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Numerical Methods HW #3

like Jacobi but use updated values as soon as possible

1.68.) Solve by Gauss-Seidel iteration:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 8 \end{bmatrix}$$

I

$2x^{(1)} - 1(0) = 5$

$x_1^{(1)} = \frac{5}{2}$

$-1(x_1^{(1)}) + 2x_2^{(1)} = 1$

$-\frac{5}{2} + 2x_2^{(1)} = 1$

$x_2^{(1)} = \frac{7}{4}$

$0(x_1^{(1)}) - x_2^{(1)} + 2x_3^{(1)} - 0 = 0$

$-\frac{7}{4} + 2x_3^{(1)} = 0$

$x_3^{(1)} = \frac{7}{8}$

$-\frac{7}{8} + 2 \cdot x_4^{(1)} = 8$

$x_4^{(1)} = 74/16 = 4.625$

$2(x_1^{(2)}) - 1(\frac{7}{4}) = 5$

$x_1^{(2)} = \frac{5 + \frac{7}{4}}{2} = 3.375$

$-3.375 + 2(x_2^{(2)}) - 1(\frac{7}{8}) = 1$

$x_2^{(2)} = 2.6265$

$-2.6265 + 2(x_3^{(2)}) - \frac{74}{16} = 0$

$x_3^{(2)} = 3.62575$

$-3.62575 + 2(x_4^{(2)}) = 8$

$x_4^{(2)} = 5.812875$

$2x_1^{(3)} - 1(2.6265) = 5$

$x_1^{(3)} = 3.81325$

$-3.81325 + 2x_2^{(3)} - 3.62575 = 1$

$x_2^{(3)} = 4.2195$

$-4.2195 + 2(x_3^{(3)}) - 5.812875 = 0$

$x_3^{(3)} = 5.0161875$

$-5.0161875 + 2(x_4^{(3)}) = 8$

$x_4^{(3)} = 6.50809$

$2(x_1^{(4)}) - 4.2195 = 5$

$x_1^{(4)} = \cancel{4.60975} = 5.632813$

$-4.60975 + 2(x_2^{(4)} - 5.0161875 = 1$ ...

4	x_1	x_2	x_3	x_4
1	2.5	1.75	0.975	4.4375
2	3.375	2.6265	3.75	~~5.812875~~ 5.875
3	3.81325	4.8001	6.265 ~~5.016875~~	6.50809
4	5.6328	9.386719	8.210937	8.105
5	7.1933	12.3076	10.2065	9.10372

...

successive over-relaxation

$0 < \omega < 2$

1.73) Solve by SOR Method with $\omega = 1.25$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 8 \end{bmatrix}$$

$$x_i^{(k+1)} = \omega \underbrace{x_i^{(k)}}_{\text{gauss seidel}} + (1-\omega)\underbrace{x_i^{(k)}}_{\text{last gauss seidel}}$$

$\omega = 1 \rightarrow$ Gauss Seidel

$\omega > 1 \rightarrow$ SOR

$\omega < 1 \rightarrow$ SUR

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$
0	0	0	0	0
1	3.125	2.578125	1.6113	6.0382
2	3.95508	4.4666	6.1432	7.3377
3	4.9278	10.2672	9.4673	9.0826
4	8.3101	15.0861	12.7386	10.6910
5	10.4767	19.3295	15.5750	12.0616

$$x_1^{(1)} = \omega\left(\frac{5 + 1(x_2^{(0)})}{2}\right) - (1-\omega)(x_1^{(0)})$$

$$x_1^{(1)} = 1.25\left(\frac{5}{2}\right) + (0.25)(0)$$

$$\underline{x_1^{(1)} = 3.125}$$

$$x_2^{(1)} = 1.25\left(\frac{1 + 3.125 + 0}{2}\right) - 0.25(0)$$

$$\underline{x_2^{(1)} = 2.578125}$$

$$x_3^{(1)} = 1.25\left(\frac{2.578125}{2}\right) - 0.25(0)$$

$$\underline{x_3^{(1)} = 1.6113}$$

$$x_4^{(1)} = 1.25\left(\frac{1.6113 + 8}{2}\right) + 0$$

$$\underline{x_4^{(1)} = 6.0382}$$

...