HW6 ISM CONNOR MCCLELLAN

1. a. $\phi_\nu(\nu) = \frac{1}{\sqrt{\pi}} \frac{1}{\nu_0} \frac{c}{b} e^{-v^2/b^2}$

from Draine, $\lambda_0 = c/\nu_0$

and $\frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0}$ from the Doppler formula

so $v = c\,\frac{\lambda - \lambda_0}{\lambda_0} = cx.$

$$\therefore \quad \phi(x) = \frac{\lambda_0}{\sqrt{\pi}\, b} e^{-(cx/b)^2}$$

neglect stim. emission

$$\tau_\nu = \frac{\pi e^2}{m_e c} f_{\ell u} N_\ell \phi_\nu \left[1 - \cancel{\frac{N_u/g_u}{N_\ell/g_\ell}}\right]$$

$$\therefore \quad \tau(x) = \frac{\pi e^2}{m_e c} f_{\ell u} N_\ell \left(\frac{\lambda_0}{\sqrt{\pi}\, b} e^{-c^2x^2/b^2}\right)$$

$$\boxed{\tau(x) = \frac{\sqrt{\pi}\, e^2 \lambda_0}{m_e c\, b} f_{\ell u} N_\ell\, e^{-c^2x^2/b^2}}$$

b. $W = \int_0^\infty \left(1 - e^{N\kappa_0 \phi_\nu}\right) \frac{d\nu}{\nu_0}$

$x = \nu_0\left(\frac{1}{\nu} - \frac{1}{\nu_0}\right) = \frac{\nu_0}{\nu} - 1 \Rightarrow dx = -\frac{\nu_0}{\nu^2} d\nu$

$\nu = \nu_0/(x+1) \quad \therefore \quad d\nu = -\frac{\nu^2 dx}{\nu_0} = \frac{-\nu_0\, dx}{(x+1)^2}$

$$\boxed{\frac{W_\lambda}{\lambda_0} = \int_{-1}^{\infty} \frac{1 - e^{-\tau(x)}}{(x+1)^2}\, dx\,.}$$