scratch work

The probability distributions for polar coordinates r and θ are

$$P(r)dr \propto rdr \tag{1}$$

$$P(\theta)d\theta \propto d\theta \tag{2}$$

Their respective cumulative probability distributions are

$$\xi = \frac{\int_0^r r'dr'}{\int_0^R rdr} = \frac{r^2}{R^2} \tag{3}$$

$$\zeta = \frac{\int_0^\theta d\theta'}{\int_0^{2\pi} d\theta'} = \frac{\theta}{2\pi} \tag{4}$$

for two uniform-random numbers between 0 and 1, ξ and ζ. Solving for r and θ,

$$r = R\sqrt{\xi} \tag{5}$$

$$\theta = 2\pi\zeta \tag{6}$$