Nick McCullough - ID 950337388 - Lab Section F

Cpr E 281 HW01

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Boolean Algebra/Circuit Synthesis
Assigned: Week 2
Due Date: Jan. 29, 2023

P1 (10 points): A given circuit takes V, a 6-bit binary number, divides V by 9, and stores the quotient and remainder into Q and R, respectively. (e.g. if V=21, then Q=2 and R=3).
A: How many bits are needed to represent all possible values of Q?
B: How many bits are needed to represent R?

P2 (10 points): Draw the circuit for the following expressions:
$F = \bar{a}\bar{b} + \overline{(ab)}$ $\qquad$ $G = \overline{(w + \bar{x} + y + \bar{z})(\bar{w} + x + \bar{y} + z)}$

P3 (15 points): Use the Venn diagram to prove the following:
A: $X + YZ = (X + Y)(X + Z)$
B: $(x_1 + x_2 + x_3).(x_1 + x_2 + \overline{x_3}) = x_1 + x_2$
C: $\overline{x + y} = \bar{x} + \bar{y}$

P4 (15 points): Use Boolean Algebra to simplify the following expressions:
A: $\quad w + x + \bar{w} + x$
B: $\quad wx\bar{y}z + wx\bar{y}\bar{z} + wxy\bar{z} + wxyz + w\bar{x}y\bar{y}$
C: $\quad (\bar{p} + \bar{q} + r)(\bar{q} + r + \bar{s})(\bar{p} + q + r)(\bar{q} + \bar{r} + \bar{s})$
D: $\quad w + wx\bar{y} + wx\bar{z} + w\bar{x}y + w\bar{x}z$

P5 (10 points): Using truth table method, validate the following logic expression:
$$(x_1 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\bar{x}_1 + \bar{x}_3)$$
$X \quad Z \quad \bar{X} \quad \bar{Y} \quad \bar{Z} \quad \bar{X} \quad \bar{Y} \quad X \quad Y \quad Y \quad Z \quad \bar{X} \quad \bar{Z}$

P6 (20 points): Use Boolean Algebra to prove the following expressions as equivalent, and show each rule of Boolean Algebra used to perform each step:
I: $XY\bar{Z} + X\bar{Y}Z + XYZ + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} = X + Y\bar{Z}$
II: $(\bar{A} + B + \bar{B}\bar{C})(\bar{A}\bar{B} + B + C) = \bar{A} + B + \bar{A}C$
III: $\overline{\overline{(p + q)}} + r + \overline{x_3} + \overline{x_2} + \overline{x_1} = (p + \bar{\bar{q}})\bar{r} + \overline{(x_3 x_2 x_1)}$

P7 (20 points): Given the following expression $G = \bar{x}\bar{y}(w + z) + x\bar{y}(\bar{w} + z) + xy(\bar{w} + \bar{z})$:
A. Let the circuit cost be defined as the number of gates plus the number of gate inputs. Draw the circuit for G, then show that the cost of this circuit is 33. You may have to reuse a gate to reduce the cost; the circuit should be drawn appropriately to reflect the cost.
B. Use Boolean algebra to simplify the expression for G.
C. Draw the circuit for G and state the new cost of the circuit.

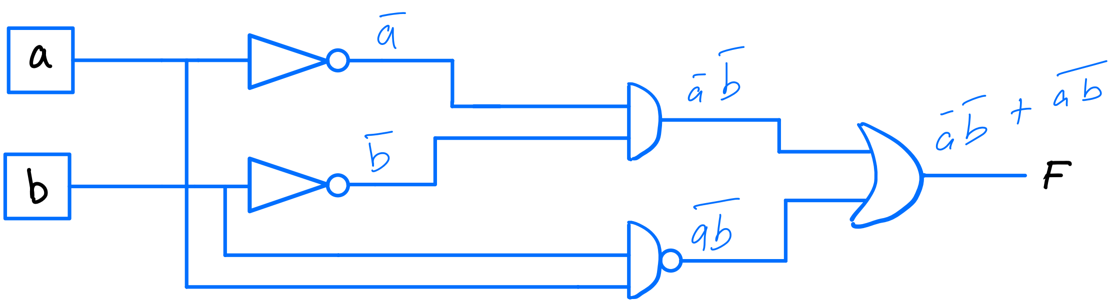
P1) $111111_2 = 63$ six-bit binary $= 2^6 = 64 - 1 = 63$

$63/9 = 7$ no remainder

a) $7 = 111_2$, so three bits

b) $62/9 = 6\frac{8}{9} \to R_{max}$ $1000_2 = 8$ four bits

P2) $F = \bar{a}\bar{b} + \overline{(ab)}$



$G = \overline{(w + \bar{x} + y + \bar{z})(\bar{w} + x + \bar{y} + z)}$

$\overline{w + \bar{x} + y + \bar{z}} + \overline{\bar{w} + x + \bar{y} + z}$

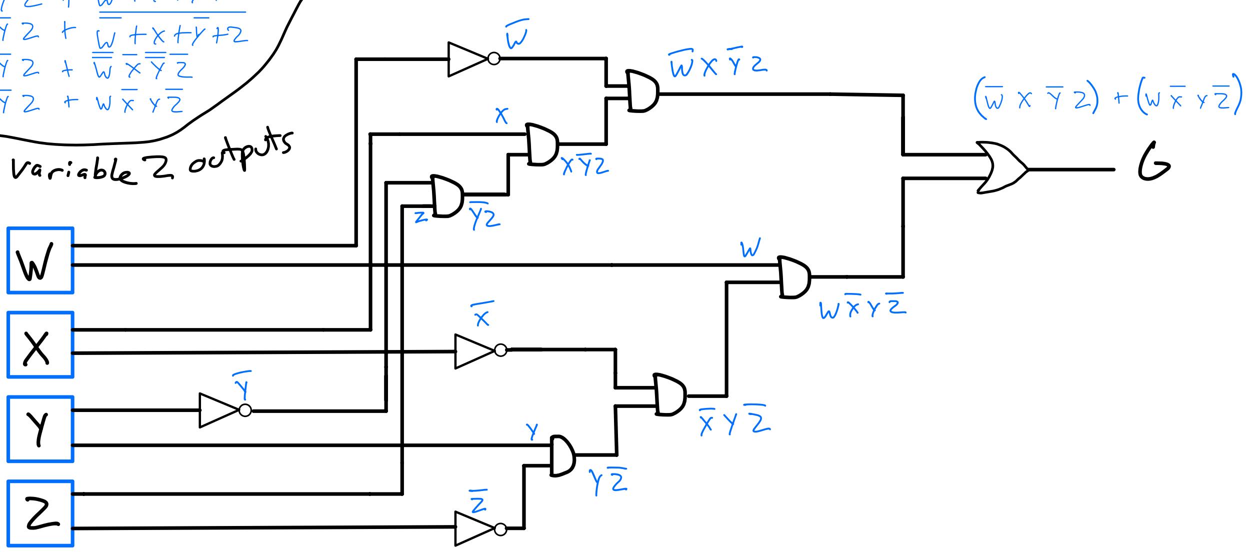
$\bar{w}\,\bar{\bar{x}}\,\bar{y}\,\bar{\bar{z}} + \overline{\bar{w} + x + \bar{y} + z}$

$\bar{w}\,x\,\bar{y}\,z + \overline{\bar{w} + x + \bar{y} + z}$

$\bar{w}\,x\,\bar{y}\,z + \bar{\bar{w}}\,\bar{x}\,\bar{\bar{y}}\,\bar{z}$

$\bar{w}\,x\,\bar{y}\,z + w\,\bar{x}\,y\,\bar{z}$

each variable 2 outputs

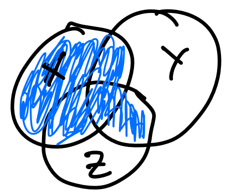


P3) A) $X + YZ = (X+Y)(X+Z)$

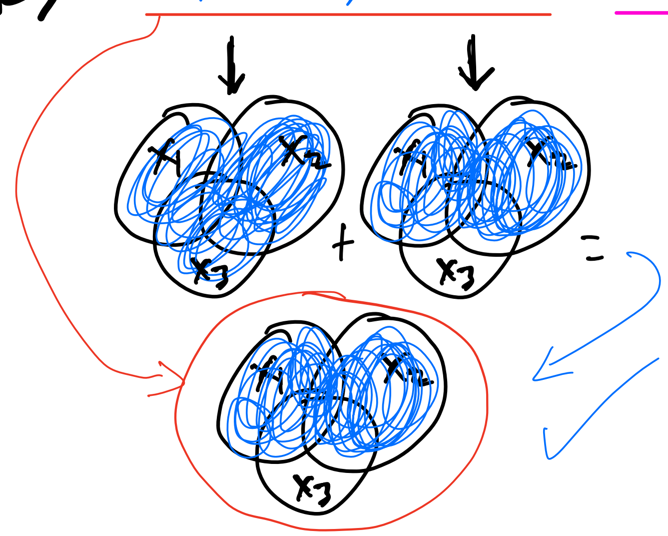
$X+YZ$

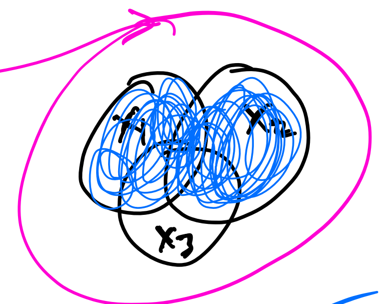
=

$(X+Y)(X+Z)$

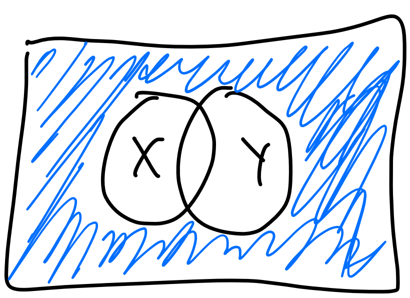


B) $(X_1+X_2+X_3)(X_1+X_2+\overline{X_3}) = X_1+X_2$

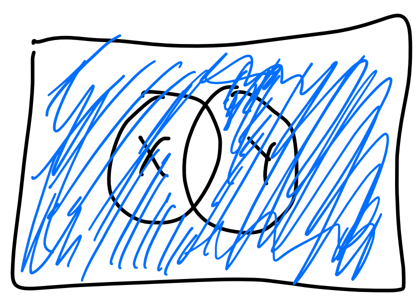




C) $\overline{X+Y} = \overline{X} + \overline{Y}$



$\neq$



P4) a) $w + x + \bar{w} + x$

Compliment law. $A + \bar{A} = 1$

$x + x + 1$

Identity Law. $A + 1 = 1$

$x + x + 1 = 1$

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b) $wx\bar{y}z + wx\bar{y}\bar{z} + wxy\bar{z} + wxyz + \underbrace{w\bar{x}y\bar{y}}_{=0}$

$wx\bar{y}z + wx\bar{y}\bar{z} + wxy\bar{z} + wxyz + 0$

$wx\bar{y}z + wx\bar{y}\bar{z} + wxy\bar{z} + wxyz$

$wx\bar{y}(z+\bar{z}) + wxy\bar{z} + wxyz$ $\quad A + \bar{A} = 1$

$wx\bar{y}1 + wxy\bar{z} + wxyz$

$wx\bar{y} + wxy\bar{z} + wxyz$

$wx(y\bar{z} + \bar{y}) + wxyz$

$wx(\bar{z} + \bar{y}) + wxyz$ $\quad$ distribute

$wx\bar{z} + wx\bar{y} + wxyz$

$wx\bar{y} + wx(yz + \bar{z})$

$wx\bar{y} + wx(y + \bar{z})$ $\quad$ redundancy law

$wx\bar{y} + wxy + wx\bar{z}$

$wx(\bar{y} + y) + wx\bar{z}$ $\quad A + \bar{A} = 1$

$wx + wx\bar{z}$ $\quad A + AB = A$

P4) c) $\underline{(\bar{p}+\bar{q}+r)(\bar{q}+r+\bar{s})(\bar{p}+q+r)(\bar{q}+\bar{r}+\bar{s})}$

$$((\bar{p}+r)+\bar{q}\cdot q)((q+\bar{s})+(\bar{r}\cdot r))$$

$$(\bar{p}+r)(\bar{q}+\bar{s})$$

$\bar{p}p=0$
inverse

$$\bar{p}\bar{q}+\bar{p}\bar{s}+r\bar{q}+r\bar{s}$$

P4) d) $w + wx\bar{y} + wx\bar{z} + w\bar{x}y + w\bar{x}z$

$w + wx\bar{z} + w\bar{x}y + w\bar{x}z$

$w + w\bar{x}y + w\bar{x}z$

$w + w\bar{x}z$

$= w$

absorption Law

$A + AB = A$

P5) Truth Table

$(x_1 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\bar{x}_1 + \bar{x}_3)$

x_1	x_2	x_3	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$\neq$

x_1	x_2	x_3	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

P6) I)

$$XY\bar{Z} + X\bar{Y}Z + XYZ + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$

$$XY(\bar{Z} + Z) + X\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$

$$XY(1) + X\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$

$$XY + X\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$

$$X(\bar{Y}Z + Y) + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$ Absorption

$$X(Z + Y) + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$

$$XZ + XY + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$ distribute

$$XZ + Y(\bar{X}\bar{Z} + X) + X\bar{Y}\bar{Z}$$

$$Y(\bar{Z} + X) + X(\bar{Y}\bar{Z} + Z)$$ Absorption

$$Y(\bar{Z} + X) + X(\bar{Y} + Z)$$ distribute

$$Y\bar{Z} + YX + X(\bar{Y} + Z)$$

$$YZ + YX + X\bar{Y} + XZ$$

$$Y\bar{Z} + X(Y + \bar{Y}) + XZ$$

$$Y\bar{Z} + X(1) + XZ$$

$$Y\bar{Z} + XZ + X$$ Absorption

$$Y\bar{Z} + X$$ ✓

II) $(\bar{A} + B + \bar{B}\bar{C})(\bar{A}\bar{B} + B + C)$

$(\bar{A} + B + \bar{C})(\bar{A}\bar{B} + B + C)$

$(\bar{A} + B + \bar{C})(\bar{A} + B + C)$ distribute

$(\bar{A} + B + C)\bar{A} + (\bar{A} + B + C)B + (\bar{A} + B + C)\bar{C}$

does not equal $\bar{A} + B + \bar{A}C$

III) $\overline{(\overline{p+q})+r}+\overline{x_3}+\overline{x_2}+\overline{x_1}$

$= (p+\bar{\bar{q}})\,\bar{r} + \overline{(x_3 x_2 x_1)}$?

$\overline{\overline{\overline{P+Q}}+r+\overline{x_3}+\overline{x_2}+\overline{x_1}}$ demorgan

$(P+Q)\,\bar{r}+\overline{x_3}+\overline{x_2}+\overline{x_1}$ involution

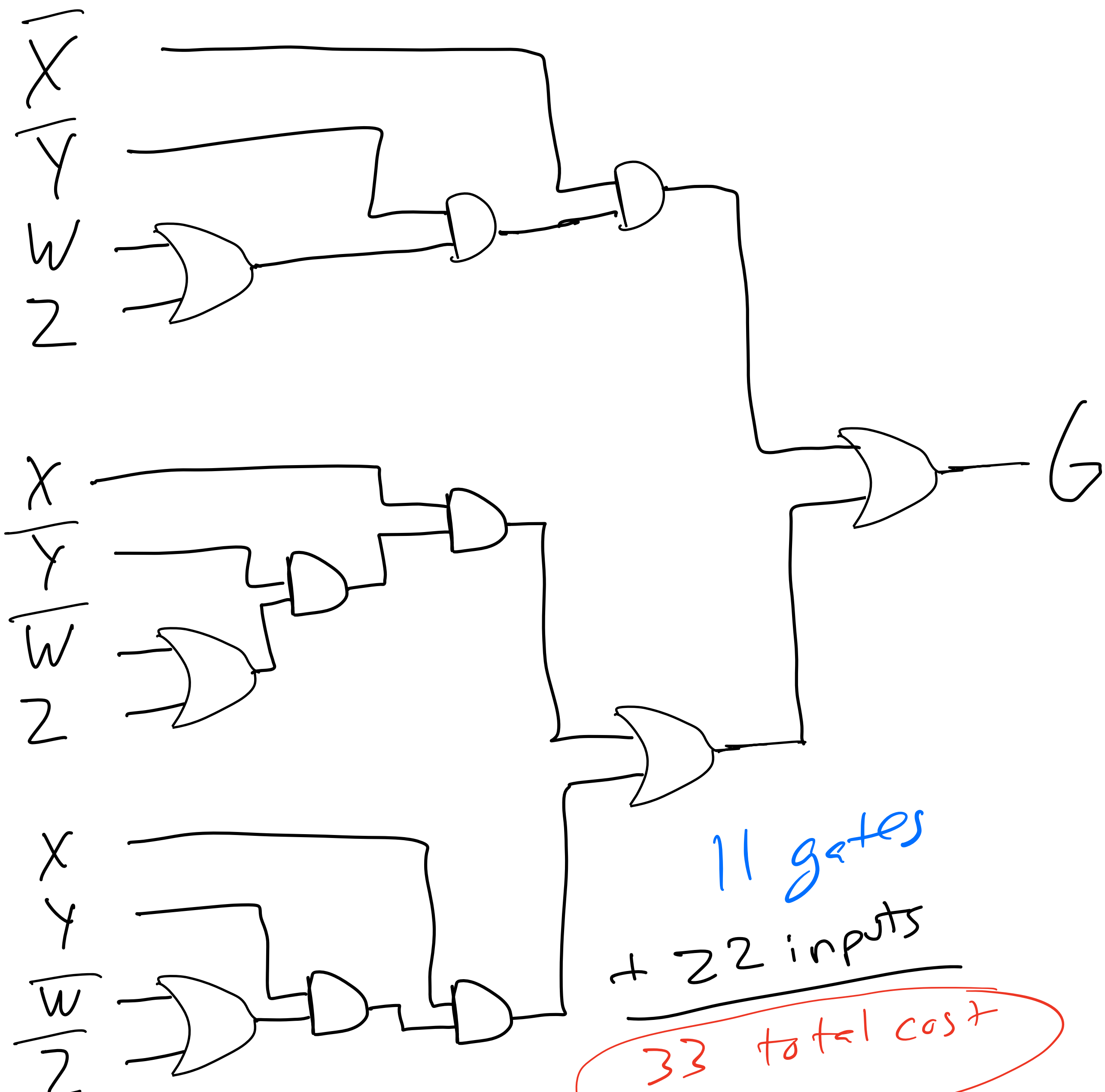
$\bar{r}P+\bar{r}Q+\overline{x_3}+\overline{x_2}+\overline{x_1}$ distribute

does not reduce to

$(p+\bar{\bar{q}})\,\bar{r} + \overline{(x_3 x_2 x_1)}$

P7) $G = \bar{X}\bar{Y}(W+Z) + X\bar{Y}(\bar{W}+Z) + XY(\bar{W}+\bar{Z})$

A)



11 gates

+ 22 inputs

33 total cost

B) $\bar{X}\bar{Y}(W+Z) + X\bar{Y}(\bar{W}+Z) + XY(\bar{W}+\bar{Z})$

$\bar{X}\bar{Y}W + \bar{X}\bar{Y}Z + X\bar{Y}(\bar{W}+Z) + XY(\bar{W}+\bar{Z})$

$\bar{X}\bar{Y}W + \bar{X}\bar{Y}Z + X\bar{Y}\bar{W} \quad X\bar{Y}Z + XY(\bar{W}+\bar{Z})$

$\bar{X}\bar{Y}W + \bar{Y}Z(\bar{X}+X) + X\bar{Y}\bar{W} + XY(\bar{W}+\bar{Z})$

$\bar{X}\bar{Y}W + \bar{Y}Z + X\bar{Y}\bar{W} + XY(\bar{W}+\bar{Z})$

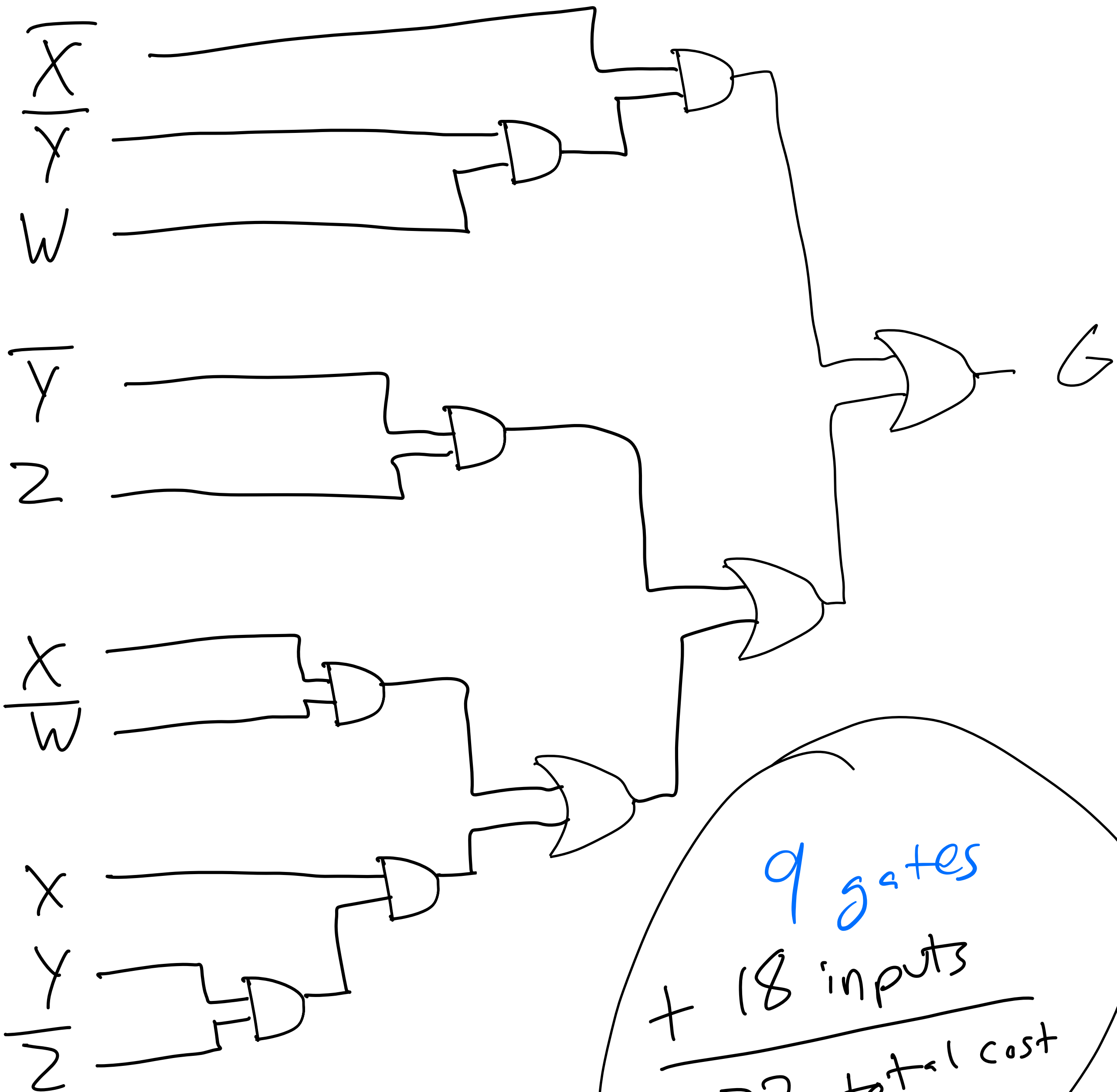
$\bar{X}\bar{Y}W + \bar{Y}Z + X\bar{Y}\bar{W} + XY\bar{W} + XY\bar{Z}$

$\bar{X}\bar{Y}W + \bar{Y}Z + X\bar{W}(\bar{Y}+Y) + XY\bar{Z}$

$\bar{X}\bar{Y}W + \bar{Y}Z + X\bar{W}(1) + XY\bar{Z}$

$\bar{X}\bar{Y}W + \bar{Y}Z + X\bar{W} + XY\bar{Z}$

c) $\bar{X}\bar{Y}W + \bar{Y}Z + X\bar{W} + XY\bar{Z}$



9 gates

+ 18 inputs

27 total cost