1

Explicit vs implicit

$$\frac{\partial n_H}{\partial t} = +\Gamma n_H - \alpha n_p^2$$

Present time : n_H, n_p
Future time : n_H^+, $n_p^+ = (n - n_H^+)^2$
n = constant = ρ / m_p as hydro constant

Explicit

$$\frac{n_H^+ - n_H}{\Delta t} = +\Gamma n_H - \alpha n_p^2$$

$$n_H^+ = n_H + \Delta t \left(\Gamma n_H - \alpha n_p^2 \right)$$

Pro : easy
Con : if Δt too big, could way overshoot equilibrium

Implicit

For fully implicit, use n_H^+ & n_p^+ in RHS.

$$\frac{n_H^+ - n_H}{\Delta t} = \Gamma n_H^+ - \alpha {n_p^+}^2$$

$$= \Gamma n_H^+ - \alpha (n - n_H^+)^2$$

Quadratic for n_H^+.

(2)

$$\alpha\left(n_H^{+2} - 2nn_H^+ + n^2\right) - \Gamma n_H^+ + \frac{n_H^+ - n_H}{\Delta t} = 0$$

$$n_c \equiv \Gamma/\alpha \quad . \quad n_r \equiv \frac{1}{\alpha\Delta t}$$

$$n_H^{+2} - n_H^+\left(2n + n_c - n_r\right) + \left(n^2 - n_r n_H\right) = 0$$

$$n_H^+ = +\frac{1}{2}\left(2n + n_c - n_r\right)$$

$$\pm\frac{1}{2}\left[\left(2n + n_c - n_r\right)^2 - 4\left(n^2 - n_r n_H\right)\right]^{\frac{1}{2}}$$

$$= n + \frac{n_c - n_r}{2} \pm \left[\frac{1}{4}\left\{\cancel{4n^2} + 4n(n_c - n_r) + (n_c - n_r)^2 - \cancel{4n^2} + 4n_r n_H\right\}\right]^{\frac{1}{2}}$$

$$= n + \frac{n_c - n_r}{2} \pm \left[n(n_c - n_r) + \frac{(n_c - n_r)^2}{4} + n_r n_H\right]^{\frac{1}{2}}$$

$\Delta t \to \infty \Rightarrow$ equilibrium, $n_r \to 0$ gives

$$n_H = n + \frac{n_c}{2} - \left[n n_c + \frac{n_c^2}{4}\right]^{\frac{1}{2}}$$

Large n: $n_H \simeq n$

Small n: $n_H \simeq \cancel{n} + \cancel{\frac{n_c}{2}} - \frac{n_c}{2}\left(\cancel{1} + \cancel{2}\frac{n}{n_c} + \frac{1}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{4n}{n_c}\right)^2\right)$

$$\simeq \frac{n^2}{n_c}$$

(3)

small time step: $n_H^+ \simeq n_H + O(\Delta t)$.

$n_r \rightarrow \infty$

$${n_H^+}^2 - n_H^+ (2n + n_c - n_r) + (n^2 - n_r n_H) = 0$$

$$n_H^+ \simeq n_H$$