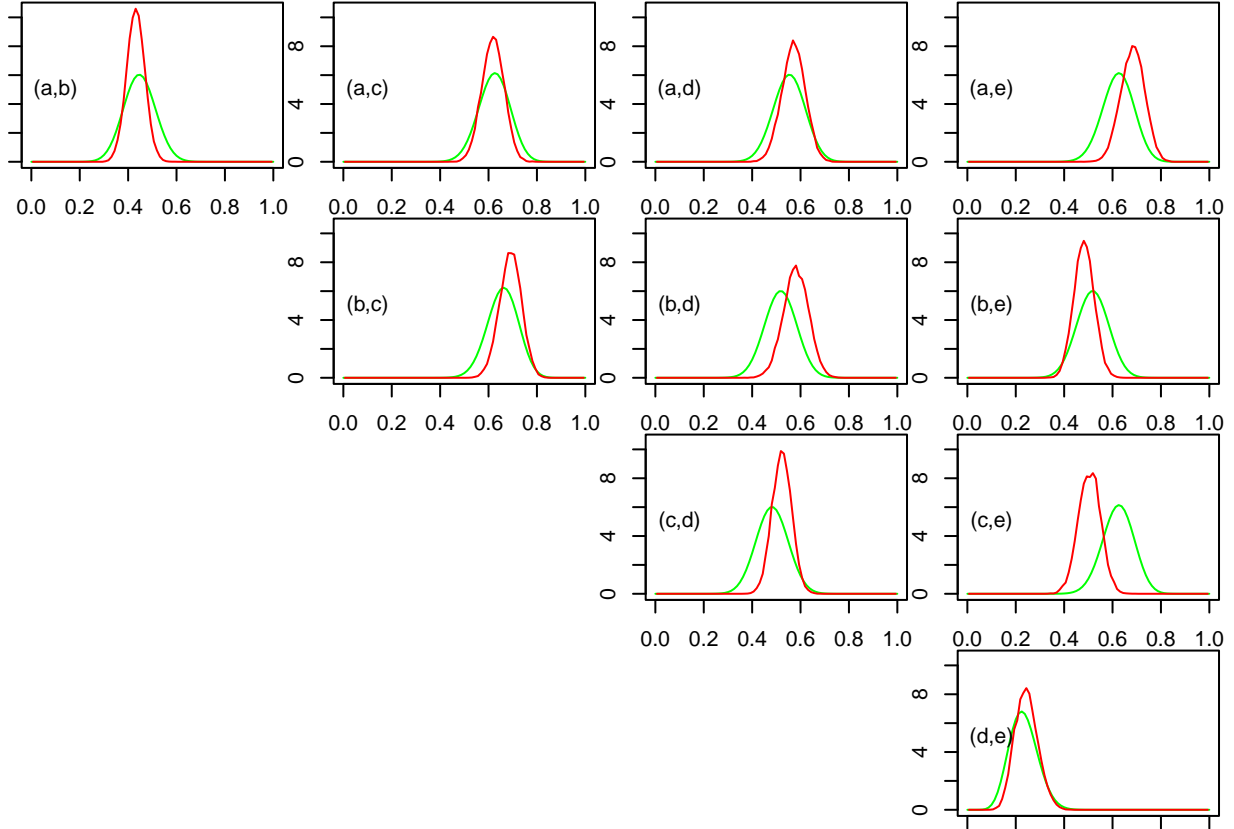
The next two figures show numerical estimates of the log marginal likelihood  $\log f_\lambda(y)$  of the hybrid model indexed by  $\lambda$ , as a function of  $\lambda$ .  $\lambda = 0$  gives the Dirichlet RC model and  $\lambda = 1$ , the Dirichlet RP model. The first figure shows a numerical estimate based on all  $J$  groups of SMC particles (black) as well as the same numerical estimate plus and minus the numerical standard error of the estimate (both in green). If there appears to be a single green curve, this indicates that the error is imperceptible at the resolution of the graph.

The second figure shows  $J$  estimates in grey, one for each group  $j = 1, \dots, J$  of particles, to illustrate the numerical variation across groups. A red line indicates the numerical estimate using all  $J$  groups.



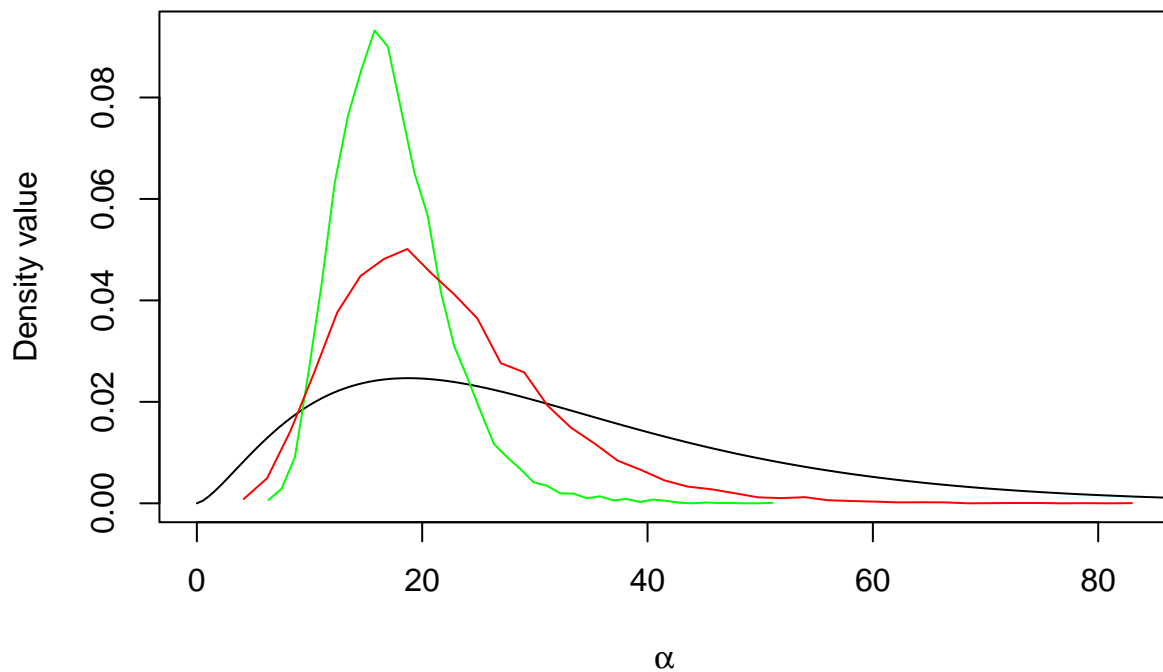
### Posterior densities of binary choice probabilities

The following figure shows posterior densities of binary choice probabilities, for the ten doubleton menus in the domain. Each panel corresponds to a doubleton menu: the ordered pair lists the choice options in the menu and the plot shows posterior densities for the probability of choosing the first option of the pair. For example, the top left panel, labelled  $(a, b)$ , gives posterior densities of the probability  $P_{\{a,b\}}(a)$ . In each panel, the green curve is the density for the Dirichlet RC model ( $\lambda = 0$ ) and the red curve is the density for the Dirichlet RP model ( $\lambda = 1$ ).

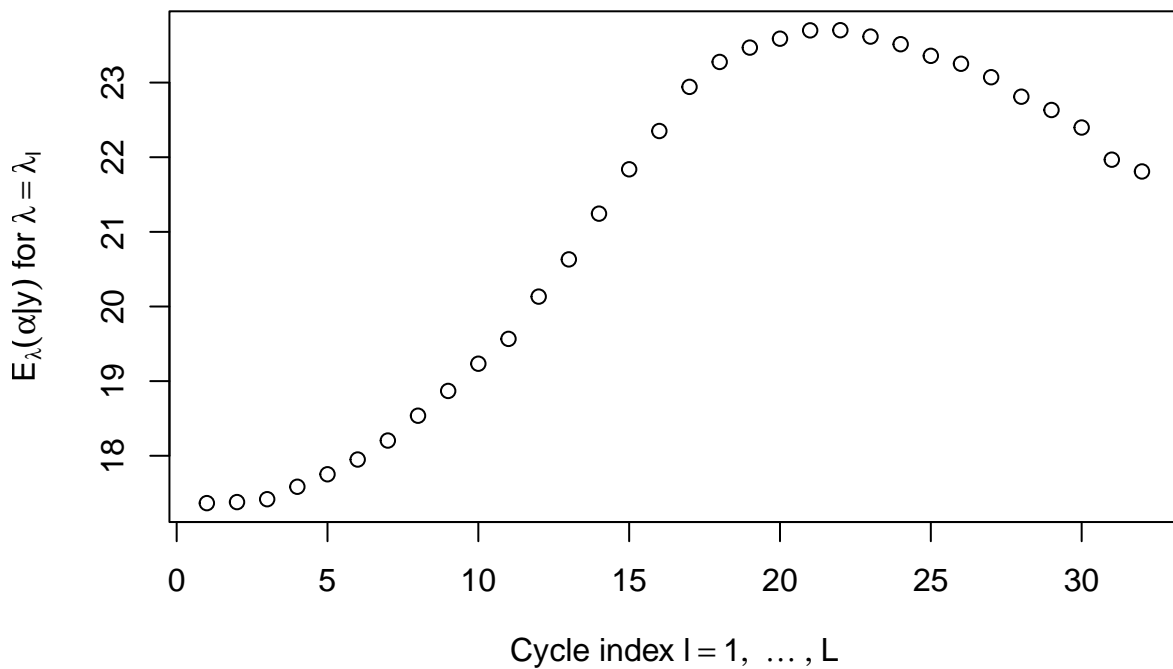


### Prior and posterior densities of $\alpha$

The following figure shows the prior density of  $\alpha$  in black. It shows the posterior density for the Dirichlet RC model ( $\lambda = 0$ ) in green and the posterior density for the Dirichlet RP model ( $\lambda = 1$ ) in red.

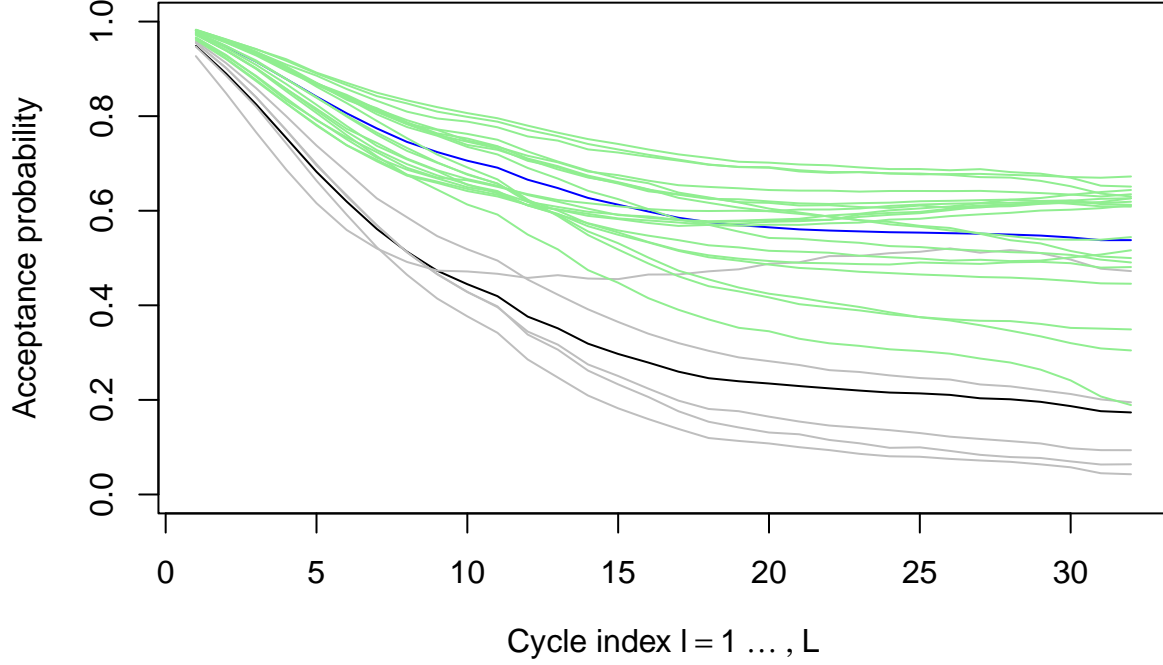


The following figure shows the mean of  $\alpha$  for the hybrid model with  $\lambda = \lambda_l$ , based on the posterior sample available at the end of the M phase,  $l = 1, \dots, L$ .

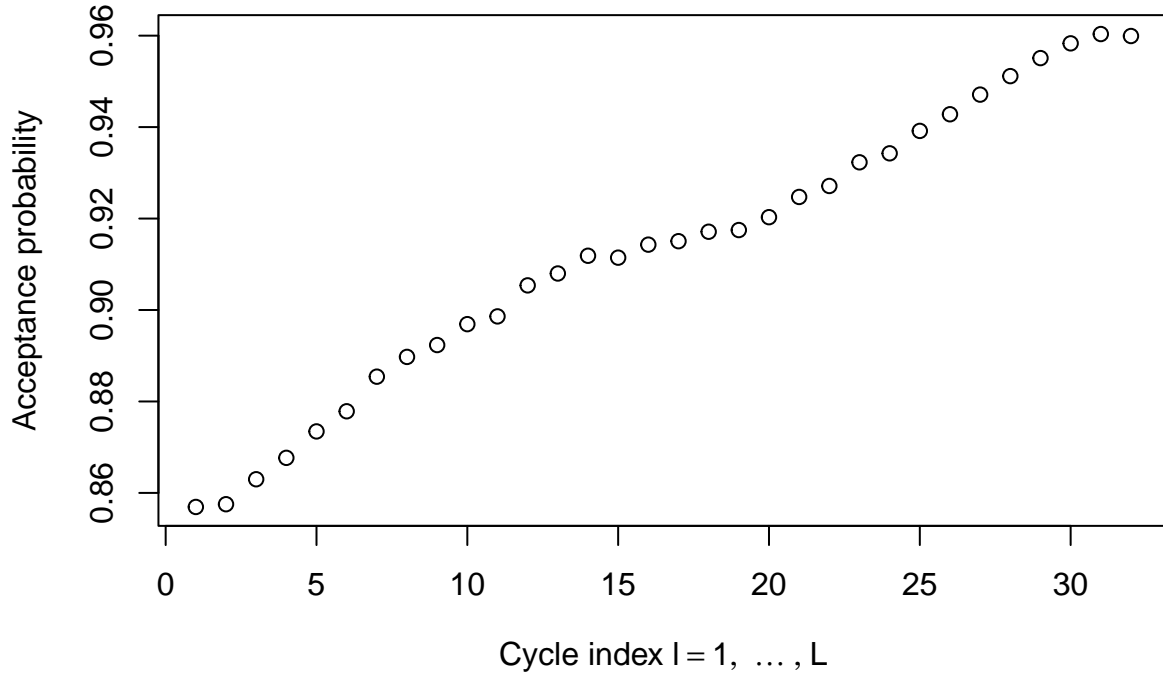


### Metropolis-Hastings acceptance probabilities

The following figure shows acceptance probabilities for Metropolis-Hastings updates of  $\gamma$  blocks, at the  $M$  phase of cycle  $l$ ,  $l = 1, \dots, L$ . The black line shows the average acceptance probability over the  $n = 5$  blocks of size  $(n - 1)! = 24$ ; the grey lines show the acceptance probability for each of these blocks. The blue line shows the average acceptance probability over the  $n(n - 1) = 20$  blocks of size  $(n - 2)! = 6$ ; the light green lines show the acceptance probability for each of these blocks.

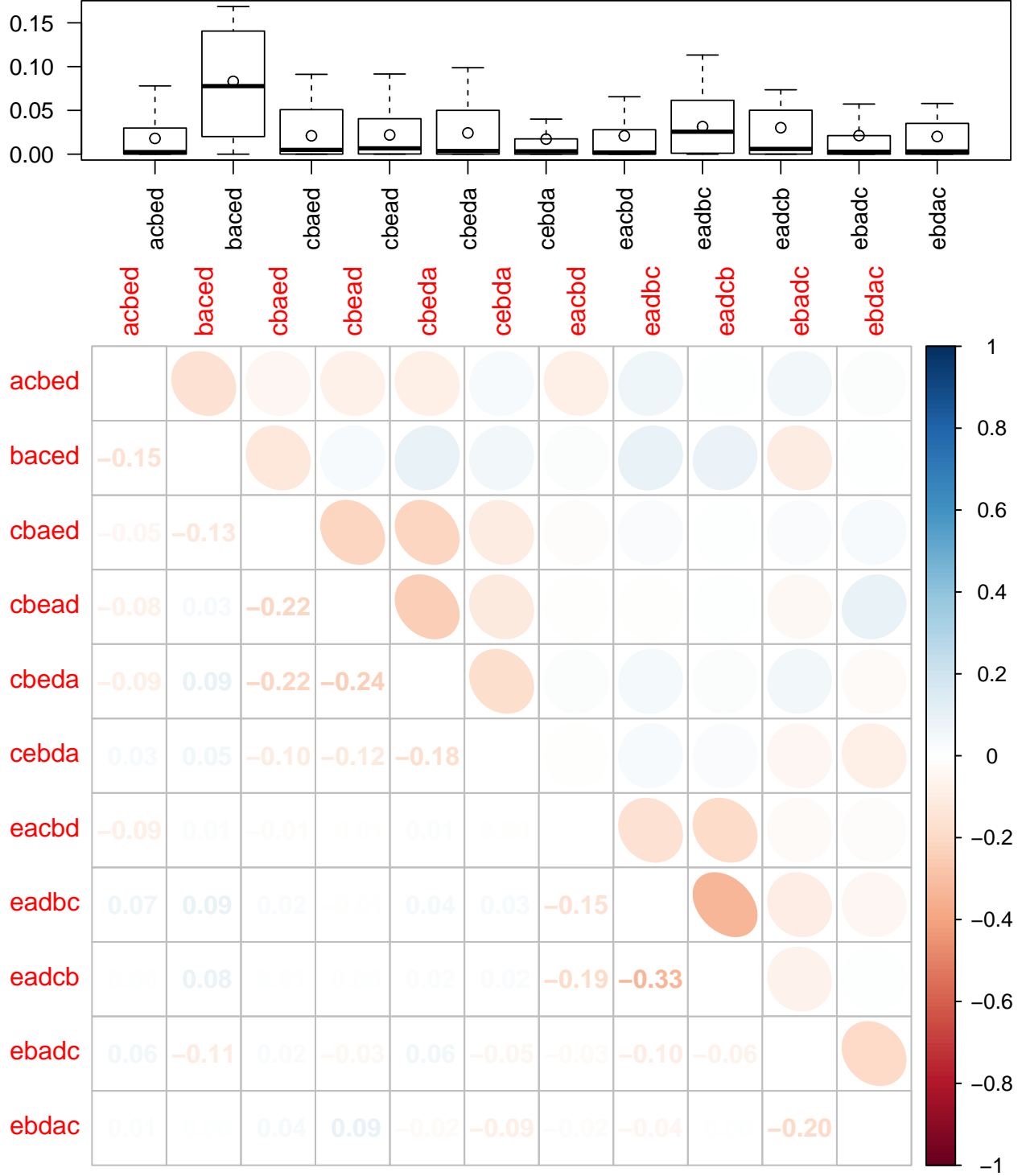


The following figure shows the acceptance probability for the Metropolis-Hastings update of  $\alpha$ , at the  $M$  phase of cycle  $l$ ,  $l = 1, \dots, L$ .



### Posterior statistics for $\pi$

The following two plots illustrate features of the posterior distribution  $\pi|y$  for the Dirichlet RP model. Of the  $n! = 120$  elements of  $\pi$ , 11 have a posterior mean greater than  $2/n!$ , for a total probability of 0.3094959. For these preference probabilities, the first plot shows the posterior mean (circle) and posterior quantiles, for probabilities 0.25, 0.5, 0.75 and 0.9. The second plot shows posterior correlations among them, indicated numerically (lower triangle), by ellipse eccentricity (upper triangle) and by colour (both triangles).



Box plot showing the distribution of the number of nodes in the minimum spanning tree for 24 different permutations of the nodes a, b, c, d. The y-axis represents the number of nodes, ranging from 0.00 to 0.15. The x-axis lists the permutations. The 'baced' permutation shows the highest median value, around 0.08, while most other permutations have medians around 0.02. The 'baced' permutation also has the largest interquartile range.

