$$\frac{V_c^2}{r} = \frac{GM(r)}{r^2} \rightarrow \boxed{M(r) = \frac{rV_c^2}{G}}$$

$$M(1\text{ kpc}) = 5.978 \times 10^9\ M_\odot$$
$$M(10\text{ kpc}) = 1.188 \times 10^{11}\ M_\odot$$
$$M(20\text{ kpc}) = 2.373 \times 10^{11}\ M_\odot$$

$$\omega = \frac{V_c}{r}\ ;\ T = \frac{2\pi}{\omega} \rightarrow \boxed{T = \frac{2\pi r}{V_c}}$$

$$T(1\text{ kpc}) = 3.832 \times 10^7\ \text{yr}$$
$$T(10\text{ kpc}) = 2.718 \times 10^8\ \text{yr}$$
$$T(20\text{ kpc}) = 5.439 \times 10^8\ \text{yr}$$

Exponential disk density model: $\rho = \rho_0 e^{-r/R_h}$
where R_h is the exponential scale length (e-folding) of 3 kpc.

→ Integrate to find mass as a function of radius (enclosed)

$$M(r) = \int_0^r \int_0^{2\pi} \rho_0 e^{-r'/R_h} r'dr'd\theta$$

$$= 2\pi\rho_0 \int_0^r r'e^{-r'/R_h} dr'$$

$$\begin{array}{ccc} r' & \overset{+}{\searrow} & e^{-r'/R_h} \\ 1 & \overset{-}{\longrightarrow} & -R_h e^{-r'/R_h} \end{array}$$

→ Integrate by parts

$$= 2\pi\rho_0 \left(-r'R_h e^{-r'/R_h} \Big|_0^r + \int_0^r R_h e^{-r'/R_h} dr' \right)$$

$$= 2\pi\rho_0 \left(-rR_h e^{-r/R_h} - R_h^2 e^{-r'/R_h} \Big|_0^r \right)$$

$$\boxed{M(r) = 2\pi\rho_0 \left(-rR_h e^{-r/R_h} - R_h^2 e^{-r/R_h} + R_h^2 \right)}$$

EXPONENTIAL DISK MODEL

$$\boxed{V_c = \sqrt{\frac{GM(r)}{r}}}$$

$$M_{DM}(r) = M_{DATA}(r) - M_{MODEL}(r) \rightarrow V_{c,DM} = \sqrt{\frac{GM_{DM}(r)}{r}}$$