$$\frac{V_c^2}{r} = \frac{GM(r)}{r^2} \rightarrow \boxed{M(r) = \frac{rV_c^2}{G}}$$

$$M(1\ \text{kpc}) = 5.978 \times 10^9\ M_\odot$$
$$M(10\ \text{kpc}) = 1.188 \times 10^{11}\ M_\odot$$
$$M(20\ \text{kpc}) = 2.373 \times 10^{11}\ M_\odot$$

$$\omega = \frac{V_c}{r}\ ; \quad T = \frac{2\pi}{\omega} \rightarrow \boxed{T = \frac{2\pi r}{V_c}}$$

$$T(1\ \text{kpc}) = 3.832 \times 10^7\ \text{yr}$$
$$T(10\ \text{kpc}) = 2.718 \times 10^8\ \text{yr}$$
$$T(20\ \text{kpc}) = 5.439 \times 10^8\ \text{yr}$$

Exponential disk density model: $\rho = \rho_0 e^{-r/R_h}$
where R_h is the exponential scale length (e-folding) of 3 kpc.

→ Integrate to find mass as a function of radius (enclosed)

$$M(r) = \int_0^r \int_0^{2\pi} \rho_0 e^{-r'/R_h} r' dr' d\theta$$

$$= 2\pi\rho_0 \int_0^r r' e^{-r'/R_h} dr'$$

→ Integrate by parts

$$\begin{array}{ccc} r' & \xrightarrow{+} & e^{-r'/R_h} \\ 1 & \xrightarrow{-} & -R_h e^{-r'/R_h} \end{array}$$

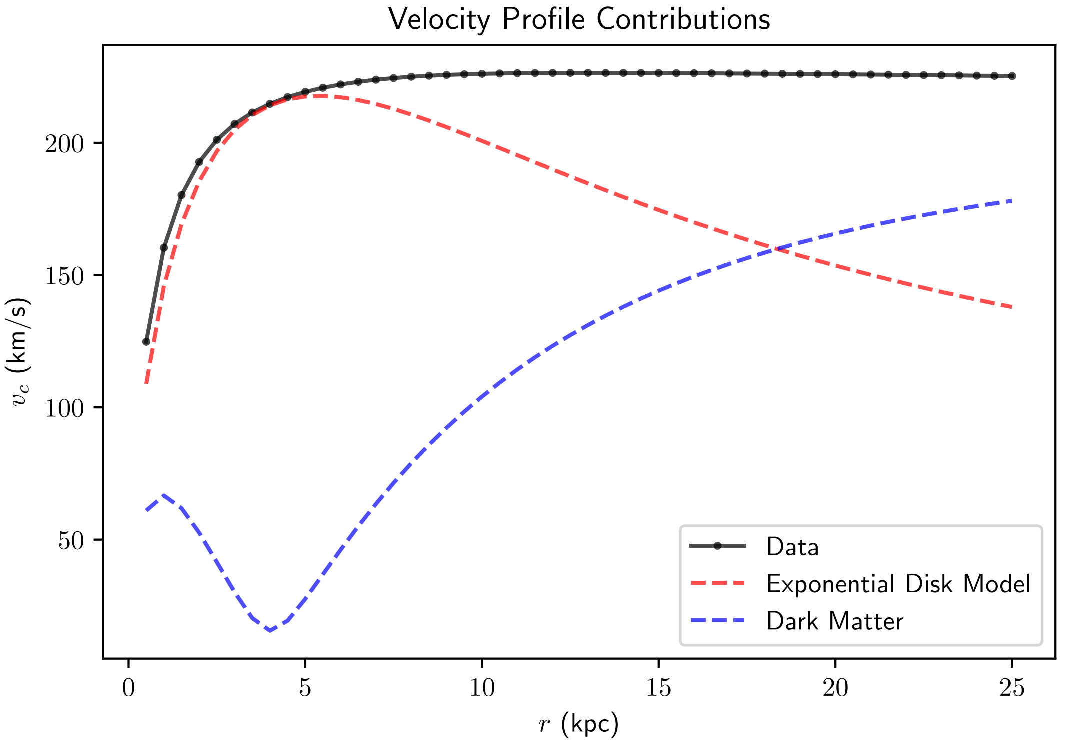
$$= 2\pi\rho_0 \left(-r' R_h e^{-r'/R_h} \Big|_0^r + \int_0^r R_h e^{-r'/R_h} dr' \right)$$

$$= 2\pi\rho_0 \left(-rR_h e^{-r/R_h} - R_h^2 e^{-r'/R_h} \Big|_0^r \right)$$

$$\boxed{M(r) = 2\pi\rho_0 \left(-rR_h e^{-r/R_h} - R_h^2 e^{-r/R_h} + R_h^2 \right)} \qquad \boxed{V_c = \sqrt{\frac{GM(r)}{r}}}$$

EXPONENTIAL DISK MODEL

$$M_{DM}(r) = M_{DATA}(r) - M_{MODEL}(r) \rightarrow V_{c,\,DM} = \sqrt{\frac{GM_{DM}(r)}{r}}$$



Velocity Profile Contributions
v_c (km/s)
200
150
100
50
0
5
10
15
20
25
r (kpc)
Data
Exponential Disk Model
Dark Matter

```python
import numpy as np
import astropy.units as u
import astropy.constants as c
import matplotlib.pyplot as plt
from matplotlib import rc
rc('text', usetex=True)

# Load in the velocity and radius data
a = np.loadtxt('hw1_data.dat', skiprows=1)
radius = a[:, 0] * u.Unit('kpc')
vcirc = a[:, 1] * u.Unit('km/s')


### QUESTION 1 ###
# Find interior mass to a certain radius
M_int = radius * vcirc ** 2. / c.G.cgs
M_int = M_int.to('g')

# Print the answers to the first question
     ("Mass inside 1 kpc: {}".format(M_int[radius == 1.*u.kpc]/c.M_sun.cgs))
     ("Mass inside 10 kpc: {}".format(M_int[radius == 10.*u.kpc]/c.M_sun.cgs))
     ("Mass inside 20 kpc: {}".format(M_int[radius == 20.*u.kpc]/c.M_sun.cgs))


### QUESTION 2 ###
# Calculate the orbital period as a function of circular velocity and distance
T = 2. * np.pi * radius / vcirc
T = T.to('year')
     ("Orbital period at 1 kpc: {}".format(T[radius == 1.*u.kpc]))
     ("Orbital period at 10 kpc: {}".format(T[radius == 10.*u.kpc]))
     ("Orbital period at 20 kpc: {}".format(T[radius == 20.*u.kpc]))


### QUESTION 3 ###
Rh = 3 * u.Unit('kpc')  # Exponential Scale Length
# Density at the center --- ish. Fitting coefficient added
rho_0 = (M_int[0] / (np.pi * radius[0]**2.)).to(u.Unit('g/cm2')) * 0.85
# Exponential disk model for mass density at a certain radius
rho = rho_0 * np.exp(-radius / Rh)
# Enclosed mass
M_int_exp = (((-Rh * radius - Rh**2.) * np.exp(-radius / Rh) +
              Rh**2.) * 2. * np.pi * rho_0).to('g')
# Circular velocity from exponential model
vcirc_exp = (np.sqrt(c.G.cgs * M_int_exp / radius)).to('km/s')
# Circular velocity of difference between data and exponential model
M_diff = M_int - M_int_exp
vcirc_diff = (np.sqrt(c.G.cgs * M_diff / radius)).to('km/s')


# Make plots
plt.plot(radius, vcirc, 'k-', marker='o', ms=2, label='Data', alpha=0.7)
plt.plot(radius, vcirc_exp, 'r--', label='Exponential Disk Model', alpha=0.7)
plt.plot(radius, vcirc_diff, 'b--', label='Dark Matter', alpha=0.7)
plt.legend()
plt.title('Velocity Profile Contributions')
plt.xlabel('$r$ (kpc)')
plt.ylabel('$v_{c}$ (km/s)')
plt.show()
```