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CS590.04 — Spring 2014 **Homework 1** Due: 28 February, 2014

Problem 1

a $\boxed{\text{Borda, Copeland, and Simpson are rules that satisfy the OCO criterion.}}$

For Borda, let $B(c_i)$ denote the total Borda score for candidate c_i, and $B_j(c_i)$ denote the portion of the Borda score for c_i resulting from the j^{th} voter's ranking. With two opposite votes $v_1 = (c_1, c_2, \ldots, c_m)$ and $v_2 = (c_m, c_{m-1}, \ldots, c_1)$ the Borda scores are:

$$
\begin{aligned}
B(c_1) &= B_1(c_1) + B_2(c_1) = (m-1) + 0 = m-1 \\
B(c_2) &= B_1(c_2) + B_2(c_2) = (m-2) + 1 = m-1 \\
&\ldots \\
B(m) &= B_1(m) + B_2(m) = 0 + (m-1) = m-1
\end{aligned}
$$

Thus, the Borda scores for opposite votes cancel out and do not affect the winner when they are combined with another set of votes.

The Copeland and Simpson rules both depend on pairwise comparisons. For all pairs of opposite votes and each pair of candidates $c_i \neq c_j$, c_i and c_j will each win one pairwise comparison and lose the other. This causes the two rules to satisfy OCO.

$\boxed{\text{Approval, Plurality, and Single Transferrable Vote (STV) do not satisfy the OCO criterion.}}$

In an approval vote setting, the two opposite voters will not necessarily both approve all candidates preferred to their median candidate, $c_{m/2}$. If the first voter approves her preferred k candidates and the second voter approves his preferred l candidates ($l > k$ without loss of generality), then $l - k$ candidates ($c_{m+(k-l)/2}, \ldots, c_{m+(l-k)/2}$ will receive approval votes from both voters, which could affect the outcome of the overall election.

When plurality voting is being used, as long as the winner without the opposite votes $c_{-1,-m}$ is not either of the opposite voters most preferred candidates (c_1 and c_m, respectively) and there are at least two non-oppositve voters, the opposite votes will not affect the election. This is because $c_{-1,-m}$ will have two or more votes and a single vote from each of the two opposite voters cannot tip the scales. By similar logic, both of the opposite voters must prefer $c_{-1,-m}$ to their opposite's most preferred candidate, so this candidate will still beat c_1 and c_m in some elections using STV.

b $\boxed{\text{Borda, Copeland, and Simpson are rules that satisfy the CCO criterion,}}$ for similar reasons as those discussed above.

$\boxed{\text{Approval, Bucklin, and STV do not satisfy the CCO criterion.}}$ The reasoning for Approval is the similar to the explanation above. STV can fail to satisfy CCO because the ranks in the original election (without the addition of cycles) may be different from the cyclic votes. For example, if the cycles are over $c_1 > c_2 > \ldots > c_m$, some of the other voters may prefer $c_m > c_2 < c_1 > \ldots$, which could affect the outcome when using STV.

To see that the Bucklin rule can fail to satisfy CCO, suppose that there are three candidates c_1, c_2, c_3. In the original election (without adding sets of voters with cyclical preferences), suppose x voters rank the candidates c_1, c_2, c_3 and y voters rank the candidates c_2, c_1, c_3. Candidate c_1 wins as long as $x + y > y$, i.e. if $y > 1$. Suppose we add n cycles of votes (i.e. $3n$ new votes). Candidate c_2 now wins as long as $n > x$.

Problem 2