

Exercise 7.1

Show

$$\frac{\partial U'}{\partial t} + \frac{\partial F'}{\partial z} + \frac{\partial G'}{\partial \eta} = 0 \text{ is equiv to } \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0$$

$$\frac{\partial}{\partial t} (d_{xy} U) + \frac{\partial}{\partial z} (F \frac{\partial y}{\partial \eta} - G \frac{\partial x}{\partial \eta}) + \frac{\partial}{\partial \eta} (-F \frac{\partial y}{\partial z} + G \frac{\partial x}{\partial z}) = 0$$

$$d_{xy} \frac{\partial U}{\partial t} + \frac{\partial F}{\partial z} \frac{\partial y}{\partial \eta} + F \frac{\partial^2 y}{\partial \eta \partial z} - \frac{\partial G}{\partial z} \frac{\partial x}{\partial \eta} - \frac{\partial F}{\partial \eta} \frac{\partial y}{\partial z} - G \frac{\partial^2 x}{\partial \eta \partial z} - \frac{\partial F}{\partial \eta} \frac{\partial y}{\partial z} - F \frac{\partial^2 y}{\partial \eta \partial z} + \frac{\partial G}{\partial \eta} \frac{\partial x}{\partial z} + G \frac{\partial^2 x}{\partial \eta \partial z} = 0$$

$$d_{xy} \frac{\partial U}{\partial t} + \frac{\partial F}{\partial z} \frac{\partial y}{\partial \eta} - \frac{\partial G}{\partial z} \frac{\partial x}{\partial \eta} - \frac{\partial F}{\partial \eta} \frac{\partial y}{\partial z} + \frac{\partial G}{\partial \eta} \frac{\partial x}{\partial z} = 0$$

$$\text{ID's: } \frac{\partial y}{\partial \eta} = \frac{1}{d_{yz}} \frac{\partial z}{\partial x}; \frac{\partial x}{\partial \eta} = \frac{-1}{d_{yz}} \frac{\partial z}{\partial y};$$

$$\frac{\partial y}{\partial z} = \frac{-1}{d_{yz}} \frac{\partial \eta}{\partial x}; \frac{\partial x}{\partial z} = \frac{1}{d_{yz}} \frac{\partial \eta}{\partial y}$$

$$d_{xy} \frac{\partial U}{\partial t} + \frac{\partial F}{\partial z} \frac{\partial z}{\partial x} \cdot \frac{1}{d_{yz}} + \frac{\partial G}{\partial z} \frac{\partial z}{\partial y} \cdot \frac{1}{d_{yz}} + \frac{\partial F}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} \cdot \frac{1}{d_{yz}} + \frac{\partial G}{\partial \eta} \frac{\partial \eta}{\partial y} \cdot \frac{1}{d_{yz}} = 0$$

$$d_{xy} \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} \cdot \frac{1}{d_{yz}} + \frac{\partial G}{\partial y} \cdot \frac{1}{d_{yz}} = 0$$

$$\frac{\partial U}{\partial t} + \frac{1}{d_{xy} d_{yz}} \left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} \right) = 0$$

$$d_{xy} d_{yz} = 1$$

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \quad \square$$

ID's used
this from text,
From def of
transformation