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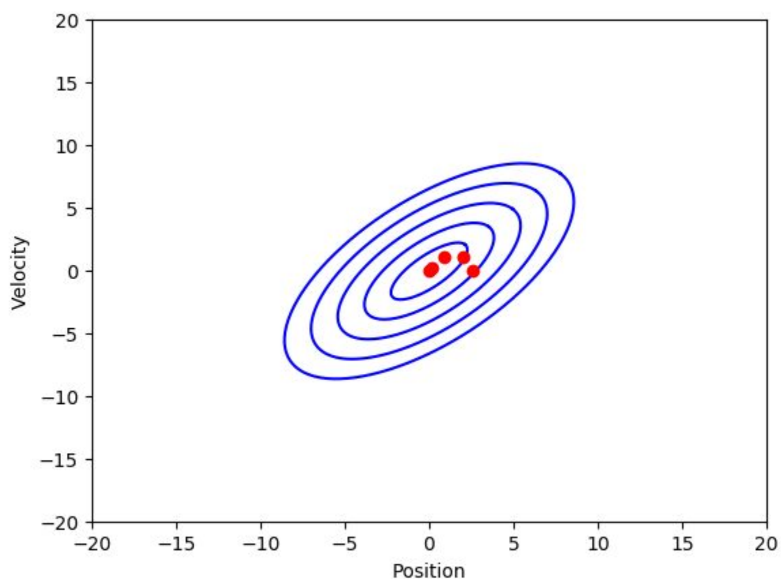
COMP-150 HW#1

1.1)
A Markov model assumes that future states depend only on the current state. The minimal state vector required to create a markovian system in the case of a 1 Dimensional Sailboat is:
X(t) = [x,dx,ddx] where x is the position, dx is the velocity and ddx is the acceleration.

1.2)
$P(x_t|u_t,x_{t-1}) = A*x_{t-1} + B*u_t + R$
R = $\mathcal{N}(0.0,1.0)$
A = [[1 , 0]
[0, 1]]
B = [[0.5],
[1]]

The state transition function relates the current state vector to the previous through matrix A, and adds the effects of acceleration to each of the two states accordingly through matrix B. The covariance of noise between the two states is handled by R.
1.3)



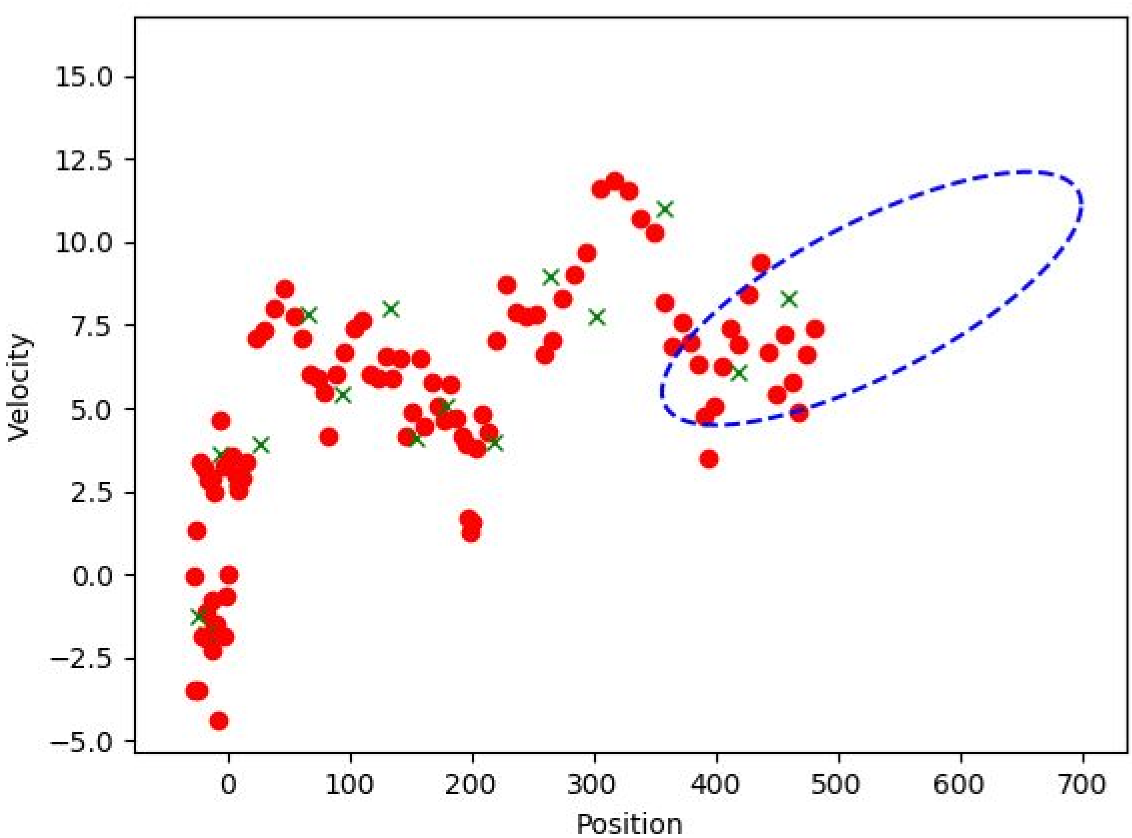
The state distributions for times 1 through 5 are pictured above. The major length of the ellipse is calculated as 2*sqrt(s*lambdaX) where s is a constant obtained from the table of Chi-Square Probabilities as a function of the number of degrees of freedom (number of timesteps since measurement) and lambdaX is the larger of two eigenvectors of the noise covariance matrix. Similarly the minor ellipse length is calculated as 2*sqrt(s*lambdaY) where lambdaY is the smaller of the two eigenvectors of the noise covariance matrix.

Upon visual inspection, the ellipse has an initial rotation angle of pi/4 due to the fact that early on it is impossible to exist in a state with a large positive velocity and a large negative position or vice versa. As the simulation continues, the bounding ellipses representing states within a single standard deviation transition to a lesser rotation angle.

2.1)

The location of the sailboat is measured once every n timesteps. Measurements taken by the GPS have standard deviation of 8 units from the actual position. In order to obtain an estimate for the position of the boat, a weighted average is taken between the most recently measured GPS position and the most up to date prediction of where the boat is likely to be at the current timestep. The weights of these two terms are calculated as a function of the variance of each respectively. C is the measurement covariance matrix, which represents how much error in the measurement of one state will affect values of another.
Q represents the process noise covariance matrix and is calculated using a Chi-Squared table. In this case, the longer the system goes without GPS measurement, the larger Q will grow.

2.2)



In this visualization, the red dots represent the boat's location at each timestep and each green x marks a point in which the GPS measured a location. Because the GPS is incapable of measuring velocity, the velocity of the sample point was estimated as the distance traveled since the last GPS measurement divided by the number of timesteps since. This average velocity value was fed forward into the the model and was used to update the position of the estimation ellipse by adding the estimated velocity to the position for each following timestep. This allowed the ellipse to move relatively smoothly despite the chaotic GPS measurements.

Additionally, just as calculated in the first part of this exercise, the longer the boat moves without a GPS measurement to track its location, the larger the ellipse will grow. The angle of rotation of the ellipse is determined by the arctan of the eigenvectors of the noise covariance matrix.

2.3)

In situations in which there is a high probability of the weather obscuring the GPS signal, estimations trend towards the those seen in the first part of this exercise with no GPS at all. One issue, however, is that infrequent GPS readings can result in highly erratic velocity measurements. This can lead to extremely large error as a perceived sudden increase in velocity can shift all predictions going forward. For that reason, the weights placed on the initial position of (0,0) must have a larger scaling factor than those attached to the GPS measurements such that over time, old GPS measurements hold less and less meaning.