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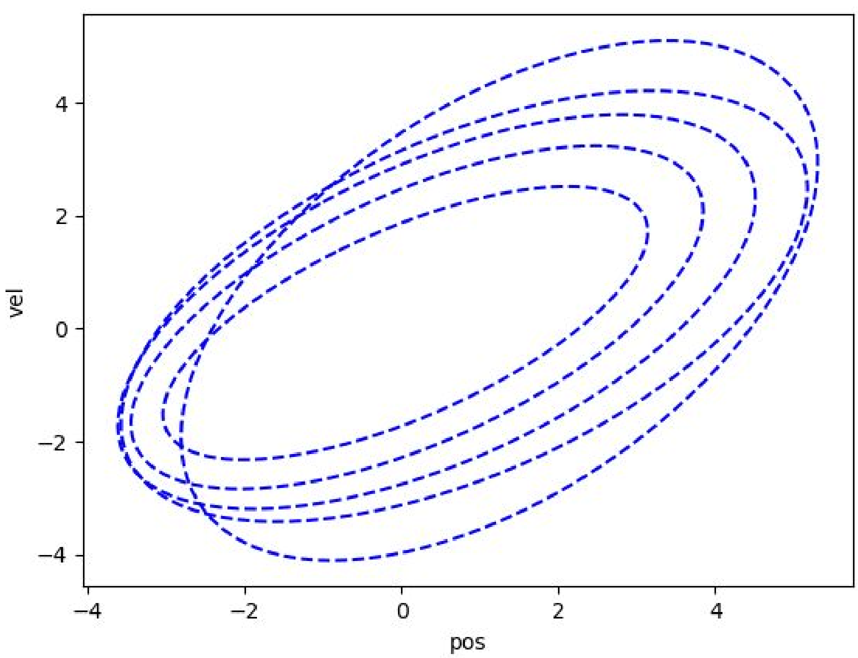
HW1 Corrections

1.1)
A Markov model assumes that future states depend only on the current state. The minimal state vector required to create a markovian system in the case of a 1 Dimensional Sailboat is:
X(t) = [[x],
[xdot]] where x is the position and xdot is the velocity.

1.2)
$P(x_t|u_t,x_{t-1}) = A*x_{t-1} + B*u_t + R$
R = Process_Noise*[[0.5**N*(0.0,1.0),
[*N*(0.0,1.0)]]
A = [[1 , 1]
[0, 1]]
B = [[0.5],
[1]]
The state transition function relates the current state vector to the previous through matrix A, and adds the effects of acceleration to each of the two states accordingly through matrix B.

1.3)



The state distributions for times 1 through 5 are pictured above. At each timestep, the length of the major axis of the ellipse is calculated as 2*sqrt(lambdaX) lambdaX is the larger of the two eigenvalues of the noise covariance matrix P. Similarly the minor ellipse length is calculated as 2*sqrt(lambdaY) where lambdaY is the smaller of the two eigenvalues of the error covariance matrix *P*. The angle of rotation of each ellipse is calculated as arctan(a) where a is the eigenvector associated with the larger eigenvalue.

2.1)

Matrix C is used to get the value of position from the state vector x. Because position is the first element of the state vector, C is simply [1,0], thus computing the dot product of C and X and adding some gaussian noise to account for noise in equipment will sufficiently simulate a GPS measurement.

Matrix Q describes the process noise covariance or how confident we are that the model describes the real world process it represents. In the equation for calculating Error Covariance, $P_k = AP_{k-1}A^T + Q$, Q exists to add a baseline uncertainty to the error covariance, meaning that as Q is increased, the value for P_k becomes less certain and thus the range in which the true state vector is most likely to exist within must be increased.

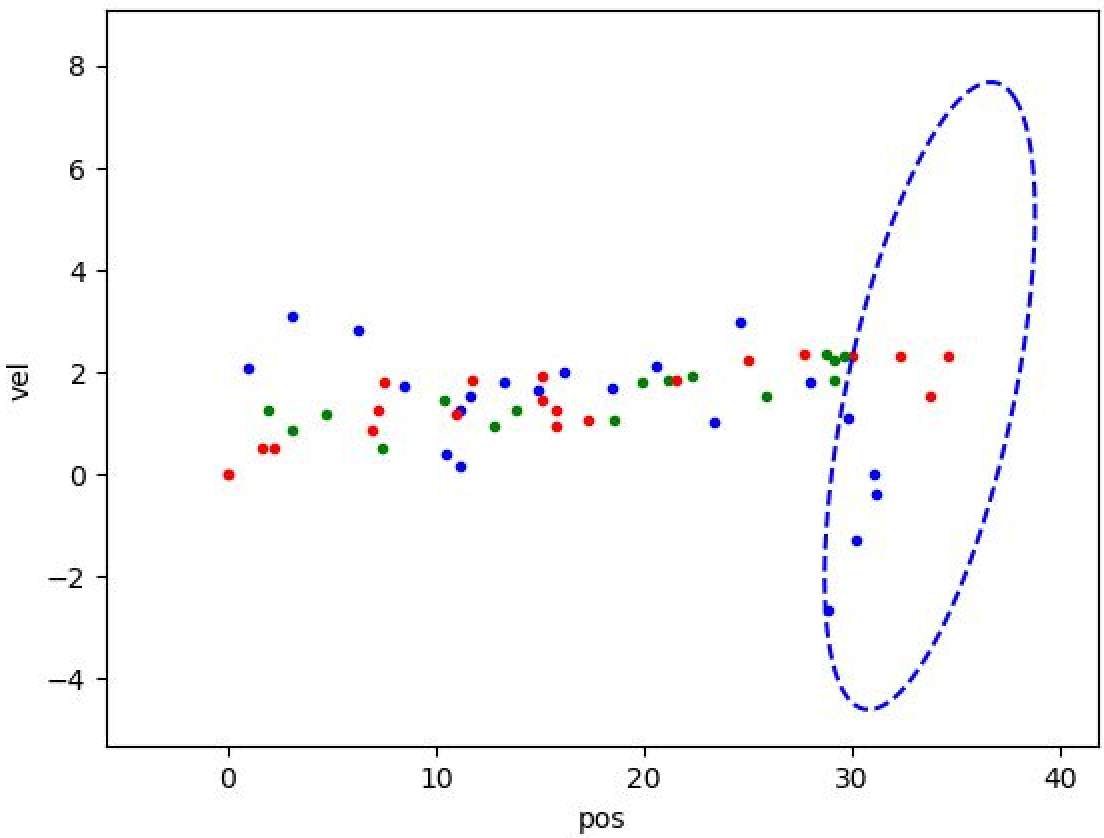


Fig. 2- Position vs Velocity, no control input given.
Blue = actual position, Green = GPS Measurement, Red = Estimated states

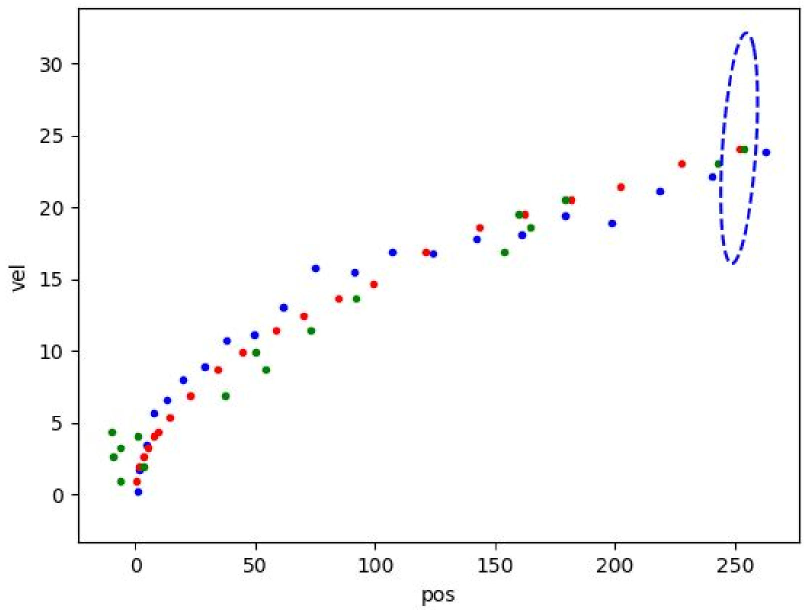
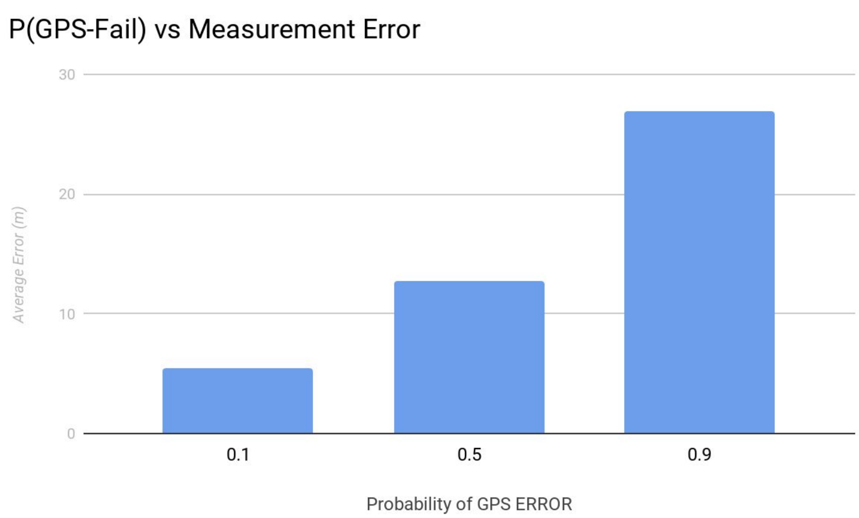


Fig. 3- Position vs Velocity, constant control input u = 1
Blue = actual position, Green = GPS Measurement, Red = Estimated states

2.3)



The simulation was repeated with the GPS sensor having a random chance of not collecting a measurement at each timestep. At the end of 20 timesteps, the final error between the estimated position and actual position was recorded and averaged across 10 trials for each respective GPS failure probability. The large error in the $P_{GPS\text{-}Fail}$=0.9 trials makes sense because it is very difficult for the algorithm to estimate the state vector of the boat when on average there are only 2 measurements across all 20 timesteps.