HW6 ISM CONNOR MCCLELLAN

1. a. $$\phi_\nu(\nu) = \frac{1}{\sqrt{\pi}}\frac{1}{\nu_0}\frac{c}{b}e^{-\nu^2/b^2}$$

from Draine. $\lambda_0 = c/\nu_0$

and $\frac{v}{c} = \frac{\lambda-\lambda_0}{\lambda_0}$ from the Doppler formula

so $$v = c\frac{\lambda-\lambda_0}{\lambda_0} = cx.$$

$$\therefore \quad \phi(x) = \frac{\lambda_0}{\sqrt{\pi}\,b}e^{-(cx/b)^2}$$

neglect stim. emission

$$\tau_\nu = \frac{\pi e^2}{m_e c} f_{lu} N_l \phi_\nu \left[1 - \frac{N_u/g_u}{N_l/g_l}\right]$$

$$\therefore \quad \tau(x) = \frac{\pi e^2}{m_e c} f_{lu} N_l \left(\frac{\lambda_0}{\sqrt{\pi}\,b}e^{-c^2x^2/b^2}\right)$$

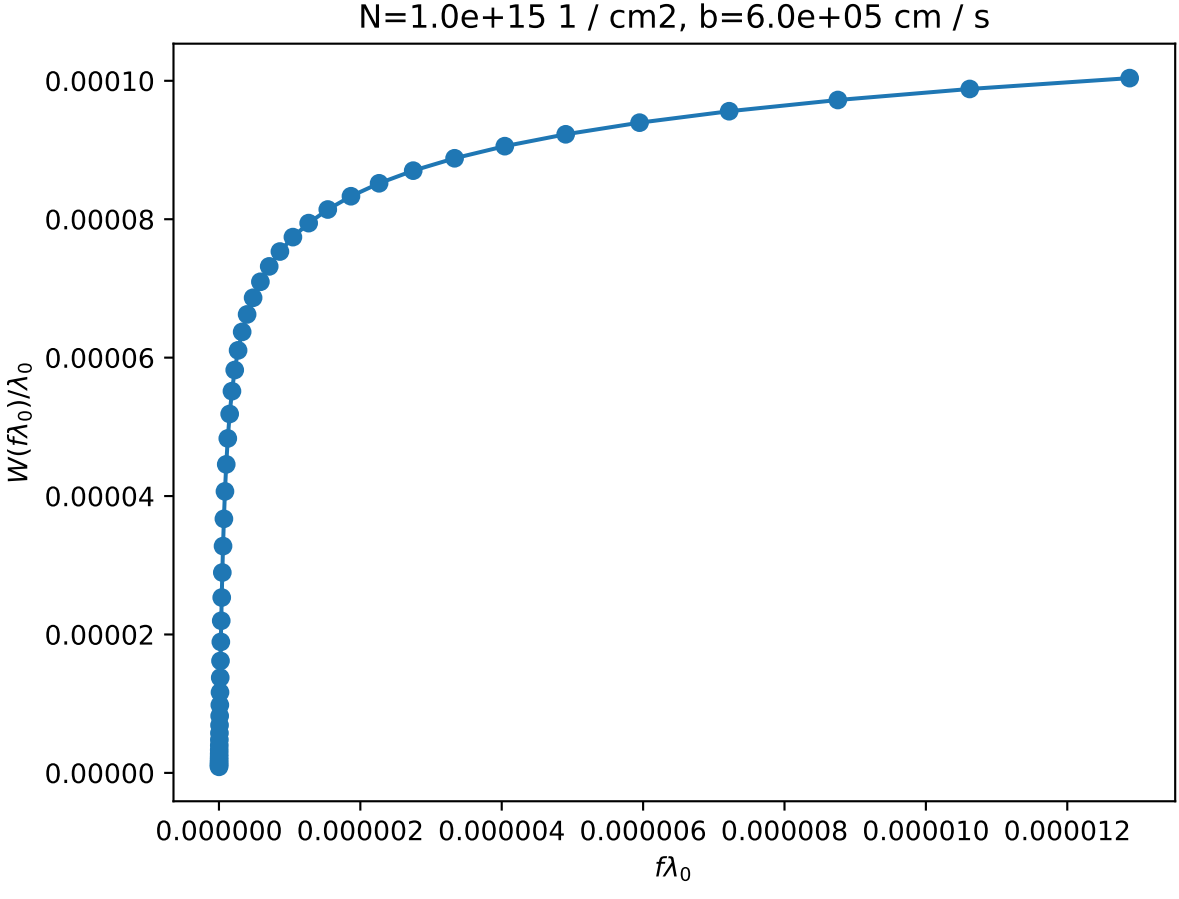
$$\boxed{\tau(x) = \frac{\sqrt{\pi}e^2\lambda_0}{m_e c\, b} f_{lu} N_l e^{-c^2x^2/b^2}}$$

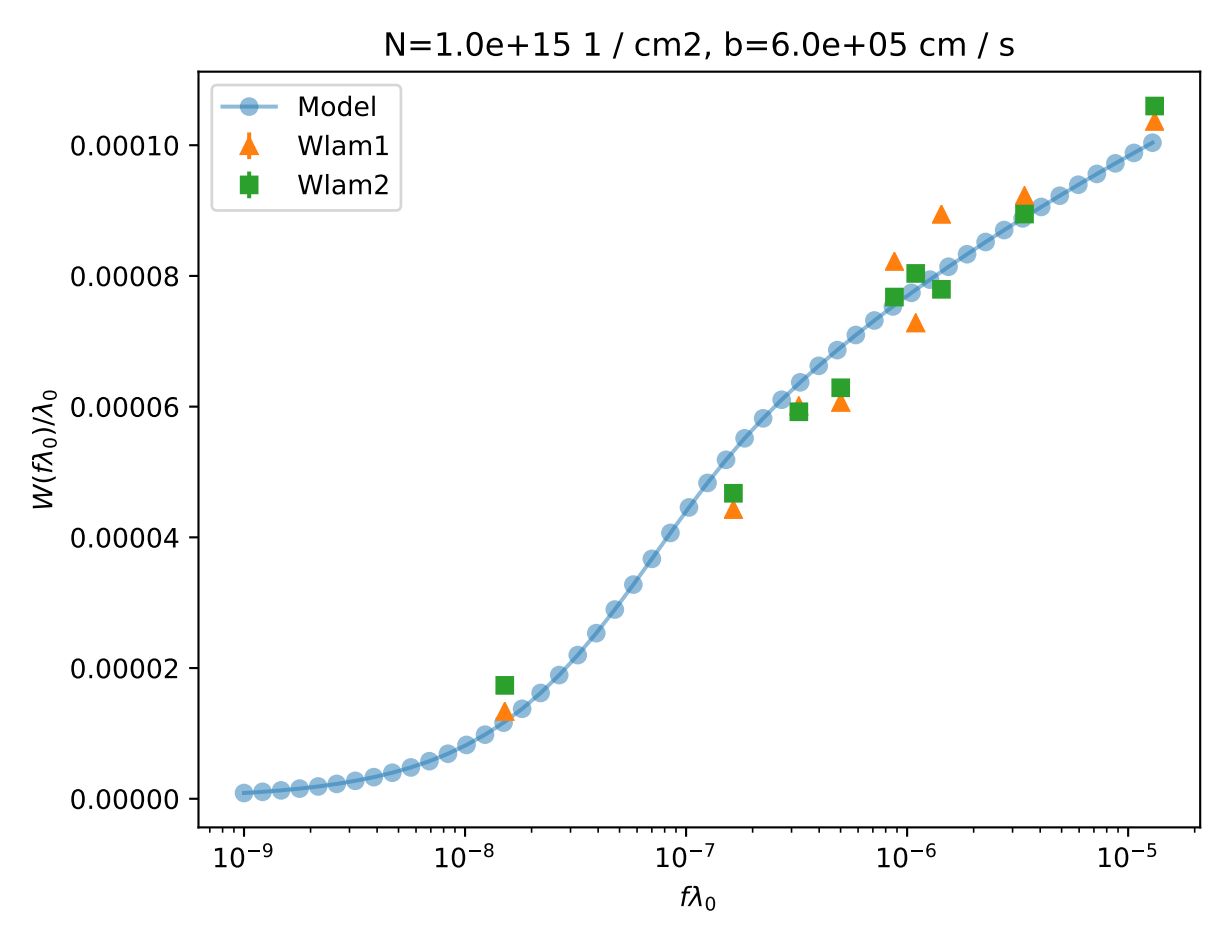
b. $$W = \int_0^\infty \left(1 - e^{N\kappa_0\phi_\nu}\right)\frac{d\nu}{\nu_0}$$

$$x = \nu_0\left(\frac{1}{\nu} - \frac{1}{\nu_0}\right) = \frac{\nu_0}{\nu} - 1 \quad \Rightarrow \quad dx = -\frac{\nu_0}{\nu^2}d\nu$$

$$\nu = \nu_0/(x+1) \qquad \therefore \quad d\nu = -\frac{\nu^2 dx}{\nu_0} = \frac{-\nu_0\,dx}{(x+1)^2}$$

$$\boxed{\frac{W_\lambda}{\lambda_0} = \int_{-1}^{\infty} \frac{1 - e^{-\tau(x)}}{(x+1)^2}dx}$$


N=1.0e+15 1 / cm2, b=6.0e+05 cm / s
0.00010
0.00008
0.00006
0.00004
0.00002
0.00000
$W(f\lambda_0)/\lambda_0$
0.000000
0.000002
0.000004
0.000006
0.000008
0.000010
0.000012
$f\lambda_0$


N=1.0e+15 1 / cm2, b=6.0e+05 cm / s
Model
Wlam1
Wlam2
0.00010
0.00008
0.00006
0.00004
0.00002
0.00000
$W(f\lambda_0)/\lambda_0$
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
$f\lambda_0$

```
import astropy.constants as c
import astropy.units as u
import numpy as np
import matplotlib.pyplot as plt
import pdb

N = 10**15 * u.cm**-2
b = (6 * u.km/u.s).cgs
x = np.linspace(-5, 5) * (b/c.c.cgs).value
flam = np.logspace(-9, -4.89)

    tau(x, flam):
    prefac = np.sqrt(np.pi)*c.e.esu**2/c.m_e.cgs/c.c.cgs
           (prefac * flam * N / b * np.exp(-(c.c.cgs*x/b)**2)).value

    W(x, flam):
    dx = np.diff(x)[0]
    sol_array = np.zeros(np.shape(flam))
        i, f    enumerate(flam):
        sol_array[i] = np.sum((1 - np.exp(-tau(x, f)))/(x+1)**2*dx)
           sol_array

plt.plot(flam, W(x, flam), marker='o')
plt.xlabel('$f\lambda_0$')
plt.ylabel('$W(f\lambda_0) / \lambda_0$')
plt.title('N={:.1e}, b={:.1e}'.format(N, b))
plt.tight_layout()
plt.show()


data = np.loadtxt('cog.data.txt', skiprows=1)
lam, f, wlam1, wlam1_err, wlam2, wlam2_err = data.T * u.angstrom.to(u.cm)
f = f/u.angstrom.to(u.cm)

plt.plot(flam, W(x, flam), marker='o', label='Model', alpha=0.5)
plt.errorbar(f*lam, wlam1/lam, yerr=wlam1_err, marker='^', linestyle='None',
label='Wlam1')
plt.errorbar(f*lam, wlam2/lam, yerr=wlam2_err, marker='s', linestyle='None',
label='Wlam2')
plt.xlabel('$f\lambda_0$')
plt.ylabel('$W(f\lambda_0) / \lambda_0$')
plt.title('N={:.1e}, b={:.1e}'.format(N, b))
plt.xscale('log')
plt.legend()
plt.tight_layout()
plt.show()
```