Quiz 1 problem bank

(Review material)

1. Find all roots for $x^5 + 2x^3 - 8x = 0$.
2. Find a and b so that $\dfrac{7x+7}{(x-2)(x+5)} = \dfrac{a}{x-2} + \dfrac{b}{x+5}$.
3. Find s and t so that $3s + 2t = 4$ and $8s + 5t = 9$.
4. Find $\dfrac{d}{dt}\left(\ln\left(\dfrac{(t+7)^4(t-5)^2}{(t^2+3t+6)^3}\right)\right)$.
5. Find $\int \cos(2t)\sec(t)\,dt - \int \sin(2t)\sec(t)\,dt$.

(New material)

6. Verify that $x = e^{2t}$ is a solution of the differential equation $(8t^2 - 6t)x'' - (16t^2 - 3)x' + (24t - 6)x = 0$.
7. Verify that $y = 2 + C(t^2+1)^{-3/2}$ is a solution to the differential equation $(t^2+1)y' + 3ty = 6t$; and find C so $y(\sqrt{3}) = 5$.
8. Find all values b so that $y = e^{bx}$ is a solution of the differential equation $2y'' - 5y' + 3y = 0$.
9. Find all values r so that $y = x^r$ is a solution of the differential equation $3x^2y'' + xy' - 5y = 0$.
10. Set up (but do not solve) a differential equation in terms of x and y satisfied by the curve $y = f(x)$ with the property that any *normal* line to the curve at the point (x, y) passes through the point $(0, -y)$. *Include a relevant figure with your answer.*

The quiz will consist of two randomly chosen problems (with possibly small variations; so make sure to learn processes and not just memorize answers).

Key ideas and processes

- *Verifying* a differential equation means to plug a function into a differential equation and checking that the relationship is satisfied. This also can help solve a differential equation if we have an approximate guess for what the answer looks like by plugging that expression in and seeing if/what would make the equation satisfied.
- *Modeling* or *setting up* a differential equation works by figuring out what is changing and then relating the change in that quantity to what is currently happening. When in doubt, draw a picture.
- *General solutions* will usually involve one (or more) constants (though they may not always be a "+C" on the end). *Specific solutions* or *solutions to initial value problems* look for a single function which both satisfy the differential equation and some other constraint.

Comments and (partial) answers on problems

1. $0, \pm\sqrt{2}, \pm 2i$
2. $a = 3$, $b = 4$
3. $s = -2$, $t = 5$
4. $\dfrac{4}{t+7} + \dfrac{2}{t-5} - \dfrac{6t+9}{t^2+3t+6}$
5. $2\sin(t) + 2\cos(t) - \ln|\sec(t) + \tan(t)| + C$
6. Take derivatives of x and plug and check
7. Take derivative of y and plug and check; $C = 24$
8. $b = 1$, $b = 3/2$
9. $r = 5/3$, $r = -1$
10. $2yy' = -x$; a relevant picture would include a sketch of line from generic (x, y) to $(0, -y)$

① $x^5 + 2x^3 - 8x = 0$

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① factor an x out

$$x(x^4 + 2x^2 - 8) = 0$$

almost quadratic

$$x((x^2)^2 + 2(x^2) - 8) = 0$$

$$y = x^2$$

$$x((y)^2 + 2(y) - 8) = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-2 \pm \sqrt{4 - 4(1)(-8)}}{2}$$

$$= \frac{-2 \pm \sqrt{36}}{2}$$

$$= \cancel{\frac{-2 \pm 6}{2}}$$

$$y = -1 \pm 3$$

$y = 2$ or $y = -4$

$$x[y-2][y+4] = 0$$

$$x\{x^2 - 2\}\{x^2 + 4\} = 0$$

$$x\{x^2-2\}\{x^2+4\}=0$$

$$x^2-2=0 \rightsquigarrow \pm\sqrt{2}$$

$$x^2+4=0 \rightsquigarrow \pm\sqrt{-4}=\pm 2i$$

$$x=0$$

roots $0, \pm\sqrt{2}, \pm 2i$

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② $$\frac{7x+7}{(x-2)(x+5)}=\frac{a}{x-2}\quad\frac{b}{x+5}$$

① Multiply both sides by $(x-2)(x+5)$

$$7x+7=a\frac{\cancel{(x-2)}(x+5)}{\cancel{x-2}}+b\frac{(x-2)\cancel{(x+5)}}{\cancel{x+5}}$$

$$7x+7=a(x+5)+b(x-2)$$

② plug in a value such as $x=2$

$$7(2)+7=a((2)+5)$$

$$21=7a$$

$$3=a$$

③ Now Find B, plug in $x = -5$

$$7(-5) + 7 = b((-5) - 2)$$
$$-28 = -7b$$
$$4 = b$$

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③ Find s and t so that $3s + 2t = 4$ and $8s + 5t = 9$.

① use method of elimination.

$$(3s + 2t = 4)5 \qquad (8s + 5t = 9)-2$$
$$15s + 10t = 20 \qquad (-16s - 10t) = -18$$

② add them together

$$\begin{array}{r} 15s + 10t = 20 \\ -16s - 10t = -18 \\ \hline -s = 2 \end{array}$$
$$\boxed{s = -2}$$

$$3(-2) + 2t = 4$$
$$-6 + 2t = 4$$
$$2t = 10$$
$$\boxed{t = 5}$$

④ Find $\frac{d}{dt}\left(\ln\left(\frac{(t+7)^4(t-5)^2}{(t^2+3t+6)^3}\right)\right)$

$\ln(a \cdot b) = \ln a + \ln b$
$\ln(a/b) = \ln a - \ln b$
$\ln a^b = b \ln a$

$$\frac{d}{dt}\left[\ln(t+7)^4 + \ln(t-5)^2 - \ln(t^2+3t+6)^3\right]$$

$$\frac{d}{dt}\left[4\ln(t+7) + 2\ln(t-5) - 3\ln(t^2+3t+6)\right]$$

the 1 is for $\frac{d}{dt}(t-5)=1$

$$\frac{4(1)}{t+7} + \frac{2(1)}{(t-5)} - \frac{3(2t+3)}{t^2+3t+6}$$

answer

!! always be aware of chain rule !!

☆ no +C, because not integration

5) Find $\int \cos(2t)\sec(t)\,dt - \int \sin(2t)\sec(t)\,dt$.

$\cos 2t = 2\cos^2 t - 1$

$= 1 - 2\sin^2 t$

$= \cos^2 t - \sin^2 t$

$\sin 2t = 2\sin t \cos t$

$\sec t = 1/\cos t$

These are provided on exams!

$$\int \left[2\cos^2 t - 1\right]\sec t\,dt - \int \left[2\sin t\cos t\right]\sec t\,dt$$

simplify

$$\int \left[2\cos t - \sec t\right]dt - \int 2\sin t\,dt$$

now integrate

$$= 2\sin t - \ln|\sec t + \tan t| + 2\cos t + C$$

⑥ Find first & second derivative & plug it in.

$x = e^{2t}$

$x' = 2e^{2t}$

$x'' = 2 \cdot 2e^{2t} = 4e^{2t}$

$(8t^2 - 6t)(4e^{2t}) - (16t^2 - 3)(2e^{2t}) + (24t - 6)(e^{2t})$

$= 32t^2e^{2t} - 24te^{2t} - 32t^2e^{2t} + 6e^{2t} + 24te^{2t} - 6e^{2t}$

these all cancel out

$= 0$

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$$y = 2 + c(t^2+1)^{-3/2}$$

$$y' = \frac{-3}{2} c\left[t^2+1\right]^{-5/2}(2t)$$

$$(t^2+1)\left[-3c+\left[t^2+1\right]^{-5/2}+3t\left[2+c\left[t^2+1\right]^{-3/2}\right]\right]$$

$$=(t^2+1)^{1-5/2}\left(-3ct\right)+6t+3ct\left[t^2+1\right]^{-3/2}$$

cancels out

$$=\underline{(t^2+1)^{-3/2}\left(-3ct\right)}+6t+\underline{3ct\left[t^2+1\right]^{-3/2}}$$

$$=6t$$

$$y(t)=5$$

$$y(\sqrt{3})=5$$

$$5=2+c\left[(\sqrt{3})^2+1\right]^{-3/2}$$

$$c=24$$

$$3=c\left[3+1\right]^{-3/2}$$

$$3=c\left[4\right]^{-3/2}$$

$$3=c\left[4\right]^{\frac{1}{2}\cdot 3\cdot -1}=c\left[2\right]^{3\cdot -1}=c\left[8^{-1}\right]$$

$$3=\frac{c}{8}$$

8) $2y'' - 5y' + 3y = 0$

1 $y = e^{bx}$

2 $y' = be^{bx}$

3 $y'' = b \cdot b e^{bx} = b^2 e^{bx}$

$$2(b^2 e^{bx}) - 5(be^{bx}) + 3(e^{bx}) = 0$$

$$e^{bx}\Big[\underbrace{2b^2 - 5b + 3}_{\text{quadratic}}\Big] = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{5 \pm \sqrt{25-24}}{4}$$

$$b = \frac{5 \pm 1}{4} = \boxed{b = 3/2 \text{ or } b = 1}$$

(9) $3x^2y'' + xy' - 5y = 0 \qquad y = x^r$

Find derivatives

1 $\quad y = x^r$

2 $\quad y' = r x^{r-1}$

3 $\quad y'' = r(r-1) x^{r-2}$

$$3x^2\left[r(r-1)x^{r-2}\right] + x\left[r x^{r-1}\right] - 5x^r = 0$$

$$3r(r-1)x^r + r x^r - 5x^r = 0$$

$$3\left[r^2 - r\right]x^r + r x^r - 5x^r = 0$$

$$x^r\left[3r^2 - 3r + r - 5\right] = 0$$

$$x^r\left[3r^2 - 2r - 5\right] = 0$$

use quadratic formula

$$x^r\left[3r - 5\right]\left[r + 1\right] = 0$$

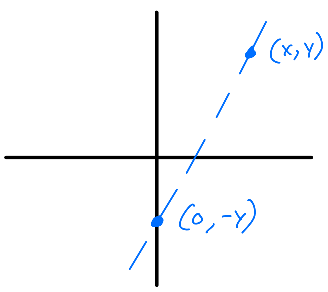
$$\boxed{r = \frac{5}{3} \ \ \& \ \ r = -1}$$

(10)

$y' =$ slope of tangent line

slope of normal line is perpendicular to slope of tangent line. So,

$-\frac{1}{y'} =$ slope of normal line



slope $= \frac{\text{rise}}{\text{run}}$

slope $= \frac{2y}{x}$

$$-\frac{1}{y'} = \frac{2y}{x}$$

$$\boxed{-x = 2yy'}$$