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Cpr E 281 HW03

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

NAND/NOR Gates with Synthesis Minimization and Karnaugh Maps

Assigned: Week 4

Due Date: Feb. 12, 2023

P1 (12 points): Given the expression $G(W,X,Y,Z) = \sum m(0,1,3,7,8,15)$, implement this function using no more than 8 NAND gates. The NAND gates that you use may have any number of inputs. You may not use NOT gates to create this circuit.

P2 (16 points): Given the expression $H(A,B,C) = \sum m(0,1,5,7)$, perform the following:

a. Write the expression for H as a simplified SOP expression.
b. Write the expression for H as a simplified POS expression.
c. Implement H using exactly five NOR gates and no other gates.
d. Did you use the SOP expression or the POS expression to implement the circuit? Why?

P3 (9 points): Produce the simplified sum-of-products (SOP) expressions for the following K-maps:

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

WX \ YZ	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	0	0
10	0	0	0	0

WX \ YZ	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	1	0	0	0
10	1	0	0	1

P4 (21 points): For each shorthand expression below, derive the simplest SOP expression:
A: $F_1(w,x,y,z) = \sum m(4,6,10,11,14,15)$
B: $F_2(x,y,z) = \sum m(2,5,6,7)$
C: $F_3(w,x,y,z) = \sum m(0,2,3,7,8,10,11,15)$

P5 (21 points): For each expression below, derive the simplest POS expression:
A: $G_1(a,b,c,d) = \prod M(0,1,2,3,4,5,10,11)$
B: $G_2(a,b,c) = \prod M(0,2,3,6)$
C: $G_3(a,b,c,d) = \prod M(0,2,5,9,13)$

P6 (21 points): Use Karnaugh Maps to convert the following expressions to simplified SOP expressions:
I: $H_1(A,B,C,D) = \bar{C}D + B\bar{C} + BC\bar{D} + AC\bar{D} + A\bar{B}C$
II: $H_2(A,B,C,D) = (A + C + D)(B + C + \bar{D})(\bar{A} + B + \bar{C})$
III: $H_3(A,B,C,D) = \prod M(0,2)$

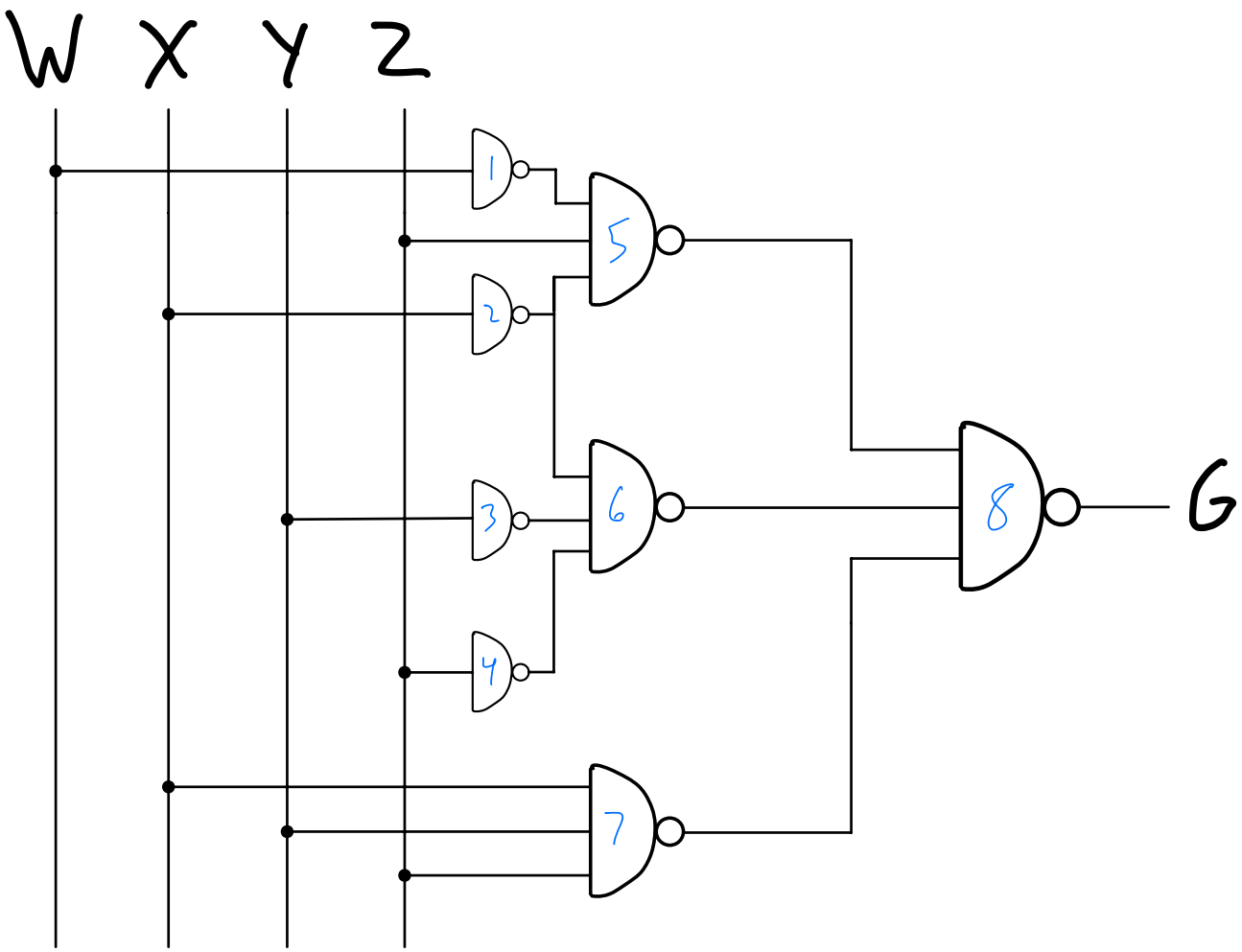
P1) $G = (W, X, Y, Z) = \sum m(0, 1, 3, 7, 8, 15)$

* using no more than 8 NAND gates & no NOR gates

G

WX \ YZ	00	01	11	10
00	1 (0)	1 (1)	1 (3)	(2)
01	(4)	(5)	1 (7)	(6)
11	(12)	(13)	1 (15)	(14)
10	1 (8)	(9)	(11)	(10)

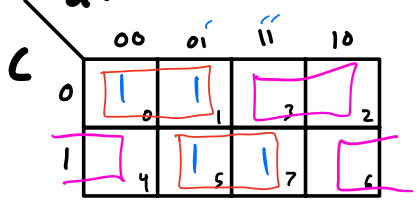
$G = \bar{X}\bar{Y}\bar{Z} + \bar{W}\bar{X}Z + XYZ$



P2) $H(A,B,C) = \Sigma m(0,1,5,7)$

a) Simplified SOP | b) Simplified POS | c) Implement using 5 NOR | d) POS or SOP? Why?

H

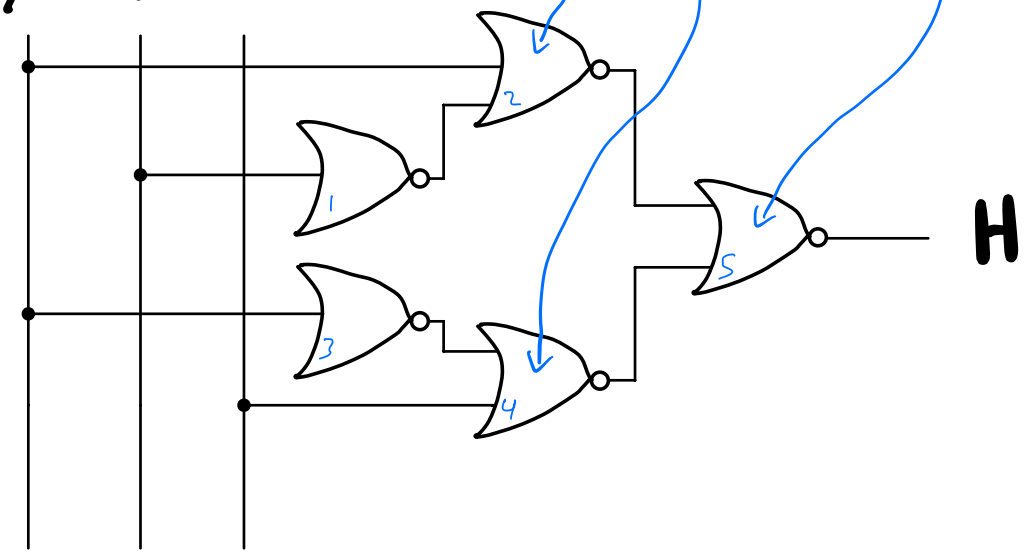


a) $\bar{a}\bar{b} + ac$ SOP

b) $(a+\bar{b})(\bar{a}+c)$ POS

c) A B C

$\overline{\overline{(a+\bar{b})(\bar{a}+c)}} =$



d) POS is far easier to implement using NOR gates compared to SOP, as shown above, $\Rightarrow \overline{\overline{(a+\bar{b})(\bar{a}+c)}}$

P3) 1)

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$= \bar{B}$

2)

WX \ YZ	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	0	0
10	0	0	0	0

$= (X\bar{Y}) + (\bar{W}\,\bar{X}\,\bar{Z})$

3)

WX \ YZ	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	1	0	0	0
10	1	0	0	1

$= (W\bar{Y}\bar{Z}) + (W\bar{X}\bar{Z}) + (\bar{W}X\bar{Y}Z)$

P4) A) $F_1(w,x,y,z) = \sum m(4,6,10,11,14,15)$

SOP B) $F_2(x,y,z) = \sum m(2,5,6,7)$

C) $F_3(w,x,y,z) = \sum m(0,2,3,7,8,10,11,15)$

A) F_1

wx \ yz	00	01	11	10
00	0	1	3	2
01	1 (4)	5	7	1 (6)
11	12	13	15	1 (14)
10	8	9	1 (11)	1 (10)

$= (w\bar{x}y) + (\bar{w}x\bar{z}) + (xy\bar{z})$

B) F_2

x \ yz	00	01	11	10
0	0	1	3	1 (2)
1	4	1 (5)	1 (7)	1 (6)

$= (xz) + (y\bar{z})$

C) F_3

wx \ yz	00	01	11	10
00	1 (0)	1	1 (3)	1 (2)
01	4	5	1 (7)	6
11	12	13	15	14
10	1 (8)	9	1 (11)	1 (10)

3 groups

$= (\bar{x}y) + (\bar{x}\bar{z}) + (\bar{w}yz)$

P5) A) $G_1(a,b,c,d) = \prod M(0,1,2,3,4,5,10,11)$

POS B) $G_2(a,b,c) = \prod M(0,2,3,6)$

C) $G_3(a,b,c,d) = \prod M(0,2,5,9,13)$

A)

G_1

ab \ cd	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	1 (8)	1 (9)	0 (11)	0 (10)

$= \sum m(6,7,8,9,12,13,14,15)$ SOP

2 groups POS

$= (a+c)(b+\bar{c})$

B)

G_2

	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

$= \sum m(1,4,5,7)$ SOP

3 groups POS

$= (a+\bar{b})(a+c)(\bar{b}+c)$

C)

G_3

ab \ cd	00	01	11	10
00	0 (0)	1 (1)	1 (3)	0 (2)
01	1 (4)	0 (5)	1 (7)	1 (6)
11	1 (12)	0 (13)	1 (15)	1 (14)
10	1 (8)	0 (9)	1 (11)	1 (10)

Cannot group 3

$= \sum m(1,3,4,6,7,8,10,11,12,14,15)$ SOP

3 groups POS

$= (a+b+d)(\bar{a}+c+\bar{d})(\bar{b}+c+\bar{d})$

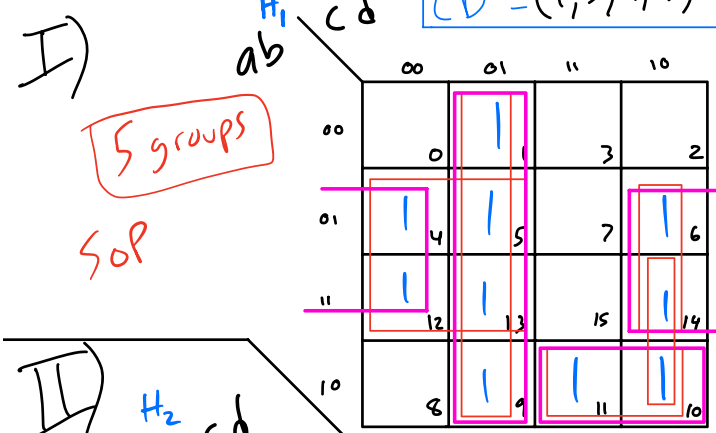
P6) Use K MAPS to convert to simplified SOP expressions

I) $H_1(A,B,C,D) = \bar{C}D + B\bar{C} + BC\bar{D} + AC\bar{D} + A\bar{B}C$

II) $H_2(A,B,C,D) = (A+C+D)(B+C+\bar{D})(\bar{A}+B+\bar{C})$

III) $H_3(A,B,C,D) = \prod M(0,2)$

I)



5 groups

SOP

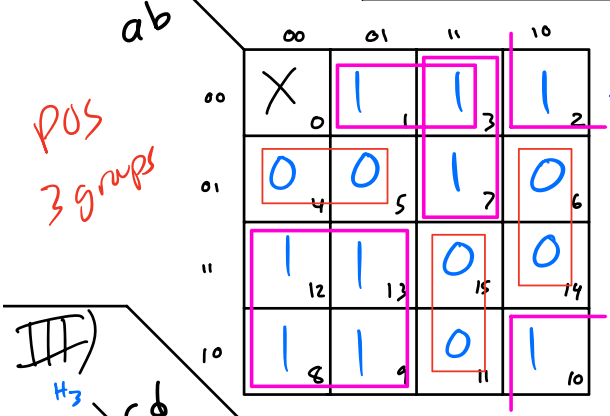
$\bar{C}D = (1,5,9,13)$	$B\bar{C} = (4,5,12,13)$	$AC\bar{D} = (10,14)$
$BC\bar{D} = (6,14)$	$A\bar{B}C = (10,11)$	

$\Sigma m(1,4,5,6,9,10,11,12,13,14) = SOP\ H_1 =$

$= B\bar{C} + B\bar{D} + C\bar{D} + A\bar{B}C$

reduces to only 4 groups

II)



POS

3 groups

$POS\ H_2 = \Sigma m(4,5,6,11,14,15)$

$SOP\ H_2 = \Sigma m(1,2,3,7,8,9,10,12,13)$

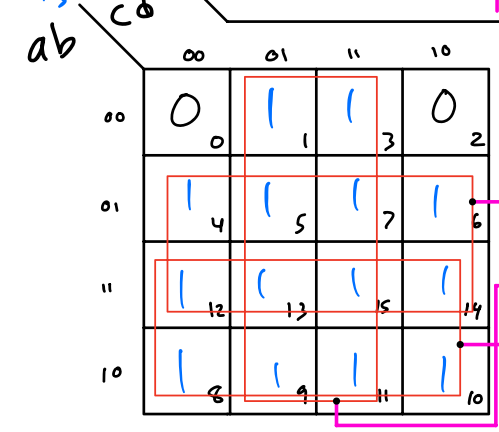
4 SOP groups =

$A+C+D = (11,15)$	
$B+C+D = (6,14)$	
$\bar{A}+B+\bar{C} = (4,5)$	

$SOP\ H_2 =$

$A\bar{C} + \bar{A}\bar{B}D + \bar{B}C\bar{D} + CD\bar{A}$

III)



$\prod M(0,2) = \Sigma m(1,3,4,5,6,7,8,9,10,11,12,13,14,15,16)$

reduces to 3 SOP groups

$SOP\ H_3(A,B,C,D) =$

$A + B + D$