

**P1 (12 points):** Convert the following numbers to IEEE 754 Single-Precision Floating Point binary format:

- A. -8.125
- B. 239
- C. 19/512

**P2 (12 points):** Convert the following numbers from IEEE 754 Single-Precision Floating Point format to decimal. Note that each number is given in hexadecimal. You may leave the result as a fraction.

- A. BF000000<sub>16</sub>
- B. 42C80000<sub>16</sub>
- C. BD600000<sub>16</sub>

**P3 (20 points).** Consider a function  $F$  with 4 bits of input  $A_3, A_2, A_1, A_0$  such that the output of  $F$  is 1 if the unsigned binary number represented by  $A_3A_2A_1A_0$  is prime (i.e. 2, 3, 5, 7, 11, or 13). Otherwise, the output of  $F$  is 0.

- a. Write the truth table for  $F$ .
- b. Implement  $F$  using only a 16-to-1 MUX.
- c. Implement  $F$  using an 8-to-1 MUX, and some AND, OR, and NOT gates.
- d. Implement  $F$  using an 4-to-1 MUX, and some AND, OR, and NOT gates.
- e. Using Shannon's expansion, implement  $F$  using a 2-to-1 MUX, and some AND, OR, and NOT gates.

**P4 (20 points):** Given  $P(A, B, C, D) = BCD + A\bar{B}C + \overline{(A + C + D)}(B + D)$

- A. Implement this function using one 16-to-1 MUX.
- B. Implement this function using one 8-to-1 MUX and NOT gates.
- C. Implement this function using one 4-to-1 MUX with  $A$  and  $B$  as the select lines and a minimal number of AND/OR/NOT gates.
- D. Implement this function using one 2-to-1 MUX with  $C$  as the select line and some AND/OR/NOT gates. Do not implement  $P$  separately with gates and place the MUX in a trivial connection with the rest of the circuit.
- E. Implement this function using one 4-to-1 MUX with  $B$  and  $C$  as the select lines.

**P5 (14 points):** Implement the function  $G(w, x, y, z) = \sum m(5, 7, 8, 10, 13, 14, 15)$  as follows:

- a) Use a K-map to show that  $G$  can be written as  $G = xz + w\bar{x}\bar{z} + wy\bar{z}$
- b) Implement  $G$  using only a minimal number (3) of 2-1 MUXes and no other gates (NOT gates are not allowed, either). Hint: Use Shannon's Expansion Theorem a few times.

**P6 (22 points).** Implement the following functions using Shannon's expansion:

- Implement  $F = w_1w_2 + w_1w_3 + w_2w_3$  using only 2-to-1 MUXs
- Implement  $F = w_1w_2 + w_1w_3 + w_2w_3$  using only 4-to-1 MUXs

P1) A) -8.125 B) 239 C) 19/512

A) -8.125 in binary IEEE 754 Floating Point is:

1	00000100000000000000000000000000	
Sign	Exponent	Mantissa

B) 239 in binary IEEE 754 Floating Point is:

0	0000110110111000000000000000	
Sign	Exponent	Mantissa

C) 19/512 in binary IEEE 754 Floating Point is:

0	1000011011011100000000000000	
Sign	Exponent	Mantissa

P2) A) BF000000<sub>16</sub> B) 42C80000<sub>16</sub> C) BD600000<sub>16</sub>

A) BF000000<sub>16</sub>

Sign	Exponent	Mantissa
1	01111110	000 0000 0006 0000 0000 0000
-	126	0

$$(-1)^{\text{Sign}} \cdot (2^{\text{Exponent}-127}) \cdot (1) = -2^{-1} \text{ or } \boxed{-\frac{1}{2} \text{ or } -0.5_{10}}$$

B) 42C80000<sub>16</sub>

$$133-127=6=2^6$$

Sign	exp.	Mantissa
0	10000101	1001000 0000 0006 0000 0000
+	133	$\frac{1}{2} + \frac{1}{24} = 0.5625$

$$(-1)^0 \cdot 2^6 \cdot 1.5625 = 64(1.5625) = \boxed{100_{10}}$$

C) BD600000<sub>16</sub>

$$122-127=-5=2^{-5}$$

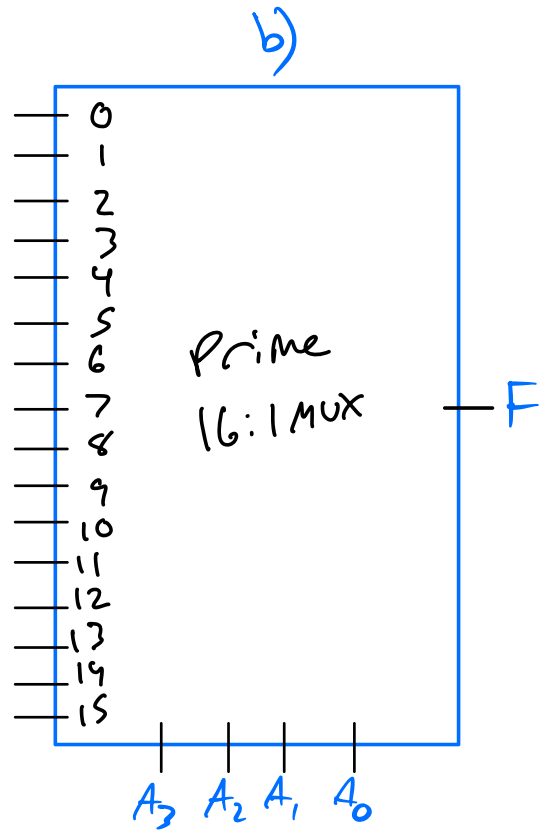
Sign	exp	Mantissa
1	01111010	110 0000 0006 0000 0000 0000
-	122	$\frac{1}{2} + \frac{1}{4} = 3/4 \text{ or } 0.75$

$$(-1)^1 \cdot 2^{-5} \cdot 1.75 = \boxed{-\frac{1.75}{32} \text{ or } -0.0546875_{10}}$$

P 3)

a)

$A_3$	$A_2$	$A_1$	$A_0$	$F$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

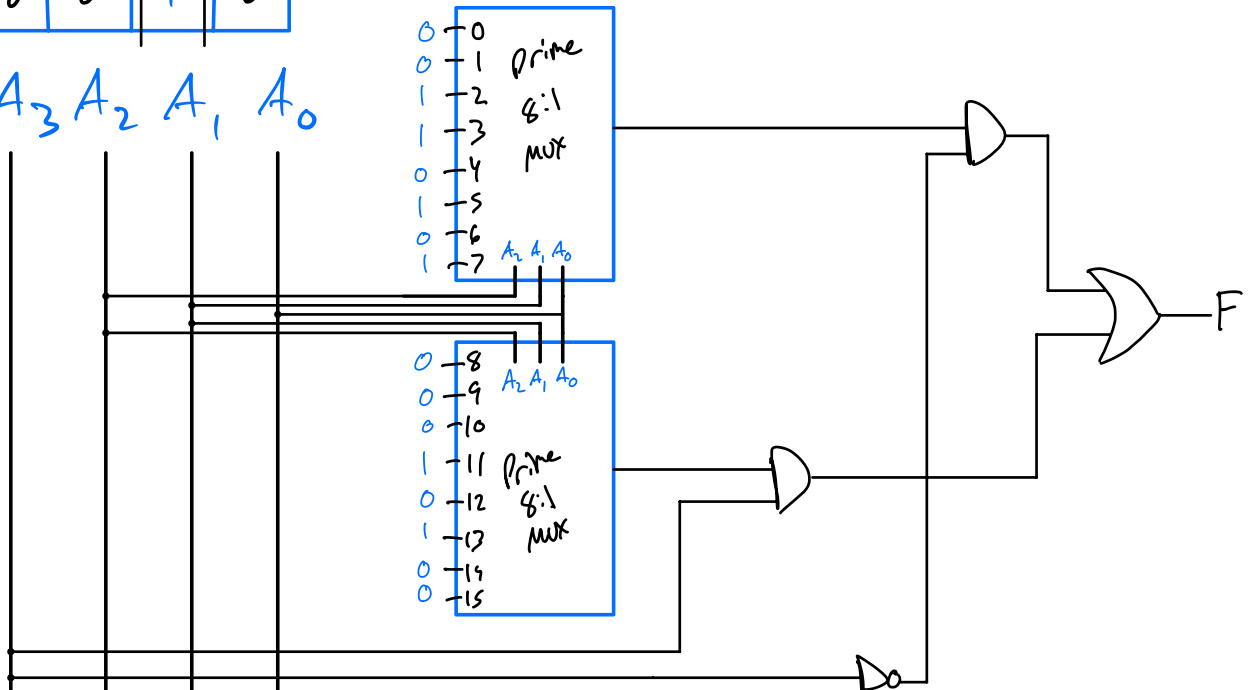


c)

$A_3 A_2$	$A_1 A_0$	$F$
00	00	0
00	01	0
00	10	1
00	11	0
01	00	1
01	01	1
01	10	0
01	11	0
10	00	0
10	01	0
10	10	0
10	11	0
11	00	0
11	01	0
11	10	0
11	11	0

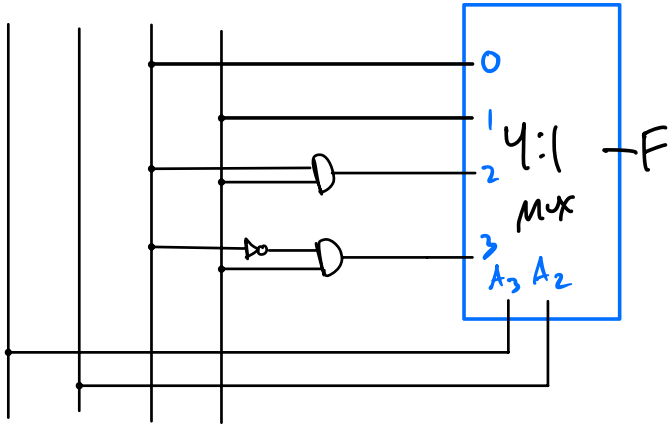
$$= \overline{A_3} \overline{A_2} A_1 + \overline{A_3} A_2 A_0 + \overline{A_2} A_1 A_0 + A_2 \overline{A_1} A_0$$

c)  $A_3 A_2 A_1 A_0$



D)

$A_3 A_2 A_1 A_0$

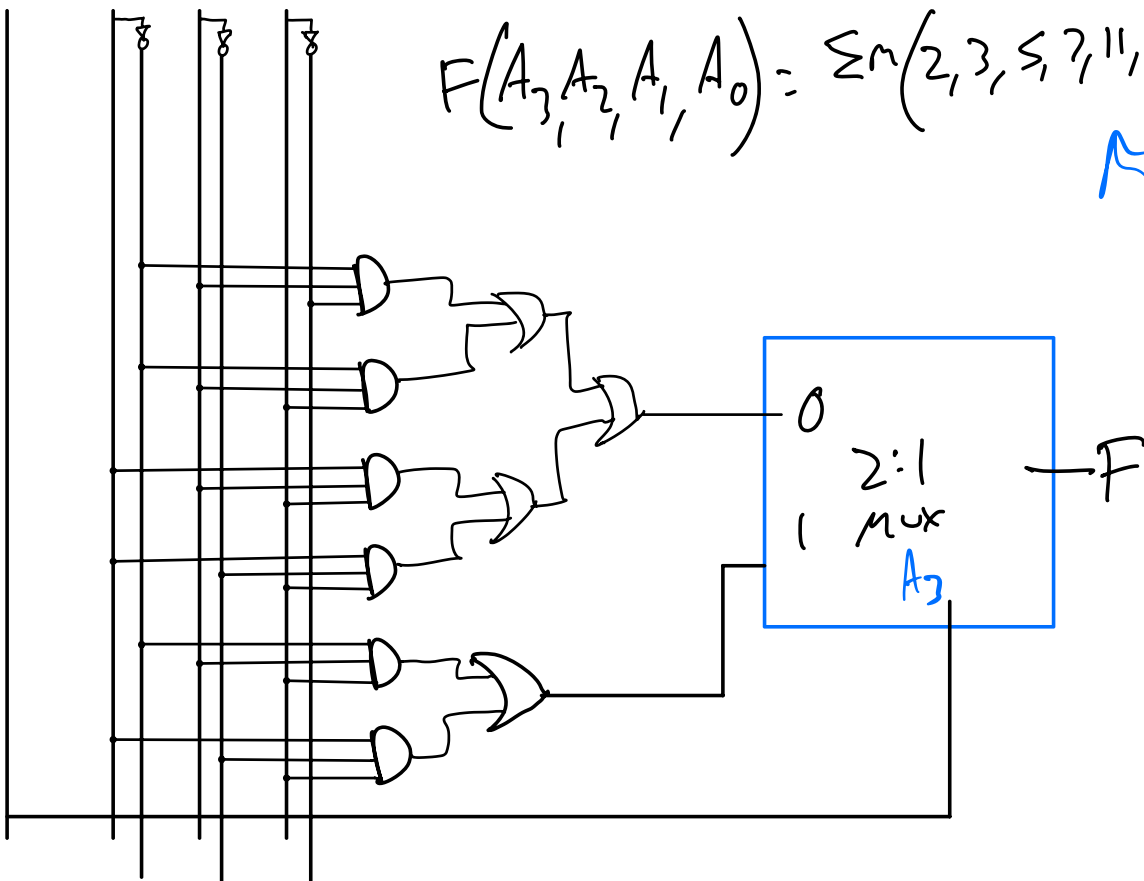


$A_3 A_2$	$N_0$	$N_1$	$N_2$	$N_3$
$A_1 A_0$	00	01	10	11
00	0	4	8	12
01	1	5	9	13
10	2	6	10	14
11	3	7	11	15

$$F(A_3, A_2, A_1, A_0) = \sum m(2, 3, 5, 7, 11, 13)$$

E)

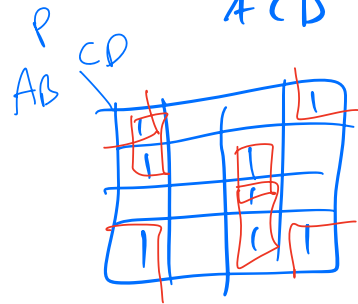
$A_3 A_2 A_1 A_0$



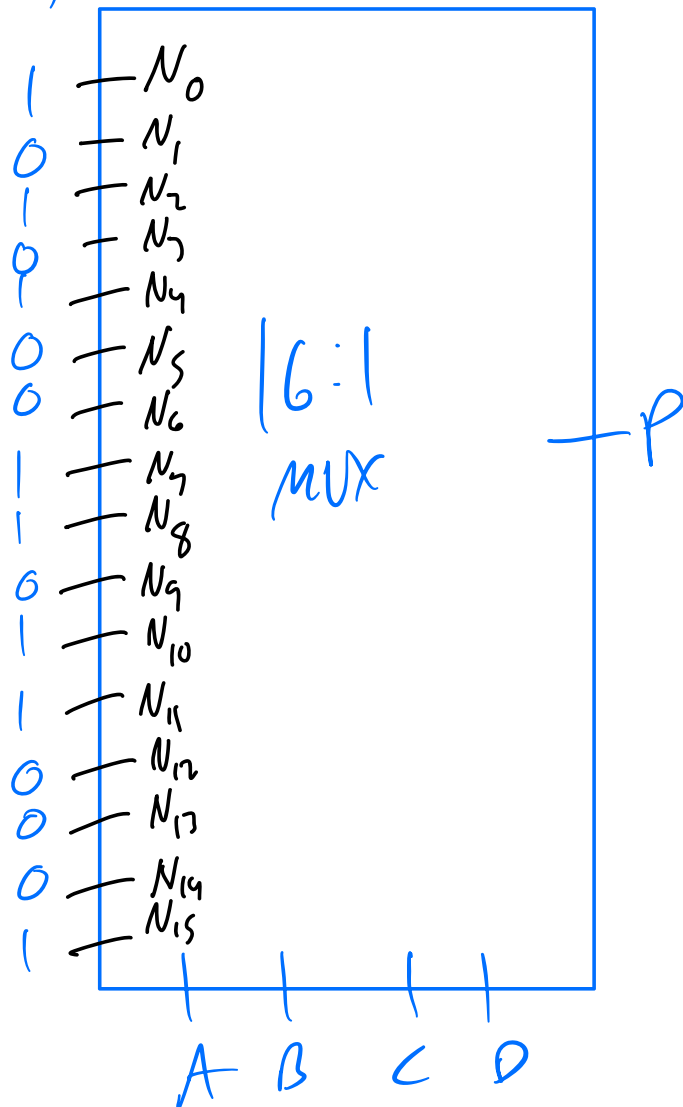
$$F(A_3, A_2, A_1, A_0) = \sum m(2, 3, 5, 7, 11, 13)$$

$$p4) P(A, B, C, D) = BCD + A\bar{B}C + \overline{(A+C+D)}(B+D)$$

$$\bar{A}\bar{C}\bar{D} + \bar{B}\bar{D}$$



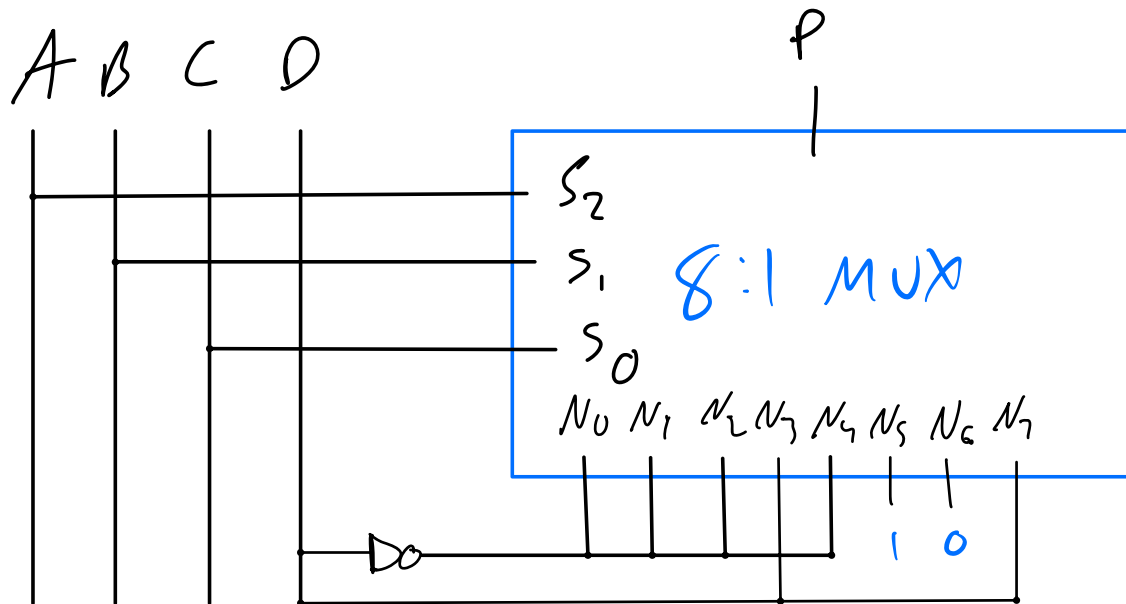
A)



$A_3$	$A_2$	$A_1$	$A_0$	$P$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

B)

$ABC$	$N_0$	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$	$N_6$	$N_7$
000	0	0	0	0	1	0	1	1
001	0	0	1	0	0	1	0	1
010	0	1	0	0	0	1	0	1
011	0	1	0	1	0	0	1	1
100	1	0	0	0	0	1	0	1
101	1	0	1	0	0	0	1	1
110	1	0	0	1	1	0	0	1
111	1	0	1	1	1	0	0	1

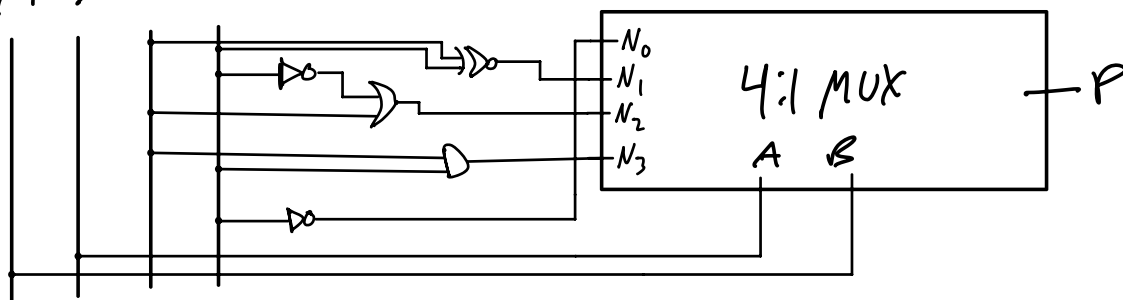


c)

AB	00	01	10	11
$\bar{C}\bar{D}$ 00	0	4	8	12
$\bar{C}D$ 01	1	5	9	13
$C\bar{D}$ 10	2	6	10	14
$CD$ 11	3	7	11	15

$$\bar{D} (00) C + \bar{D} CD = \bar{C}\bar{D} + CD$$

ABCD

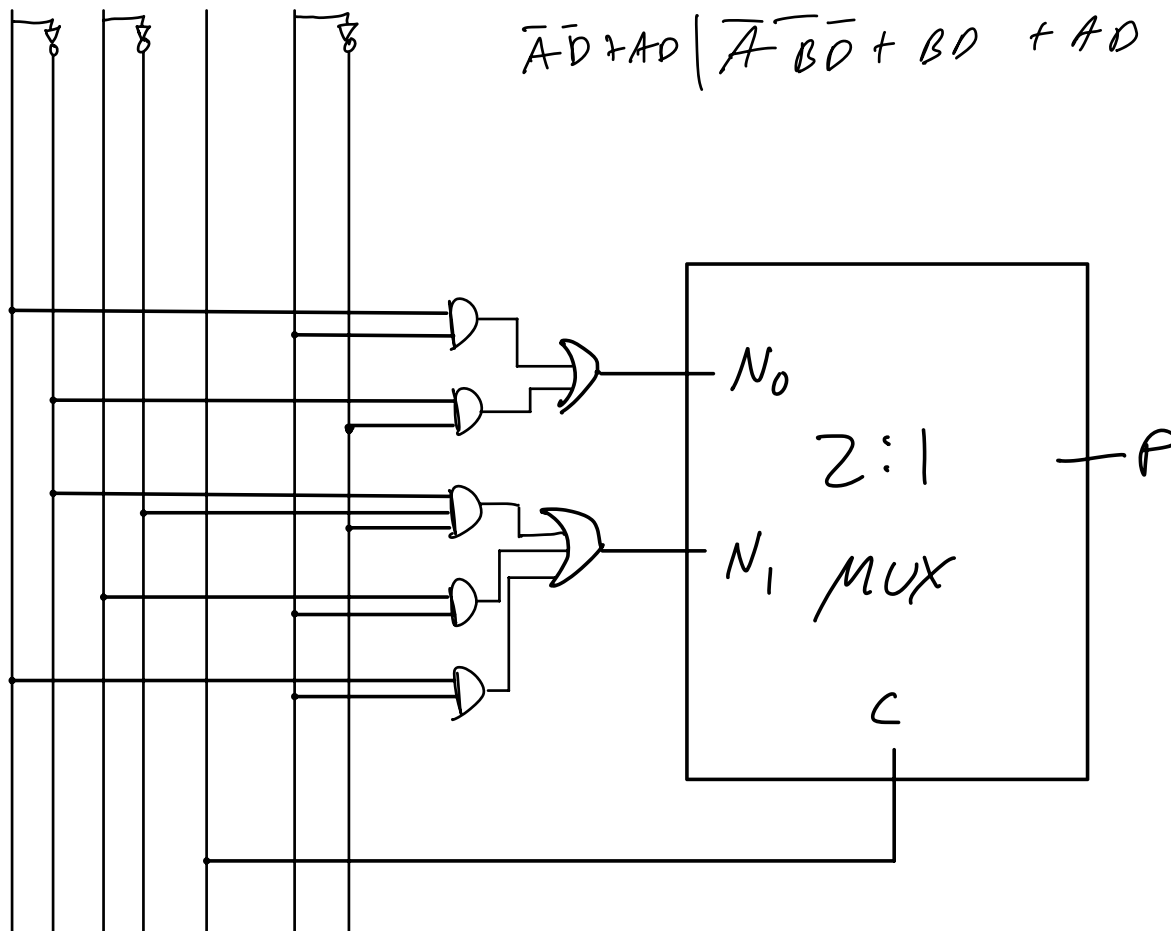


D) 2:1

	C	N <sub>0</sub> 0	N <sub>1</sub> 1
$\bar{A}\bar{B}\bar{D}$	000	<span style="border: 1px solid red;">0</span>	<span style="border: 1px solid red;">2</span>
$\bar{A}\bar{B}D$	001	1	3
$\bar{A}B\bar{D}$	010	<span style="border: 1px solid red;">4</span>	5
$\bar{A}BD$	011	6	<span style="border: 1px solid red;">7</span>
$A\bar{B}\bar{D}$	100	<span style="border: 1px solid red;">8</span>	9
$A\bar{B}D$	101	<span style="border: 1px solid red;">10</span>	<span style="border: 1px solid red;">11</span>
$AB\bar{D}$	110	12	13
$ABD$	111	14	<span style="border: 1px solid red;">15</span>

A B C D

$$\bar{A}\bar{D} + AD \mid \bar{A}\bar{B}\bar{D} + BD + AD$$

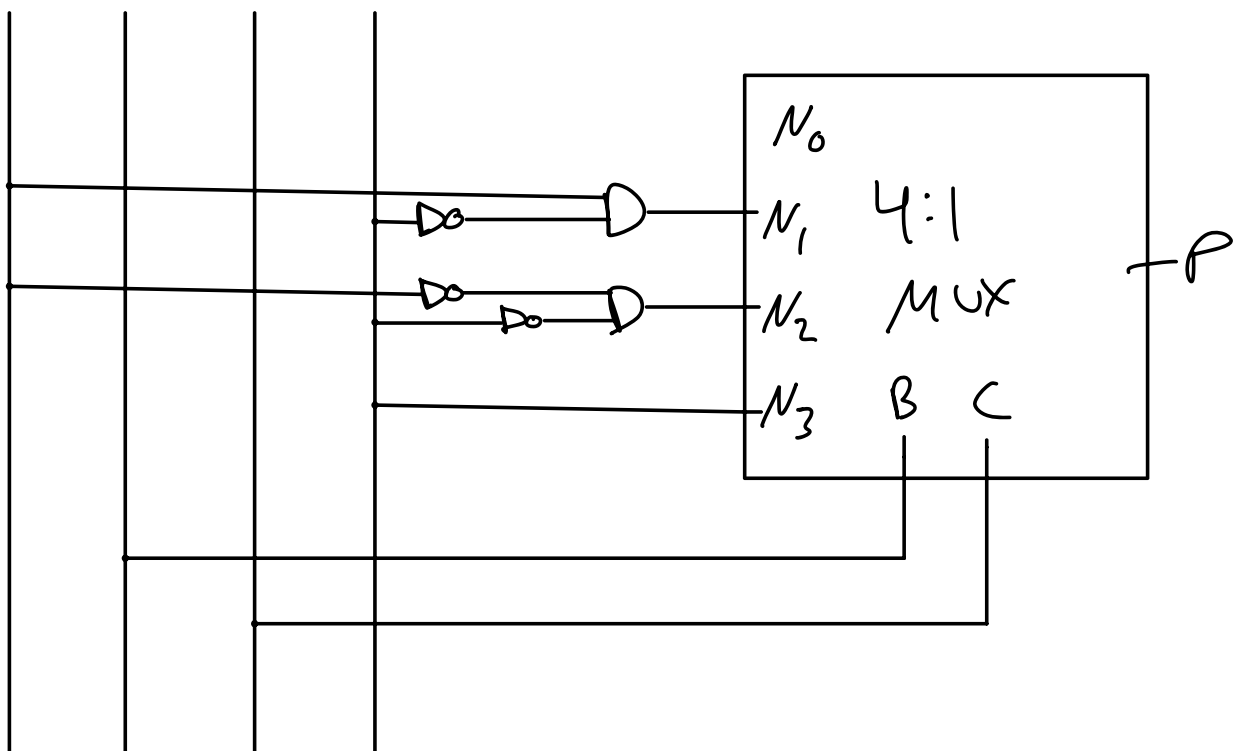




E)

BC		$N_0$	$N_1$	$N_2$	$N_3$
		00	01	10	11
$\bar{A}\bar{D}$	00	0	2	4	6
$\bar{A}D$	01	1	3	5	7
$A\bar{D}$	10	8	10	12	14
$AD$	11	9	11	13	15
		$\bar{D}$	$A+\bar{D}$	$\bar{A}\bar{D}$	$D$

ABCD



ps)

$$G(w, x, y, z) = \sum m(5, 7, 8, 10, 13, 14, 15)$$

A)

G	wx	yz			
		00	01	11	10
00		0	0	0	0
01		0	1	1	0
11		0	1	1	1
10		1	0	0	1

$xz$

$wy\bar{z}$

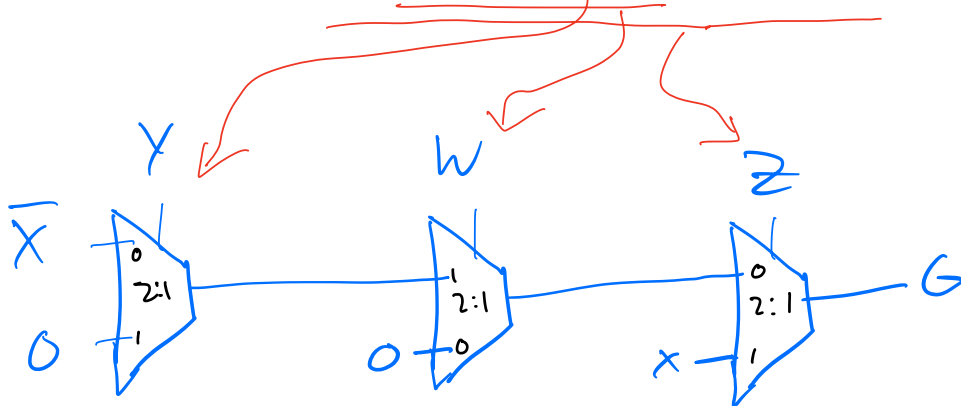
$w\bar{x}\bar{z}$

$$G = xz + wy\bar{z} + w\bar{x}\bar{z}$$

B)

factor  $\bar{z}$  out  $\bar{z}(wy + w\bar{x}) + xz$

$$G = \bar{z}(w(y + \bar{x})) + xz$$



P6)

A)  $F = w_1 w_2 + w_1 w_3 + w_2 w_3$  using 2:1 MUX

$$w_1 w_2 (w_3 + \bar{w}_3) + w_1 w_3 (w_2 + \bar{w}_2) + w_2 w_3 (w_1 + \bar{w}_1)$$

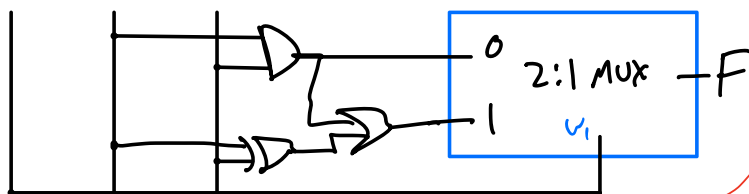
$$w + \bar{w} = 1$$

$$F = \sum m(3, 5, 6, 7)$$

$$= w_3 w_2 w_1 + w_3 w_2 \bar{w}_1 + w_3 \bar{w}_2 w_1 + \bar{w}_3 w_2 w_1$$

$$= \bar{w}_1 (w_2 w_3) + w_1 (w_2 w_3 + \bar{w}_2 w_3 + w_2 \bar{w}_3)$$

$w_1, w_2, w_3$



$$= w_1 w_2 + w_2 w_3 + w_1 w_3$$

$$\sum m(3, 5, 6, 7)$$

