CHEM 571A - Problem Set #2

Due Monday September 9th

1. $McQuarrie\ 3-3$: In each case, show that f(x) is an eigenfunction of the operator given. Find the eigenvalue.

$$\frac{\hat{A}}{(a) \frac{d^2}{dx^2}} \qquad \qquad f(x)$$

(b)
$$\frac{d}{dt}$$
 $e^{i\omega t}$

(c)
$$\frac{d^2}{dx^2} + 2\frac{d}{dx} + 3 e^{\alpha x}$$

(d)
$$\frac{\partial}{\partial y}$$
 $x^2 e^{6y}$

- 2. McQuarrie 3-5: Write out the operator for \hat{A}^2 for $\hat{A} =$
 - (a) $\frac{d^2}{dx^2}$
 - (b) $\frac{d}{dx} + x$
 - (c) $\frac{d^2}{dx^2} 2x\frac{d}{dx} + 1$
- 3. McQuarrie 4-7: Calculate the values of $\sigma_E^2 = \langle E^2 \rangle \langle E \rangle^2$ for a particle in a box in the state described by

$$\psi(x) = \left(\frac{630}{a^2}\right)^{1/2} x^2 (a-x)^2 \qquad 0 \le x \le a$$

4. McQuarrie 4-9: The momentum operator in two dimensions is

$$\hat{P} = -i\hbar \left(\mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} \right).$$

Using the wave function for a free particle constrained to move over a rectangular region $0 \le x \le a$ and $0 \le y \le b$, given as

$$\psi_{n_x,n_y}(x,y) = \left(\frac{4}{ab}\right)^{1/2} \sin \frac{n_x \pi x}{a} \sin \frac{n_y \pi y}{b} \quad \begin{cases} n_x = 1, 2, 3, \dots \\ n_y = 1, 2, 3, \dots \end{cases},$$

1

calculate $\langle p \rangle$ and $\sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2$.