Examen final Q4.

$$M(t) = \sum_{t=1}^{\infty} (1-p)^{k-1} p e^{tk} = e^{t} \sum_{k=1}^{\infty} (1-p)^{k-1} p e^{t(k-1)} = pe^{t} \sum_{k=0}^{\infty} \left((1-p)e^{t}\right)^{k}$$

$$= \frac{pe^{t}}{1-(1-p)e^{t}} = \frac{p}{e^{-t}-(1-p)} = \frac{p}{y-(1-p)} \quad \text{où } y = e^{-t}$$

$\left|(1-p)e^{t}\right| < 1$
pour $t>0$ suffisament
près de 0.

$$\frac{dM(t)}{dt} = \frac{\partial M}{\partial y}\frac{dy}{dt} \qquad \frac{d^2M(t)}{dt^2} = \frac{\partial^2 M}{\partial y^2}\left(\frac{dy}{dt}\right)^2 + \frac{\partial M}{\partial y}\frac{d^2y}{dt^2}$$

$$\frac{\partial M}{\partial y} = \frac{-p}{(y-(1-p))^2} \qquad \left.\frac{\partial M}{\partial y}\right|_{y=1} = \frac{-1}{p} \qquad (\text{quand } t=0,\ y=1)$$

$$\frac{\partial^2 M}{\partial y^2} = \frac{2p}{(y-(1-p))^3} \qquad \left.\frac{\partial^2 M}{\partial y^2}\right|_{y=1} = \frac{2}{p^2}$$

$$\frac{dy}{dt} = -e^{-t} \qquad \left.\frac{dy}{dt}\right|_{t=0} = -1$$

$$\frac{d^2y}{dt^2} = e^{-t} \qquad \left.\frac{d^2y}{dt^2}\right|_{t=0} = 1$$

$$\left.\frac{dM(t)}{dt}\right|_{t=0} = \frac{-1}{p}\cdot -1 = \frac{1}{p} = E[X]$$

$$\left.\frac{d^2M(t)}{dt^2}\right|_{t=0} = \frac{2}{p^2}\cdot 1 + \frac{-1}{p}\cdot 1 = \frac{2-p}{p^2} = E[X^2]$$

La moyenne est $\frac{1}{p}$, la variance est $\frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$