Neural Network Discussion

> "Machine learning is essentially a form of applied statistics with increased emphasis on the use of computers to statistically estimate complicated functions and a decreased emphasis on proving confidence intervals around these functions"
>
> – Ian Goodfellow, et al[1]

Introduction

If we discuss Neural Networks, we should first consider the system we hope to emulate. Let us start with a simple count of neuronal cells in various organisms along the earth's phylogenetic or evolutionary tree. We might get a better idea of the type of "computing power" these living creatures possess. See table 5.1.

Table 5.1: Organisms Vs Number of Neurons In Each (Wikipedia)

Organism	Common Name	Approximate Number of Neurons
C. elegans	roundworm	302
Chrysaora fuscescens	jellyfish	5,600
Apis linnaeus	honey bee	960,000
Mus musculus	mouse	71,000,000
Felis silvestris	cat	760,000,000
Canis lupus familiaris	dog	2,300,000,000
Homo sapien sapien	humans	100,000,000,000

This table portrays a high-level overview of the computing power of neuronal clusters and brains produced throughout evolution. However, there is one missing number worth noting. The table above does not describe the connectivity between neurons. The connectivity of neurons varies greatly from lower to higher organisms. For example, some simple animals have only "four to eight separate branches"[2], per nerve cell. While human neurons may have approximately 10^4 inter-connected synaptic junctions per neuron, thus resulting in a total of approximately 600 trillion synapses per human brain.[3]

Although neurons have differing morphologies, neurons in the human brain are extremely diverse. Indeed, size and shape may not be the definitive way of classifying neurons but instead by what neurotransmitters the cells secrete. "Neurotransmitters can be classified as either excitatory or inhibitory."[4] Currently the NeuroPep (version 1.0, 2014-11-26) database "holds 5949 non-redundant neuropeptide entries originating from 493 organisms belonging to 65 neuropeptide families."[5]

[1] Ian Goodfellow, Yoshua Bengio, Aaron Courville, 'Deep Learning', MIT Press, 2016, http://www.deeplearningbook.org
[2] https://www.wormatlas.org/hermaphrodite/nervous/Neuroframeset.html
[3] Shepherd, G. M. (2004), The synaptic organization of the brain (5th ed.), Oxford University Press, New York.
[4] https://www.kenhub.com/en/library/anatomy/neurotransmitters
[5] http://isyslab.info/NeuroPep/home.jsp

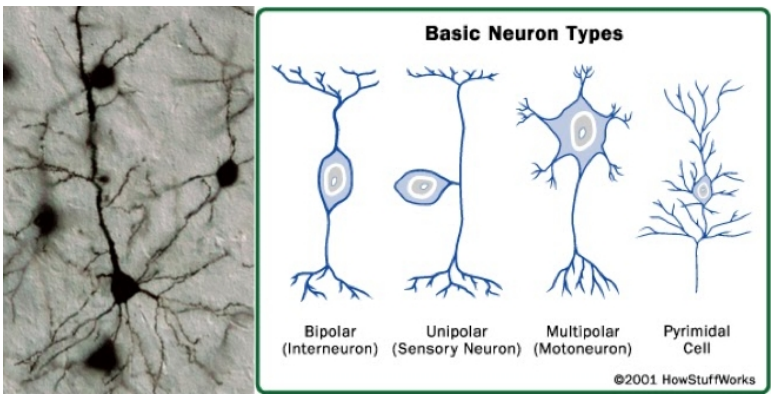

Figure 1: Basic Neuron Types and S.E.M. Image

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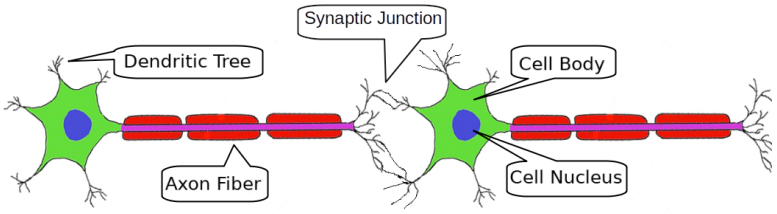


Figure 2: Two Neuron System (Image From The Public Domain)

Given an order of operation via:

Dendrite(s) $\Longrightarrow$ Cell body $\Longrightarrow$ Fibrous Axon $\Longrightarrow$ Synaptic Junction or Synaptic Gap $\Longrightarrow$ Dendrite(s) ... Ad infinitum.

However, nature is more subtle and intricate than to have neurons in a series, only blinking on and off, firing or not. Neural networks are often programmed to classify dangerous road objects, as is the case of Tesla cars. The goal of a Tesla auto-piloted car is to use all available sensors to correctly classify all the conceivable circumstances on the road. On the road, a Tesla automobile uses dozens of senors which the computer needs to evaluate and weigh the values of all these sensors to formulate a 'decision.' The altitude of the auto, derived from the GPS, may weigh less heavily than the speed of the vehicle or Lidar estimates on how close objects are. However, our goal of safe driving can be thwarted when an artificial intelligence system decides a truck is a sign and does not apply the brakes.[7]

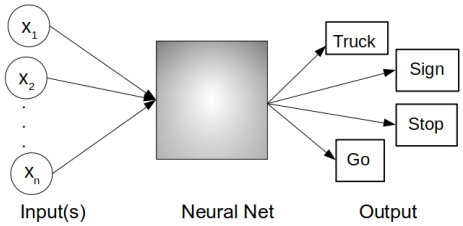


Figure 3: Goal of a Tesla Neural Networks is to generate the correct repsonses for its environment.

[6]https://www.howstuffworks.com/
[7]https://arstechnica.com/cars/2019/05/feds-autopilot-was-active-during-deadly-march-tesla-crash/

The One Neuron System

If we investigate a one neuron system, *our* neuron could be diagrammed in four sections.[8]

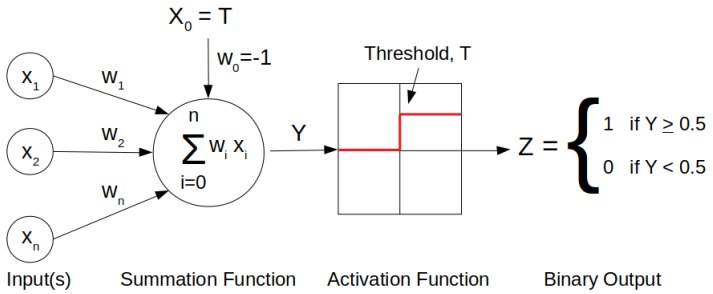


Figure 4: One Neuron Schema

If we investigate one neuron for a moment, we find two separate mathematical functions incorporated into a single nerve cell.

Summation Function

The first segment is a summation function. It receives the real number values from, x_1 to x_n, all the branches of the dendritic trees, and multiplies them by a set of weights. These X inputs are multiplied by a set of corresponding unique weights from w_1 to w_n. An analogy I prefer is of small or large rivers joining giving a total current. The current moves through the branches giving a total signal or current of sodium ions. Interestingly the summation in each neuron, while dealing with the vectors of inputs and weights, is carrying out the dot product of these vectors, such that;

Initially, the neural networks used the Heaviside-Threshold Function, as shown in figure 4, the 'One Neuron System.' The benefits of step functions were their simplicity and high signal to noise ratio. While the detriments were, it is a discontinuous function, therefore not able to be differentiated and a mathematical problem.

Let us take into account the product, $x_0 \cdot w_0$. If we assign $x_0 = T$ and $w_0 = -1$ this simply becomes a bias. This bias allows us the ability to shift our Activation Function and its inflection point in the positive or negative x-direction.

$$Y \;=\; X^T \cdot W - Bias \;\equiv\; \sum_{i=0}^{n} x_i w_i - T \tag{1}$$

Activation Functions

The second function is called an Activation Function. Once the Summation Function yields a value, its result is sent to the *Activation Function* or *Threshold Function.*

$$Z^{(1)} = f\left(\sum_{i=0}^{n} x_i w_i - T\right) = \{0, 1\} \tag{2}$$

[8]Tom Mitchell, Machine Learning, McGraw-Hill, 1997, ISBN: 0070428077

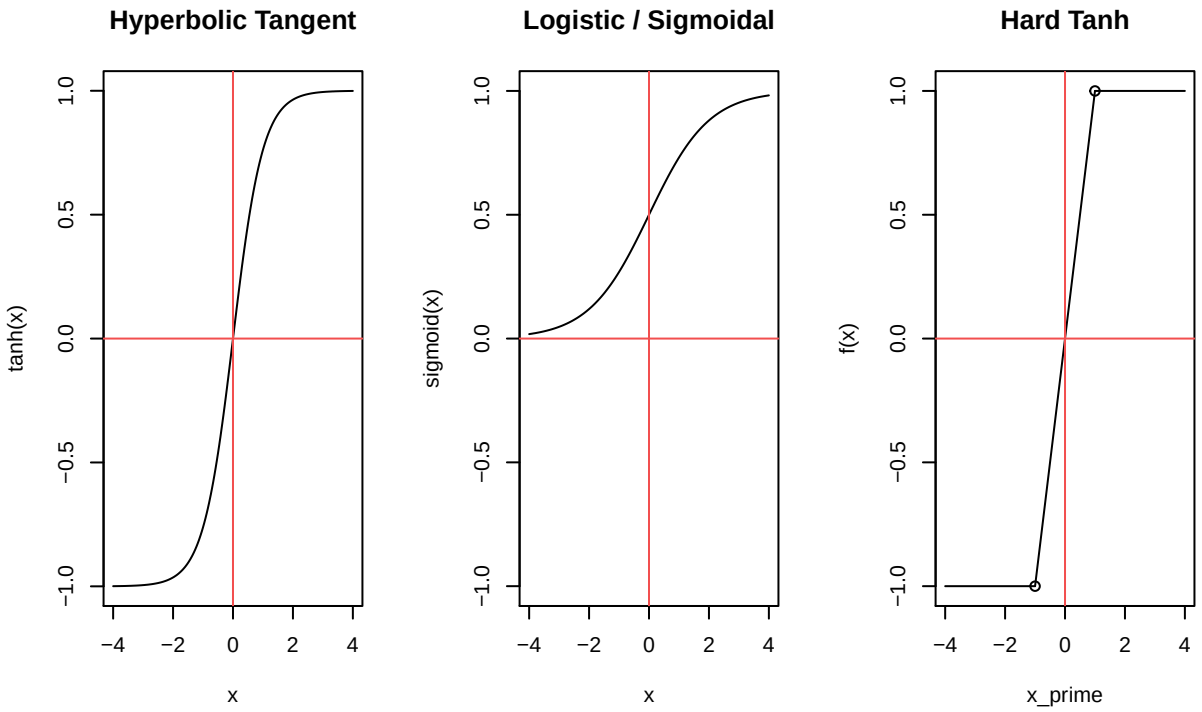
The function displayed in figure #4 (One Neuron Schema) is a step function. However this step function has a problem mathematically, namely it is a discontinuous and therefore not differentiable. This fact is important.

Therefore several functions may be used in place of the step function. One is the hyperbolic tangent (*tanh*) function, the *sigmoidal* function, a *Hard Tanh*, a reLU, and Softmax Functions. These have certain advantges, namely they simplify the hyperbolic tangent function. Not only does the *Hard Tanh* and reLU simplify calculations it is useful for increasing the gain near the asymptotic limits of the sigmoidal and tanh functions. The derivatives of the sigmoidal and tanh functions are very small near 0 and 1 while the reLU and Hard Tanh slopes are one or zero.

$$Z^{(2)} \;=\; tanh(x) = \frac{1-e^{-\alpha}}{1+e^{-\alpha}} \quad : \quad where \quad \alpha = \sum_{i=1}^{n} x_i w_i - T \tag{3}$$

$$Z^{(3)} \;=\; sigmoid(x) \;=\; \frac{1}{1+e^{-\alpha}} \tag{4}$$

$$Z^{(4)} \;=\; Hard\ Tanh(x) \;=\; \begin{cases} 1 & x > 1 \\ x & -1 \leq x \leq 1 \\ -1 & x < -1 \end{cases} \tag{5}$$



Several alternative functions are useful for various reasons. The most common of which are Softmax and reLU functions.

Rectified Linear Activation Unit, (ReLU):

$$Z^{(5)} \;=\; ReLU \;= \begin{cases} x \geqq 0 & y = x \\ x < 0 & y = 0 \end{cases} \tag{6}$$

Binary Output Or Probability

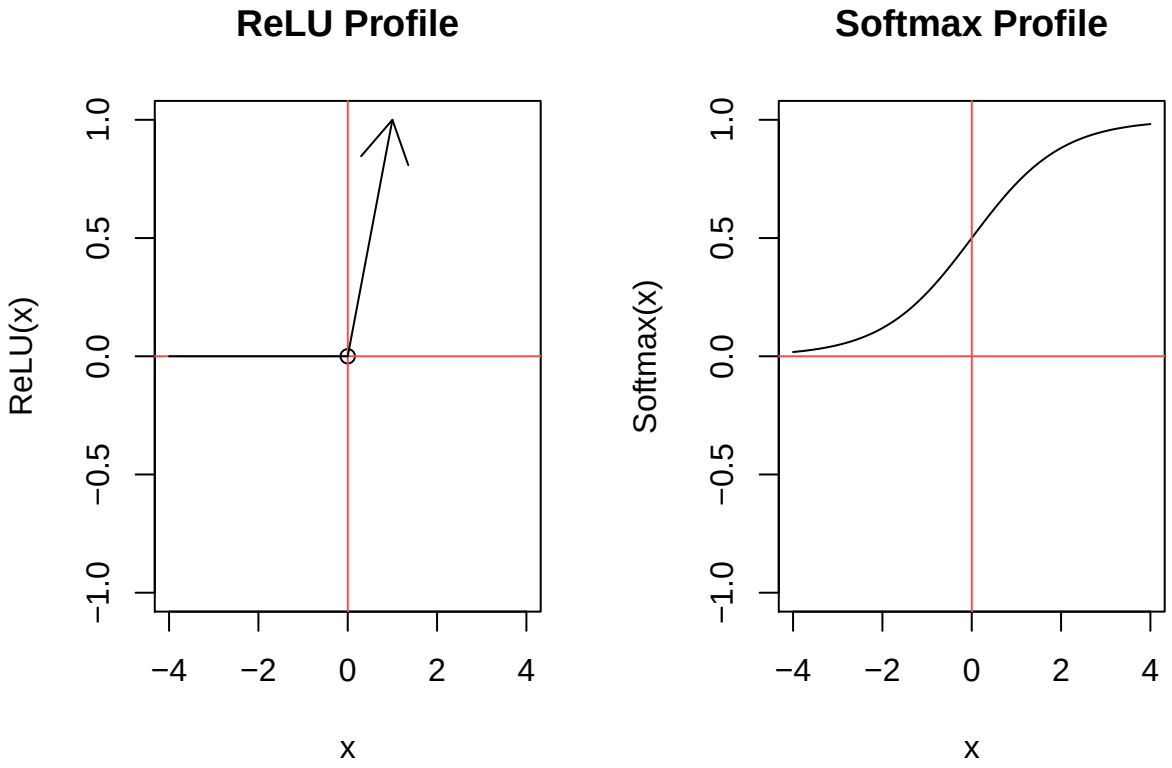
In the case of real neurons, the output is off or on, zero or one. However, in the case of our electronic model, it is advantageous to calculate a probability for greater interpretability.

The Softmax function may appear like the Sigmoid function from above but it differs in major ways.[9]

- The softmax activation function returns the probability distribution over mutually exclusive output classes.
- The calculated probabilities will be in the range of 0 to 1.
- The sum of all the probabilities is equals to 1.

Typically the Softmax Function is used in binary or multiple classification logistic regression models and in building the final output layer of neural networks.

$$Z^{(6)} = Softmax(x) = \frac{e^{\alpha_i}}{\sum_{i=1}^{n} e^{\alpha_i}} \tag{7}$$



The benefit of these activation functions is that they are now differentiable. This fact becomes important for *Back-Propagation*, which is discussed later.

[9] Josh Patterson, Adam Gibson, Deep Learning; A Practitioner's Approach, 2017, O'Rreilly

The Two Neuron System

Building up in complexity, let us could consider our first Neural Network by using *only* two neurons. In two neuron systems, let us first generalize a bit more by adding that X is an array of all the inputs as is W_1 and W_2 is also an array of weights for each neuron. See figure #5.

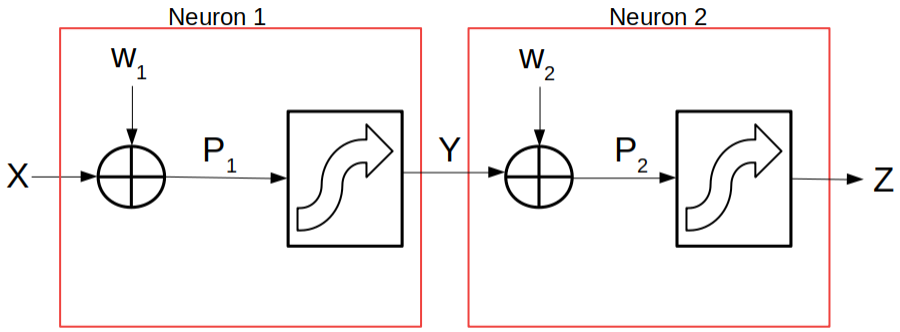


Figure 5: A Two Neuron System

Feed-Forward In A Two Neuron Network ($X \Longrightarrow Z$)

In our two neuron network, we can now write out the mathematics for each step as it progresses in a "forward" (left to right) direction.

Step #1: To move from X to P_1:

$$f^1(\vec{x}, \vec{w}) \equiv \quad P_1 = \left(X^T \cdot W_1 - T\right) \tag{8}$$

Step #2: P_1 feeds forward to Y:

$$f^2(P_1) \quad \equiv \; Y = \left(\frac{1}{1+e^{-\alpha}}\right) \quad : \quad where \quad \alpha = P_1 \tag{9}$$

Step #3: Y feeds forward to P_2:

$$f^3(\vec{y}, \vec{w}) \quad \equiv \; P_2 = \left(Y^T \cdot W_2 - T\right) \tag{10}$$

Step #4: P_2 feeds forward to Z:

$$f^4(P_2) \quad \equiv \; Z = \left(\frac{1}{1+e^{-\alpha}}\right) \quad : \quad where \quad \alpha = P_2 \tag{11}$$

Our complicated function is simply a matter of chaining one result so that it may be used in the next step.

$$Z \;=\; f^4\left(f^3\left(f^2\left(f^1\left(X, W\right)\right)\right)\right) \tag{12}$$

In our **Feed-Forward Model**, we can now take the values from any numerical system and produce zeros, ones, or probabilities. Remember, in this set of experiments, we are using the concentrations of the amino acids to provide a categorical or binary output, belongs to Myoglobin protein family, or does not.

Back-propagation ($Z \Longrightarrow X$)

Now that we have learned to calculate the output of our neurons using the Feed-Forward process, what if our final answer is incorrect. Could we build a feed back system to determine the weights needed to obtain our desired value of z? The answer is yes. The process for determining the weights is known as Back-Propagation. Back-Propagation, sometimes called error back-propagation, is crucial to understanding and tuning a neural network.

Simple stated Back-Propagation is an optimization routine which iteratively calculates the errors which occur at each stage of a neural network. Back-Propagation uses the partial derivatives of the feed forward functions, specifically the chain rule and gradient descent to determine the errors which are propagated through the network to find weights used in the summation step of a neuron.

This thumbnail sketch gives the building blocks to calculate W which can be run until we reach a value that we desire.

In the two neuron system, our first step is to generate an error or performance (Perf) function to minimize. If we call d our desired value, we can minimize the square error, a common choice.

Step #1:

$$\textbf{Performance} \quad = \quad c \cdot (d - z)^2 \tag{13}$$

Step #2:

$$\frac{d\ Performance}{dx} \quad = \quad \frac{d\left\{f^4\left(f^3\left(f^2\left(f^1\left(X, W\right)\right)\right)\right)\right\}}{dx} \tag{14}$$

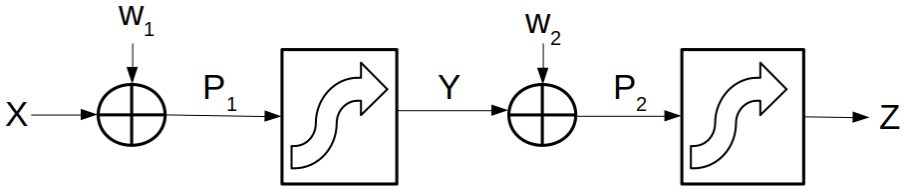


Figure 6: A Two Neuron System

Using the chain-rule and the 'Two Neuron System' figure above as a guide us through the back-propagation, we find:

Step #3: Neurons 2 $\Rightarrow$ 1

$$\frac{\delta\ Performance}{\delta w_1} \quad = \quad \frac{\delta Perf}{\delta z} \cdot \frac{\delta z}{\delta P_2} \cdot \frac{\delta P_2}{\delta y} \cdot \frac{\delta y}{\delta P_1} \cdot \frac{\delta P_2}{\delta w_1} \tag{15}$$

Step #4: Performance

$$\frac{\delta Perf}{\delta z} \quad = \quad \frac{\delta\left\{\frac{1}{2}\| \overrightarrow{d} - \overrightarrow{z} \|^2\right\}}{\delta z} \quad = \quad \overrightarrow{d} - \overrightarrow{z} \tag{16}$$

Step #5: If we substitute $P_2 = \alpha$

$$\frac{\delta z}{\delta P_2} \quad = \quad \frac{\delta\left(\left(1 + e^{-\alpha}\right)^{-1}\right)}{\delta \alpha} \quad = \quad e^{-\alpha} \cdot \left(1 + e^{-\alpha}\right)^{-2} \tag{17}$$

Step #6: If we rearrange the expression:

$$\frac{e^{-\alpha}}{\left(1 + e^{-\alpha}\right)^{-2}} \quad = \quad \frac{e^{-\alpha}}{1 + e^{-\alpha}} \cdot \frac{1}{1 + e^{-\alpha}} \tag{18}$$

Step #7: Then add 1 *and* subtract 1

$$= \frac{(1+e^{-\alpha})-1}{1+e^{-\alpha}} \cdot \frac{1}{1+e^{-\alpha}} \tag{19}$$

Step #8: Rearrange to find:

$$= \left(\frac{1+e^{-\alpha}}{1+e^{-\alpha}} - \frac{1}{1+e^{-\alpha}}\right)\left(\frac{1}{1+e^{-\alpha}}\right) = \left(1-\frac{1}{1+e^{-\alpha}}\right)\left(\frac{1}{1+e^{-\alpha}}\right) \tag{20}$$

Step #9: Therefore we find

$$\frac{\delta z}{\delta \alpha} = \frac{\delta\left((1+e^{-\alpha})^{-1}\right)}{\delta \alpha} = \left(1-\frac{1}{1+e^{-\alpha}}\right)\left(\frac{1}{1+e^{-\alpha}}\right) \tag{21}$$

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[10]Ivan N. da Silva, Danilo H. Spatti, Rogerio A. Flauzino, Luisa H. B. Liboni, Silas F. dos Reis Alves, Artificial Neural Networks: A Practical Course, DOI 10.1007/978-3-319-43162-8, 2017