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CS590.04 — Spring 2014

Homework 1

Due: 7 February, 2014

Problem 1

a $\boxed{\text{A gamble that offered \$2,000 with probability } \frac{1}{5} \text{ and \$-500 with probability } \frac{4}{5} \text{ would strictly increase Bob's expected utility.}}$

With probability $\frac{1}{5}$ he ends up with \$3,500 and has a utility of 2, and with probability $\frac{4}{5}$ he ends up with \$1,000, which has a utility of 1. Thus, Bob's expected utility is $\frac{1}{5} \cdot 2 + \frac{4}{5} \cdot 1 = \frac{6}{5} > 1$. This gamble is fair because the expected value is $\frac{1}{5} \cdot \$2,000 + \frac{4}{5} \cdot \$500 = \$400 + \$ - 400 = 0$.

b $\boxed{\text{A gamble that offered \$500 with probability } \frac{3}{4} \text{ and \$-1,500 with probability } \frac{1}{4} \text{ would strictly decrease Bob's expected utility.}}$

With probability $\frac{3}{4}$ he would end up with \$2,000 ($u(b) = 1$) and with probability $\frac{1}{4}$ Bob ends up with \$0 ($u(b) = 0$). Thus, his expected utility is $\frac{3}{4} \cdot 1 + \frac{1}{4} \cdot 0 = \frac{3}{4} < 1$. This is a fair gamble because the expected value is $\frac{3}{4} \cdot \$500 + \frac{1}{4}\$ - 1,500 = \$375 + \$ - 375 = 0$.

Problem 2

Problem 3

	Left	Center	Right
Top	5, 0	1, 2	4, 0
Middle	2, 4	2, 4	3, 5
Bottom	0, 1	4, 0	4,)

Table 1: Normal-form game for Problem 3(a) with labeled actions.

a Table 1 represents the normal-form game for this problem with the additional convenience of labeled strategies. Neither player has a strictly dominant pure strategy. Additionally, the column player does not have a strictly dominant mixed strategy: no mix over L and R can dominate C (if the row player plays T), no mix over L and C can dominate R (if the row player plays M), and no mix over C and R can dominate L (if the row player plays B).

For the row player, no mixed strategy over M and B can dominate T (if the column player plays L) and no mixed strategy over T and M can dominate B (if the column player plays C), but M is dominated by a mix over T and B (e.g. $p_T = \frac{2}{5}, p_B = \frac{3}{5}$).

We can now solve the subgame with row M eliminated, and see that a mix over L and C will dominate R for the column player. There are no dominant pure strategies in this subgame, so we must find the mixed strategy equilibrium. The column player mixes such that

$$\begin{aligned} 5p_L + 1p_C &= 0p_L + 4p_C \\ 5p_L &= 3p_C \\ p_L = \frac{3}{8} &\quad , \quad p_C = \frac{5}{8}, \end{aligned}$$

and the row player mixes such that

$$\begin{aligned} 0p_T + 1p_B = 2p_T + 0p_B \\ p_B = \frac{2}{3} \quad , \quad p_T = \frac{1}{3}, \end{aligned}$$

both of which make the other player indifferent.

The unique Nash equilibrium is the row player mixing between T and B with probability $(\frac{1}{3}, \frac{2}{3})$, and the column player mixing between L and C with probability $(\frac{3}{3}, \frac{5}{8})$.