$$D_{\{E_m\}}(\rho_1,\rho_2) = \frac{1}{2}\sum_m \left|p_m^{(1)} - p_m^{(2)}\right| = \frac{1}{2}\sum_m \left|\mathrm{tr}(E_m\rho_1) - \mathrm{tr}(E_m\rho_2)\right| \qquad \|\theta\|_1 = \mathrm{tr}\left(\sqrt{\theta^\dagger\theta}\right) = \mathrm{tr}(|\theta|)$$

$$= \frac{1}{2}\sum_m \left|\mathrm{tr}\left(E_m(\rho_1-\rho_2)\right)\right| \overset{\text{POLAR DEC.}}{\leq} \frac{1}{2}\sum_m \mathrm{tr}\left(E_m|\rho_1-\rho_2|\right) = \frac{1}{2}\mathrm{tr}\left(|\rho_1-\rho_2|\right) = D(\rho_1-\rho_2)$$

- Da dove viene la prima disuguaglianza?

$$\left|\mathrm{tr}\left(E_m(\rho_1-\rho_2)\right)\right| \overset{?}{\leq} \mathrm{tr}\left(E_m|\rho_1-\rho_2|\right)$$

$$= \left|\mathrm{tr}\left(E_m(A-B)\right)\right| \leq \left|\mathrm{tr}\left(E_m \overset{\geq 0}{(A+B)}\right)\right| = \mathrm{tr}\left(E_m \overbrace{(A+B)}^{|\rho_1-\rho_2|}\right) = \mathrm{tr}\left(E_m|\rho_1-\rho_2|\right)$$

La POVM che satura la disuguaglianza è $E_1 = P_A \qquad E_2 = P_B$