

Problem 1

- a** A gamble that offered \$2,000 with probability $\frac{1}{5}$ and \$-500 with probability $\frac{4}{5}$ would strictly increase Bob's expected utility.

With probability $\frac{1}{5}$ he ends up with \$3,500 and has a utility of 2, and with probability $\frac{4}{5}$ he ends up with \$1,000, which has a utility of 1. Thus, Bob's expected utility is $\frac{1}{5} \cdot 2 + \frac{4}{5} \cdot 1 = \frac{6}{5} > 1$. This gamble is fair because the expected value is $\frac{1}{5} \cdot \$2,000 + \frac{4}{5} \cdot \$500 = \$400 + \$ - 400 = 0$.

- b** A gamble that offered \$500 with probability $\frac{3}{4}$ and \$-1,500 with probability $\frac{1}{4}$ would strictly decrease Bob's expected utility.

With probability $\frac{3}{4}$ he would end up with \$2,000 ($u(b) = 1$) and with probability $\frac{1}{4}$ Bob ends up with \$0 ($u(b) = 0$). Thus, his expected utility is $\frac{3}{4} \cdot 1 + \frac{1}{4} \cdot 0 = \frac{3}{4} < 1$. This is a fair gamble because the expected value is $\frac{3}{4} \cdot \$500 + \frac{1}{4} \$ - 1,500 = \$375 + \$ - 375 = 0$.