SHARED LIQUIDITY AMM OF MAI PROTOCOL V3

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1. Background

MCDEX is a decentralized derivatives exchange of perpetual swaps, whose price anchors the spot index. AMM is essential for the perpetual swaps due to the following reasons: It provides an on-chain interface; it serves as the universal counterparty; and it uses funding payment to adjust the premium/discount to a user-acceptable level.

1.1 Structure

When a trader enters long or short position of ΔN contracts at a certain entry price P_{entry}, $\Delta N > 0$ indicates that the trader longs, $\Delta N < 0$ indicates that the trader shorts. The PNL (Profit and Loss) is calculated as follows:

$$(P_{mark} - P_{entry})\Delta N \tag{1}$$

P_{mark} is the mark price provided by Oracle. P_{mark} is usually equal to the index price P_{index} or is a TWAP result of P_{index} when using some decentralized Oracle like Uniswap. The profit of the MCDEX perpetual position can be withdrawn at any time, i.e. "PNL" always refers to its realized state. Trader can close position at an exit price P_{exit}. The PNL after the trader closes the position is:

$$(P_{exit} - P_{entry})\Delta N \tag{2}$$

For instance, if a trader enters 1 short position at $P_{entry} = 100$ and $\Delta N = -1$, then when the trader close the position at $P_{exit} = 90$, the PNL will be 10.

1.2 Margin Trade of Perpetual Swap

Every perpetual swap account has a net position N and cash M_c. $N > 0$ when the account holds longs, and $N < 0$ when the account holds shorts. If the fill price for this trade is P, the trader's margin account will get updated:

$$M_c \leftarrow M_c - P\Delta N \tag{3}$$

$$N \leftarrow N + \Delta N \tag{4}$$

Let the initial $M_c = 1000$, initial $N = 0$ for AMM, and AMM is the seller in a certain trade, the fill price is $P = 100$, trading volume $\Delta N = -10$, then $M_c = 2000$, $N = -10$ after the trade.

Perpetual Swaps allow leverage through calculating M_b, the margin balance and M_p, and the position value of the account. Defined as:

$$M_b = M_c + P_i N \tag{5}$$

$$M_p = |P_i N| \tag{6}$$

Thus, the leverage of the margin account is $\frac{M_p}{M_b}$.

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P_i is the index price. In the example above, if P_i =100, then $M_b = 1000$, $M_p = 1000$, the leverage is 1. In perpetual swaps, there is a safe domain for the leverage. If the leverage is too high, the margin account will be liquidated.

The margin balance M_b already accounts for the PNL of position. In the calculation above, if the index price $P_i = 90$, then $M_b = 1100$. The margin balance has increased due to the profit from going short. In such way, traders can profit before closing.

1.3 The Design Goal of AMM V3

AMM is the counterparty of all trades. Major questions to think about while designing an AMM include how to improve the capital efficiency and the expected profit of LP (Liquidity Provider).

To improve the capital efficiency, AMM should decrease the slippage to attract more traders. Our resolution is the assembling of more funds near the index price. Furthermore, MCDEX involves the concept of “shared liquidity pool” - multiple AMMs with the same collateral can use one liquidity pool.

The profit of LP has to be maximized in order to attract more LP to providing liquidity. There are two ways to increase LP profit: One is to increase trader’s trading fee, and the other one is to reduce arbitrage opportunities. The MCDEX AMM V3 is more inclined towards the second one. MCDEX AMM V3 also applies other means to increase LP profit, including but not limited to the adjustments of spread and slippage, and the funding payment which is unique in perpetual swaps.

2. The Design of AMM V3

In this chapter we will define the pricing function of MCDEX AMM V3, analyze how this function manages AMM’s risk, decreases arbitrage opportunities, and designs a funding rate such that it increases LP profit.

2.1 Definition

Variable	Definition
P_i(Index Price)	Index Price. From Oracle.
P_b(Base Price)	AMM’s base price without including additional charges.
N(Net position)	AMM Net Position. $N > 0$ means that AMM is on long position, $N < 0$ means that AMM is on short position.
M_c(Cash)	The perpetual swap margin of AMM deposited by all LPs.
M_b(Margin Balance)	Margin balance of the AMM’s account. $M_b = M_c + P_i N$.
M_p(Position value)	Position value. $M_p = \|P_i N\|$.
M(pool Margin)	The evaluation of the value of shared liquidity pool. Check formula (8).
R_f(Funding Rate)	Funding rate. When $R_f > 0$, long position pays funding fee to short position. When $R_f < 0$, short position pays funding fee to long position.
S(Share)	Total amount of pool share.
$\mathcal{M}$(Market set)	The market set in a shared liquidity pool.

m(current Market)	The current trading market in the shared liquidity pool. $m \in \mathcal{M}$.
T_{hold}(Holding Time)	Position holding time.
T_{8h}(8 hours)	8 hours.
α(Spread)	Risk parameter: half spread. $0 \le \alpha < 1$.
β_1(Open Slippage)	Risk parameter: slippage. $0 < \beta_2 \le \beta_1$. A bigger β_1 increases slippage when open position.
β_2(Close Slippage)	Risk parameter: slippage. $0 < \beta_2 \le \beta_1$. A bigger β_2 increases slippage when close position.
γ(Funding)	Risk parameter: funding rate factor. $\gamma \ge 0$. A bigger γ increases the funding rate.
Γ(maximum Funding)	Risk parameter: funding rate limit. $\Gamma \ge 0$. A bigger Γ increases the maximum funding rate.
δ (max Discount)	Risk parameter: max discount of price when close position.
ϕ(Fee)	Risk parameter: trading fee. In practice, the trading fee consists of Vault Fee, Operator Fee and LP Fee.
λ(max Leverage)	Risk parameter: target leverage of AMM's margin account.

2.2 MCDEX AMM V3 Pricing Strategy

MCDEX AMM V3 uses a price function to provide more liquidity near the index price. It uses a risk control function to adjust the spread and slippage. Meanwhile, AMM applies a virtual margin to further improve the capital efficiency. Finally, AMM provides the funding rate to adjust the premium/discount.

Price function. The AMM V3's price is based on the index price P_i and its risk exposure, AMM completes base average fill price:

$$\overline{P_b}(N_1, \Delta N) = P_i \left(1 - \beta \frac{P_i}{M} \frac{2N_1 + \Delta N}{2}\right) \tag{7}$$

$$M = \frac{1}{2}\left(M_b + \sqrt{{M_b}^2 - 2\sum_{j \in \mathcal{M}} \beta_j P_{i_j}^2 {N_1}_j^2}\right) \tag{8}$$

$$M_b = M_c + \sum_{j \in \mathcal{M}} P_{i_j} {N_1}_j \tag{9}$$

If a trader trades $-\Delta N$ contracts, AMM trades ΔN contracts. $\overline{P_b}$ is the average fill price. P_i is the index price provided by Oracle. β is the slippage parameter satisfying $\beta > 0$, in which a bigger β indicates a bigger slippage. N_1 is AMM's position. M is the pool margin. This price function helps AMM follows the spot market price without arbitrageur. The slippage automatic decreases when LPs deposit more collateral.

MCDEX enables multiple AMMs to share the same liquidity pool. $\mathcal{M}$ is the set of markets in a shared liquidity pool. Each AMM with index j has its own position ${N_1}_j$ and shares the same margin M.

Risk control function. We believe that LP takes lots of risk when holding positions. Therefore, we add several risk parameters to control the slippage, spread, trading fees and max positions.

AMM applies different slippage parameter β when the risk exposure is increasing or decreasing. When trader buys/longs against AMM, AMM is the short side ($\Delta N < 0$). If AMM holds short ($N < 0$), the risk exposure is increasing. If AMM holds long ($N > 0$), the risk exposure is decreasing. The price will be:

$$\overline{P_\beta} = \begin{cases} \overline{P_b}(\beta_1) & if\ N < 0\ and\ \Delta N < 0 \\ \max\{\overline{P_b}(\beta_2), P_i(1-\delta)\} & if\ N > 0\ and\ \Delta N < 0 \end{cases} \tag{10}$$

δ is max discount when AMM closes positions.

When trader sell/short against AMM, AMM is the long side ($\Delta N > 0$). the price will be:

$$\overline{P_\beta} = \begin{cases} \min\{\overline{P_b}(\beta_2), P_i(1+\delta)\} & if\ N < 0\ and\ \Delta N > 0 \\ \overline{P_b}(\beta_1) & if\ N > 0\ and\ \Delta N > 0 \end{cases} \tag{11}$$

Note that formula (10) and (11) contains 2 segments respectively. A trading may cross 2 segments. For example, $N_1 = 10, \Delta N = -15$. The trading can be considered as 2 parts $\Delta \mathrm{N}_{seg1} = -10, \Delta N_{seg2} = -5$ and denote prices as $\overline{P_{\beta_{seg1}}}, \overline{P_{\beta_{seg2}}}$. The average price over segments is:

$$\overline{P_{VWAP}} = \frac{\overline{P_{\beta_{seg1}}}\Delta \mathrm{N}_{seg1} + \overline{P_{\beta_{seg2}}}\Delta \mathrm{N}_{seg2}}{\Delta N} \tag{12}$$

AMM also controls a price spread. The spread is the difference between best ask and best bid price. We define the best ask/bid price as:

$$P_{best}(\beta) = \begin{cases} P_{mid}(1+\alpha) & if\ \Delta N < 0 \\ P_{mid}(1-\alpha) & if\ \Delta N > 0 \end{cases} \tag{13}$$

$$P_{mid} = P_i\left(1 - \beta\frac{P_i N_1}{M}\right) \tag{14}$$

α is the half-spread parameter. P_{mid} is the mid-price of AMM which is determined by the current AMM position N_1. The final trading price is:

$$\bar{P} = \begin{cases} \max\{\overline{P_{VWAP}}, P_{best}\} & if\ \Delta N < 0 \\ \min\{\overline{P_{VWAP}}, P_{best}\} & if\ \Delta N > 0 \end{cases} \tag{15}$$

The figure below shows the curves of the prices.

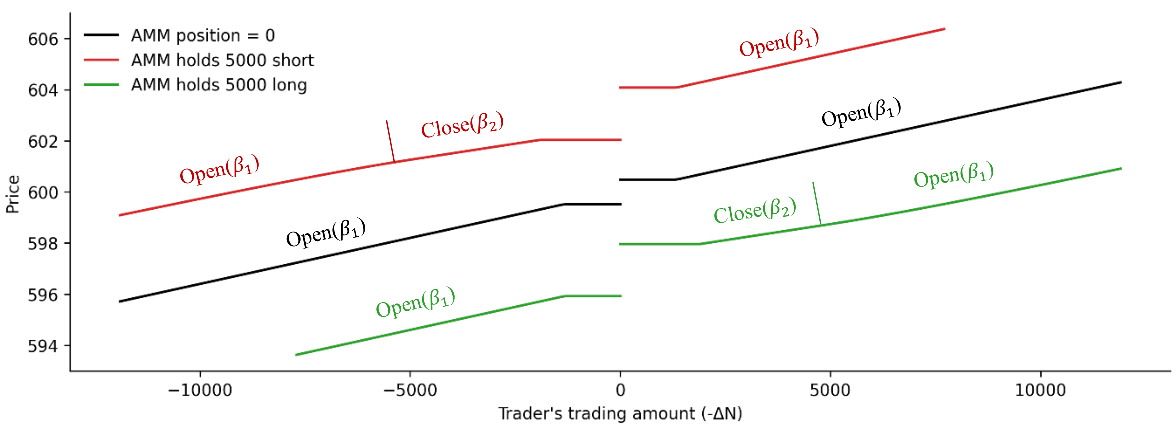


Figure 1: Fill price for Trader. The slippage increases as AMM risk increases. Vice versa, the slippage decreases as AMM risk decreases.

After adjusting the spread and slippage, the trader pays trading fee to the AMM with the fee rate ϕ. Let ΔN be the trading positions, the transaction of the fee can be described as:

$$M_c \leftarrow M_c + \phi P|\Delta N| \tag{16}$$

The maximum position of AMM is defined by calculating the AMM's position margin:

$$\sum_{j\in\mathcal{M}} \frac{P_{i_j}|N_j|}{\lambda_j} \tag{17}$$

λ_j sets AMM's max leverage of each market. AMM can open a position only if margin balance M_b is larger than the AMM's position margin. Otherwise, AMM will cease providing liquidity and opening positions.

Funding rate. AMM provides the funding rate to perpetual swap. Funding payment is a significant component when perpetual swap anchors the spot price. Funding rate is the interest rate of the funding payment. If funding rate is positive, the long side in the perpetual swap pays interest to the short side. Vice versa, if funding rate is negative, the short side pays interest to the long side.

Because AMM is the counterparty of all traders, if most traders go long, then AMM goes short ($N < 0$), and funding rate is positive. Otherwise, if most traders go short, then AMM goes long ($N > 0$), and funding rate is negative. If there is a balance in the market, then AMM has no position, and funding rate is zero. We define funding rate R_f as:

$$-\gamma\left(\frac{P_i N}{M}\right) \tag{18}$$

The funding rate R_f is hard limited by $\pm\Gamma$. γ is the coefficient of funding rate. Funding rate R_f is the ratio of position value that every position needs to collect per 8 hours. Hence if a trader holds N_1 amount of long position over ΔT amount of time, this trader needs to make a funding payment of:

$$R_f P_i N_1 \frac{\Delta T}{T_{8h}} \tag{19}$$

Among which T_{8h} =8 hours.

Bear in mind that the funding rate provided by AMM in formula (18) is always into the favorable direction for AMM. Specifically, AMM collects funding payments when it longs or shorts. AMM does not collect funding payment only when there is a tie (AMM has no position).

2.3 Features of the Pricing Strategy

In formula (7) we defined the base price of AMM V3. The formula has 3 major features: 1) The introduction of index price P_i, so that AMM's price automatically follows the market price; 2) The assembling of liquidity near the index price, decreasing the slippage; 3) The solely dependent on margin balance to avoid the situation in which LP provides two types of assets.

The introduction of index price. This function introduces index price P_i to direct AMM pricing. For instance, when the spot price increases drastically, AMM would not have to rely on arbitrageurs to give price but make use of the updated market price instead. In such way arbitrage opportunities can be reduced to avoid unnecessary loss in LP investment.

Slippage increases along with the increment of risk exposure. When the net position N held by AMM increases, the AMM risk exposure will increase as well. Thus, the AMM pricing will gradually moves away from index price P_i, and the slippage depends on the current utilization rate $\frac{P_i N}{M}$. When the utilization rate is low, the price will be close to P_i. Otherwise, the slippage will increase.

Slippage is decreased near the index price. AMM V3 introduces the slippage parameter β to drastically decrease the slippage. In the constant product AMM function, the slippage is relatively high in order to make sure that AMM covers 0 to ∞ price range. The V3 AMM function flattens the price curve near index price. While it improves user experience, capital efficiency is also maximized so that the fund provided by LP could be concentrated to fulfill the trading demand near the index price.

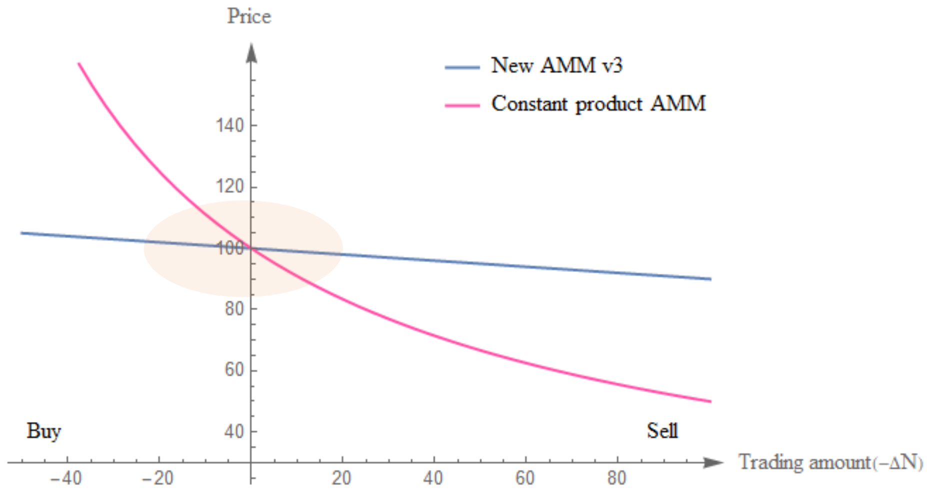


Figure 2: Comparison with the constant product formula. The blue line is the AMM V3 price which decreases the slippage.

LP only provides collateral into margin account. In perpetual swap, trader and LP only need to provide collateral into margin account but not the underlaying asset. This differentiates perpetual swap from spot trade.

In an ETH-USDC perpetual swap, the collateral is USDC, and the underlying asset is ETH. When trader longs or shorts, the fluctuation of PNL is reflected by the collateral USDC in the margin account instead of ETH.

An essential characteristic of AMM V3 is that LP only needs to provide the collateral token to add liquidity. This characteristic simplifies the operation of LP and calculation of PNL. With constant product AMM, LP needs to provide two types of tokens, hence the PNL of LP depends on the value of these two assets, making the PNL calculation of LP unclear.

Limited Market Making Depth. The market making depth of AMM V3 is limited by the max leverage. When AMM's position reaches the upper limit, AMM will only provide one side of liquidity. For example, if the long position of AMM reaches the limit, AMM will refuse the trader to sell anymore.

2.4 LP Profit and Risk Exposure

LP profit mainly comes from spread, funding payment, and transaction fee.

Spread. There is a spread between the AMM's best bid and best ask. When traders and arbitrageurs trade against AMM, this spread will cause AMM to profit. As N increases, AMM's risk increases. A higher risk is designed to result in a larger spread, which then lead to a higher profit for LP.

Funding payment. Funding payment is the interest that trader pays to LP when trader goes long or short. When the long side and the short side happen to be identical, it can be interpreted as the two sides lending money from each other and the interests cancel out. When the amount of long side and short side are different, AMM is the only asset lender. Therefore, AMM deserves the interest from net position.

Trading fee. Most AMMs charge fee at a fixed ratio when trader trades with AMM. MCDEX AMM V3 is inclined towards decreasing the trading fee and arranging the distribution of profit through the methods discussed earlier.

Other income. Technically LP receives liquidation penalty as well. But since it is a function of the perpetual swap, we will not discussion it in this passage.

The major risk of LP comes from the position it holds. If the index price P_i moves toward an unfavorable direction for AMM post trade, AMM will suffer a lost. Certain traders could take advantage of the publicity of AMM. MCDEX AMM V3 introduces the concept of an "operator". An operator is the creator of the perpetual swap as well as the one who manages risk parameters so that the risk on LP can be limited. In return, Operator receives part of LP's profit.

2.5 Trading Procedure of AMM

This section is a complete step-by-step trading procedure of AMM. If a trader trades $-\Delta N$ amount of perpetual swaps, AMM, the counterparty thus trades ΔN amount of perpetual swaps. Then:

1) Read price P_i from Oracle. Realize the funding payment using formula (19).
2) The margin M could change in circumstances like AMM has a PNL caused by the price change or AMM receives funding payment or trading fee. Use formula (36) to calculate this PNL and update the M.
3) Based on AMM position and trader's operating direction, there are 4 possibilities. Use formula (10) or (11) to choose slippage parameter β.
4) Use (7) to calculate the average fill price.
5) Make sure that the new position does not exceed the upper limit defined by formula (44).
6) Pay the fee. Use formula (16).

2.6 Peripheral

There are some peripheral issues with the pricing strategy.

2.6.1 Deposit

LP deposits collateral token to the liquidity pool to earn trading fees and market making profits. AMM's margin M increases and slippage decreases when LP deposits.

In order to track the proportion of LP in the pool, AMM mints share tokens when LP deposits into AMM. When LP withdraw from AMM, AMM burns share tokens and transfer collateral tokens back to the LP.

AMM keeps the increasing ratio of M as same as share tokens. Given the depositing collateral w and the total shares S, AMM mints s shares. The AMM's margin M satisfies:

$$\frac{M(M_c + w, N)}{M(M_c, N)} = \frac{S + s}{S} \tag{20}$$

$$s = \frac{M(M_c + w, N)}{M(M_c, N)} S - S \tag{21}$$

2.6.2 Withdrawal Penalty

Withdrawing from liquidity pool increases the slippage and causes LP to suffer losses. AMM reduces the losses by charging withdrawal penalty fees.

When LP withdraw from AMM, AMM first calculates the current margin $M_1 = M(M_c, N)$ and calculate the new margin $M_2 = M_1 \frac{S-s}{S}$. Then we can calculate the withdrawal collateral w:

$$w = M_c - M_2 + \sum_{j \in \mathcal{M}} P_{i_j} N_j - \frac{1}{2M_2} \sum_{j \in \mathcal{M}} \beta_j P_{i_j}^2 N_j^2 \tag{22}$$

If AMM's position is 0, all collateral of AMM can be withdrawn. But if the position is not 0, the w will be smaller. We consider this reduced collateral as withdrawal penalty.

3. Simulation

In order to examine the performance of AMM, a big component of which is the AMM PNL, we use the agent-base method to perform a simulation. For every experiment, LP deposits $2.5M at the beginning. There will be 43200 time steps, the duration of each time step is 1 min, and all the time steps together simulate the trend over a month. The price will be different in every time step, and each agent will make action according to the environment and their pre-designed strategy. The trading volume is $2.5M per day (1440 time steps).

3.1 Simulation of the environment

In this experiment, there are two factors that cause changes in the environment: the market price and the price given by Oracle. The changes of these two factors will result in interactions among agents.

Reference market: The reference market price P_m will update every time step, modeled by:

$$P_m \leftarrow P_m e^{\sigma X + \mu} \tag{23}$$

Here $X \sim \mathcal{N}(0,1)$ is drawn from a normal distribution and $\mu, \sigma \in \mathrm{R}$ represent the mean returns and volatility of the market when no trades are performed. We fit the σ of ETH and FIL by using the data of the past 6 months, which are 0.0008364 and 0.0059 respectively. ETH represents a coin type of conventional volatility, while FIL represents a type of high volatility.

We use the Monte Carlo method to evaluate the performance of AMM. Specifically, we have repeated 1000 random experiments to find the average income. Figure 3 and Figure 4 are samples that represent bull market, bear market, and monkey market.

Beside experimenting with simulated price, we backtested with real prices of BTC, ETH, FIL. Among them, the price of BTC and ETH over 2020/1/1 to 2020/10/26 come from CoinCap.io; the price of FIL over 2020/9/1 to 10/26 comes from CoinCap.io. Figure 5 shows the real market prices.

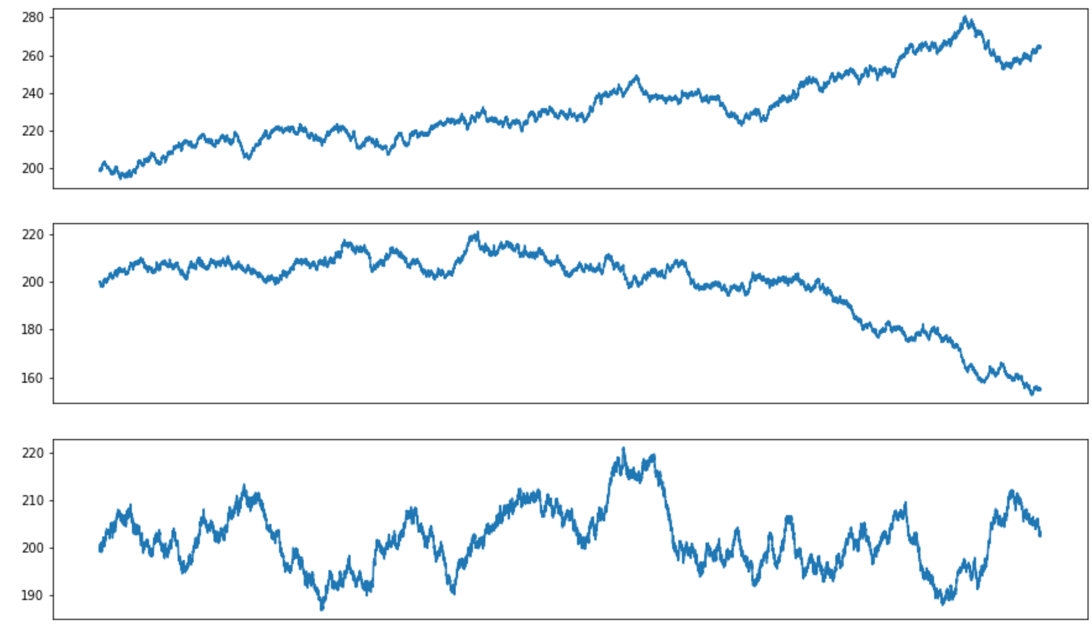


Figure 3: 3 samples of ETH simulations: Bull market, Bear market, and Monkey market

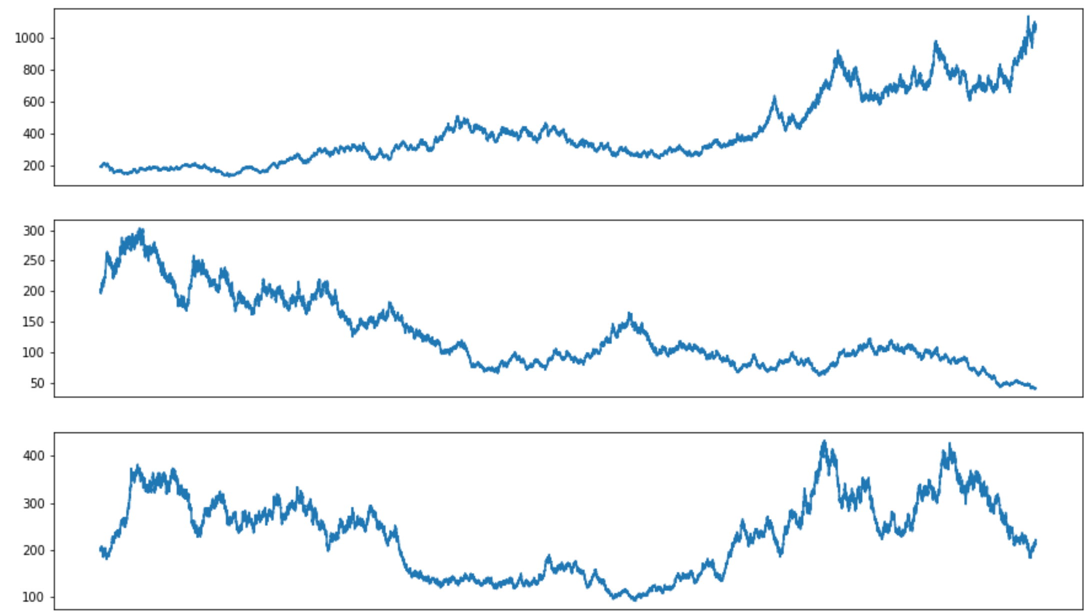
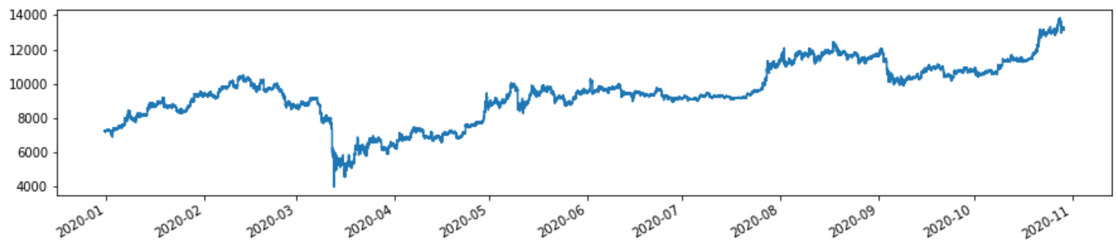


Figure 4: 3 samples of FIL simulations: Bull market, Bear market, and Monkey market



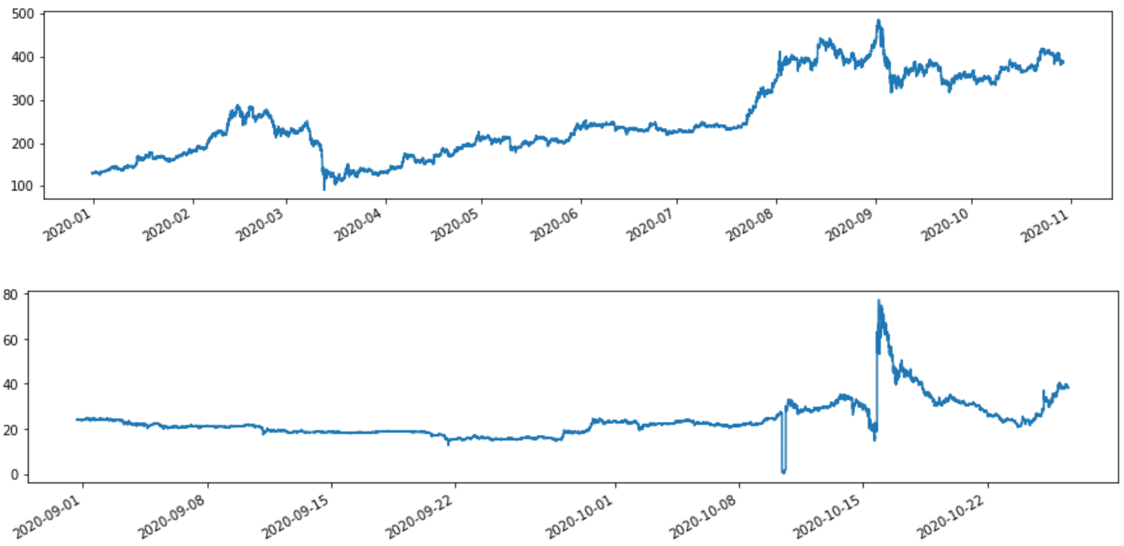


Figure 5: A top-down illustration of BTC, ETH, FIL real market price

AMM: AMM uses different configurations in markets under high volatility and conventional volatility. Table 1 shows AMM parameters we use.

Table 1: AMM parameters

Market	α	β_1	β_2	δ	γ	φ	λ
ETH	0.08%	0.008	0.0063	5%	0.5%	0.075%	3
FIL	0.2%	0.617	0.439	10%	5%		1

Oracle Price: The Oracle simulates the basic mechanism of Chainlink, which includes a derivation threshold and a heartbeat time limit. If the change of market price exceeds the derivation threshold or if the time passed since last update exceeds the time limit, the Oracle will price with the current market price. In the experiment, the derivation threshold is set to 0.1%, and the longest time limit is 3 hours.

3.2 Simulation of the agent

In this experiment, traders and arbitrageurs are both agents. Technically LP, who adds liquidity and withdraws liquidity, belongs to the agent category as well. For the sake of simplicity, we fix the behavior of LP to starting with adding liquidity and ending with withdrawing liquidity.

Trader: Trader will compare the AMM price with the market price P_m. Assume that in other markets, trades could be made at the market price, cost is $c = 0.075\%$, and an error e of 3% is allowed. If the buying price of AMM is smaller than $P_m(1+c)(1+e)$, or the selling price of AMM is bigger than $P_m(1-c)(1-e)$, there is a possibility that the trader would trade against AMM, and the trading volume follows the chi-square distribution. In a bull market, the will to buy is stronger than the will to sell; in a bear market, the will to buy is weaker than the will to sell; and in a monkey market, the two wills balance.

Arbitrageurs: When there is space for arbitrage, risk-free arbitrage activities take place in multiple markets (Here we do not take funding rate into consideration), and arbitrageurs receive their profit after closing. Meanwhile, arbitrageurs take AMM's risky position. Thus, arbitrageurs are inevitable in the system. The arbitrageurs try to solve the following optimization problem:

$$\mathrm{argmax}_{\Delta N}(f(\Delta N)) \tag{24}$$

f is the profit function that takes two situations into account. f_1 is the profit function when the arbitrageur sells in the reference market and buys from AMM. f_2 is the profit function when the arbitrageur buys in the reference market and sells to AMM. Arbitrageurs will choose the one with max profit when calculating their own profit.

$$f_1(\Delta N) = P_m \Delta N(1-c) - P(\Delta N)\Delta N \; if \; \Delta N > 0 \tag{25}$$

$$f_2(\Delta N) = P_m \Delta N(1+c) - P(\Delta N)\Delta N \; if \; \Delta N < 0 \tag{26}$$

If exists a ΔN for which $f(\Delta N)$ exceeds a minimum profit threshold, there is space for arbitrage.

3.3 Results

In this section, we simulated both single market AMM and shared liquidity AMM.

3.3.1 AMM with Isolated Liquidity Pool Simulation

The tables below show the simulation results. The "Trading Income" refers to the profit obtained from spread and slippage. The "Fee Income" refers to the profit obtained from trading fee. The "Funding Income" refers to the profit obtained from funding payment.

When there is not arbitrageur, AMM takes all the risk. We call it the "high-risk state". Table 2 and Table 4 show the PNL in this case. When there is arbitrageur, AMM takes a small part of the risk. We call it the "low-risk state". Table 3 and Table 5 show the PNL in this case. However, the reality is somewhere in between.

AMM performed well in both real markets and simulated markets under the stated conditions, so we can see its robustness to an extent. The following conclusions can be drawn: 1) AMM cannot guarantee profit only by trading income, it will only be profitable after receiving other fees. 2) AMM needs to set up its parameters according to the specific coin type with its own volatility and trading volume. In MCDEX V3, a concept of "operator" is introduced, and operator is responsible for adjusting the AMM parameters to protect the profit of LP. In the experiment, the choice of the two sets of parameters has balanced the profits of all sides.

Table 2: High Risk State in ETH Market Simulation

Market	Trading Income APY	Fee Income APY	Funding Income APY	Total Income APY
ETH real prices	-16%	15%	44%	43%
Random prices	-29%	15%	43%	29%
Simulated bull market	-27%	15%	47%	35%
Simulated bear market	-59%	15%	50%	5%
Simulated monkey market	10%	15%	15%	47%

Table 3: Low Risk State in ETH Market Simulation

Market	Trading Income APY	Fee Income APY	Funding Income APY	Total Income APY
ETH real prices	5%	15%	4%	24%
Random prices	11%	15%	4%	31%
Simulated bull market	-15%	15%	4%	4%
Simulated bear market	-3%	15%	4%	16%
Simulated monkey market	9%	15%	4%	28%

Table 4: High Risk State in FIL Market Simulation

Market	Trading Income APY	Fee Income APY	Funding Income APY	Total Income APY
FIL real prices	153%	15%	137%	305%
Random prices	220%	15%	114%	349%
Simulated bull market	103%	15%	117%	235%
Simulated bear market	-77%	15%	120%	28%
Simulated monkey market	130%	15%	68%	213%

Table 5: Low Risk State in ETH Market Simulation

Market	Trading Income APY	Fee Income APY	Funding Income APY	Total Income APY
FIL real prices	150%	15%	0.06%	165%
Random prices	11%	15%	4%	30%
Simulated bull market	190%	15%	0.01%	205%

Simulated bear market	79%	15%	0.05%	94%
Simulated monkey market	106%	15%	12%	133%

3.3.2 Shared Liquidity AMM Simulation

We simulated a ETH+BTC+FIL AMM shared the same liquidity pool. We apply the parameters of conventional volatility for the BTC and ETH; we apply the parameters of high volatility for the FIL. The behavioral model for both trader and arbitrageur remains the same as stated in section 3.2. Results are shown below, indicating a similar performance of AMM with isolated liquidity pool simulation.

Table 6: High Risk State in ETH+BTC+FIL Market Simulation

Market	Trading Income APY	Fee Income APY	Funding Income APY	Total Income APY
ETH+BTC+FIL real prices	-126%	15%	429%	318%
Random prices	-93%	15%	278%	200%

Table 7: Low Risk State in ETH+BTC+FIL Market Simulation

Market	Trading Income APY	Fee Income APY	Funding Income APY	Total Income APY
ETH+BTC+FIL real prices	19%	15%	19%	53%
Random prices	23%	15%	6%	44%

Appendix 1 The Average Fill Price

The AMM V3's price is based on the index price P_i and its risk exposure, AMM completes base price:

$$P_b(N) = P_i\left(1 - \beta \frac{P_i N}{M}\right) \tag{27}$$

During the transaction, the more important thing is the average fill price when trading ΔN positions. Let the AMM position before and after trade be N_1 and N_2. The average fill price can be considered as the used position value divided by the trading positions:

$$\bar{P}(N_1, N_2) = \frac{\int_{N_1}^{N_2} P_b(n)\, dn}{N_2 - N_1} = P_i\left(1 - \beta \frac{P_i}{M} \frac{N_1 + N_2}{2}\right) \tag{28}$$

When a trader trades $-\Delta N$ contracts, AMM trades ΔN contracts. So $N_2 = N_1 + \Delta N$. We can now express the average fill price as:

$$\bar{P}(N_1, \Delta N) = \frac{\int_{N_1}^{N_2} P_b(n)\,dn}{N_2 - N_1} = P_i\left(1 - \beta\frac{P_i}{M}\frac{2N_1 + \Delta N}{2}\right) \tag{29}$$

Appendix 2 Update M to Realize PNL Caused by Index Change

Assume no trading fees are taken, when P_i from Oracle remains unchanged, the trade does not affect the margin M, so the curve of the pricing function stays the same. But M will be affected in the following situations: AMM has the PNL when index price P_i changes; AMM receives funding payment; LP add or remove liquidity. Therefore, AMM needs to keep M updated.

For example, let's say that AMM holds long ($N > 0$), if the index price P_i decreases, there will be a loss on the long side. This loss will drag down M and the slippage will increase. If trader goes long to make AMM suffer a margin closeout, AMM will have to take another loss due to the change of slippage. These two losses will result in the loss of LP.

The mechanism of updating M is to simulate trades that make AMM close its position. The resulted M_c will be the new M. Assume the current position is N_1, and becomes $N_2 = 0$ after a simulated closing. Use (28) to calculate the change of M_c for each market. We get:

$$\begin{aligned} M &= M_c - \sum_{j\in\mathcal{M}} \int_{N_1}^{0} P_{b_j}(n)\,dn \\ &= M_c - \sum_{j\in\mathcal{M}} \left(P_{i_j}\left(1 - \beta_j\frac{P_{i_j}}{M}\frac{N_{1_j} + 0}{2}\right)\left(0 - N_{1_j}\right)\right) \end{aligned} \tag{30}$$

Solve equation (30) we get:

$$M = \begin{cases} \frac{1}{2}\left(M_b + \sqrt{\Delta}\right) & \Delta \geq 0 \quad (31) \\ \frac{1}{2}\left(M_b - \sqrt{\Delta}\right) & \Delta \geq 0 \quad (32) \\ No\ solution & \Delta < 0 \quad (33) \end{cases}$$

Among them $\Delta = M_b^2 - 2\left(\sum_{j\in\mathcal{M}} \beta_j P_{i_j}^2 N_j^2\right)$.

Due to the fact that margin balance is positive, $\Delta \geq 0$ can also be detected by checking M_c before every trading:

$$M_c \geq \sqrt{2\sum_{j\in\mathcal{M}} \beta_j P_{i_j}^2 N_j^2} - \sum_{j\in\mathcal{M}} P_{i_j} N_j \tag{34}$$

When $\Delta \geq 0$, there are 2 solutions, among which formula (32) has an unfavorable characteristic: if $N > 0$, M decreases when P_i increases, which is inappropriate for long position. So, formula (32) is disregarded and formula (31) is favored. AMM will avoid this situation by limiting the maximum position, we will discuss this method in Appendix 3.

When $\Delta < 0$, the huge loss of AMM makes it incapable of going back to the zero position status using β. The solution is to specify a β by the algorithm just like adding a slippage to the current trading market m. Meanwhile, the AMM would stop adding more position to prevent further loss. The purpose changing β is to allow AMM to provide single sided liquidity in dangerous situation. To get a safe β, let $\Delta = 0$ and we can get:

$$\beta_{safe} = \frac{M_b^2 - 2\left(\sum_{j \in \mathcal{M}, j \neq m} \beta_j P_{i_j}^2 N_j^2\right)}{2 P_{i_m}^2 N_{1_m}^2} \tag{35}$$

The β_{safe} has a minimum value $Min\{\beta_{safe}\} = 0$. For the sake of simplicity, we set $\beta = 0$. Substitute the β into the price formula (7), we get $\bar{P} = P_i$.

In summary:

If $\Delta \geq 0$, $M = \frac{1}{2}\left(M_b + \sqrt{M_b^2 - 2\left(\sum_{j \in \mathcal{M}} \beta_j P_{i_j}^2 N_j^2\right)}\right)$. Then use (7) to get $\bar{P}$.

(36)

If $\Delta < 0$, opening position is disabled. Close price $\bar{P} = P_i$.

Appendix 3 Maximum AMM Positions

AMM has a limited market making depth to keep the position under the max leverage when the position increases. The maximum position is subject to:

Condition 1: keep a positive price in (27).

Condition 2: The margin balance is limited by leverages according to (17).

Condition 3: Prevent $\Delta < 0$ in formula (33).

The first condition is already solved the first edge point:

$$N_{max1} = \frac{M}{\beta P_i} \tag{37}$$

To find the other edge points, we can simulate an opening position process assuming the AMM has no position until one of the above conditions is break. The position N_2 at this moment will be the maximum positions it can holds.

Let $N_1 = 0$, $M_{c1} = M$ to simulate the situation when AMM's position is 0. We assume only the current trading market m is increasing the position until N_2 while the other markets $j, j \in \mathcal{M}, j \neq m$ keep their positions unchanged. The final cash M_{c2} will be changed to:

$$\begin{aligned} M_{c2} = M_{c1} - &\left(P_{i_m}\left(1 - \beta_m \frac{P_{i_m}}{M} \frac{0 + N_2}{2}\right)(N_2 - 0)\right) \\ &- \sum_{j \in \mathcal{M}, j \neq m} \left(P_{i_j}\left(1 - \beta_j \frac{P_{i_j}}{M} \frac{0 + N_j}{2}\right)(N_j - 0)\right) \end{aligned} \tag{38}$$

Solving condition 2: The effect of max leverage λ can be expressed as:

$$M_b \geq \sum_{j \in \mathcal{M}} \left(\frac{P_{i_j}|N_j|}{\lambda_j}\right) \tag{39}$$

$$where\ M_b = M_{c2} + P_{i_m} N_2 + \sum_{j \in \mathcal{M}, j \neq m} \left(P_{i_j} N_j \right)$$

Solve this equation we get the maximum long position is:

$$N_{max2} = Max\left(0, \frac{M - \sqrt{M(M - 2\beta_m \lambda_m^2 x)}}{\beta_m \lambda_m P_{i_m}}\right)\ if\ M(M - 2\beta_m \lambda_m^2 x) \geq 0$$

$$where\ x = M - \sum_{j \in \mathcal{M}, j \neq m} \left(\frac{P_{i_j} |N_j|}{\lambda_j} - \frac{\beta_j P_{i_j}^2 N_j^2}{2M} \right) \tag{40}$$

The maximum short position is:

$$N_{min2} = Min\left(0, \frac{-M + \sqrt{M(M - 2\beta_m \lambda_m^2 x)}}{\beta_m \lambda_m P_{i_m}}\right)\ if\ M(M - 2\beta_m \lambda_m^2 x) \geq 0 \tag{41}$$

$$where\ x = M - \sum_{j \in \mathcal{M}, j \neq m} \left(\frac{P_{i_j} |N_j|}{\lambda_j} - \frac{\beta_j P_{i_j}^2 N_j^2}{2M} \right)$$

Solving condition 3: Let $\Delta = 0$ in formula (33) while substitute M_c as M_{c2}, we get the maximum long position is:

$$N_{max3} = \begin{cases} \frac{1}{P_{i_m}} \sqrt{\frac{2M^2 - \sum_{j \in \mathcal{M}, j \neq m} \beta_j P_{i_j}^2 N_j^2}{\beta_j}} & 2M^2 - \sum_{j \in \mathcal{M}, j \neq m} \beta_j P_{i_j}^2 N_j^2 \geq 0 \\ 0 & otherwise \end{cases} \tag{42}$$

The maximum short position is:

$$N_{min3} = \begin{cases} -\frac{1}{P_{i_m}} \sqrt{\frac{2M^2 - \sum_{j \in \mathcal{M}, j \neq m} \beta_j P_{i_j}^2 N_j^2}{\beta_j}} & 2M^2 - \sum_{j \in \mathcal{M}, j \neq m} \beta_j P_{i_j}^2 N_j^2 \geq 0 \\ 0 & otherwise \end{cases} \tag{43}$$

If $2M^2 - \sum_{j \in \mathcal{M}, j \neq m} \beta_j P_{i_j}^2 N_j^2 < 0$, the other markets j, $j \in \mathcal{M}, j \neq m$ already reach the maximum positions. In this case, the current trading market cannot open position anymore.

In summary: The maximum long position N_{max} and maximum short position N_{min} are:

$$N_{max} = \min\{N_{max1}, N_{max2}, N_{max3}\} \tag{44}$$

$$N_{min} = \max\{N_{min2}, N_{min3}\} \tag{45}$$

Appendix 4 Withdrawal Penalty

When AMM's position does not equal to 0, there is a risk-free means for LP to arbitrage: The LP can go long, withdraw, go short, he will profit without bearing any risk. To prevent this situation, we can fix M before and after withdrawal (except for the withdrawal part).

Given the amount of shares that LP wishes to burn is s and the collateral he will withdraw is w, the AMM's position is N, the total shares is S and AMM's margin is M. AMM first calculates the current margin $M_1 = M(M_c, N)$ and calculate the new margin $M_2 = M_1 \frac{S-s}{S}$. We can get the withdrawal collateral w by solving $M(M_c - w, N) = M_2$:

$$w = M_c - M_2 + \sum_{j \in \mathcal{M}} P_{i_j} N_j - \frac{1}{2M_2} \sum_{j \in \mathcal{M}} \beta_j P_{i_j}^2 N_j^2 \tag{46}$$

The withdrawal request may fail if LP removes too much liquidity and violates formular (34). We can get the minimum M in formula (31) by substitute M_c in it to formula (34). So that M_2 must larger than the minimum M to satisfies formular (34).

$$M_{safe} = \sqrt{\frac{\sum_{j \in \mathcal{M}} \beta_j P_{i_j}^2 N_j^2}{2}} \tag{47}$$

$$M_2 \geq M_{safe} \tag{48}$$