$$\frac{dP}{dr} = -\rho g(r) \longrightarrow g(r) = \frac{GM}{r^2} \; ; \; P = \frac{\rho k_b T}{\mu m_p}$$

$$\frac{kT}{\mu m_p} \frac{d\rho}{dr} = -\rho \frac{GM}{r^2}$$

$$\frac{d\rho}{dr} \frac{1}{\rho} = -\left(\frac{GM\mu m_p}{kT}\right) \frac{1}{r^2}$$

$$\frac{d}{dr} \ln \rho = -\left(\frac{GM\mu m_p}{kT}\right) \frac{1}{r^2}$$

$$\int_{r_{in}}^{r} \frac{d}{dr'} \ln \rho \, dr' = -\frac{GM\mu m_p}{kT} \int_{r_{in}}^{r} \frac{1}{r'^2} dr'$$

$$\ln \rho(r) - \ln \rho(r = r_{in}) = \frac{GM\mu m_p}{kT} \left(\frac{1}{r} - \frac{1}{r_{in}}\right)$$

$$\ln \left(\frac{\rho(r)}{\rho(r = r_{in})}\right) = \frac{GM\mu m_p}{kT} \left(\frac{1}{r} - \frac{1}{r_{in}}\right)$$

$$\boxed{\rho(r) = \rho(r = r_{in}) \, e^{\frac{GM\mu m_p}{kT}\left(\frac{1}{r} - \frac{1}{r_{in}}\right)}}$$