The Indian Buffet Process

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Outline

Example

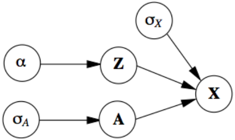


Figure: Griffiths and Ghahramani (2011) Figure 7

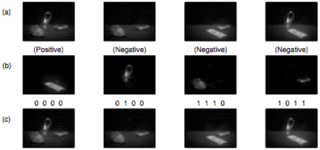


Figure: Griffiths and Ghahramani (2011) Figure 9

Motivation

When and why would we use IBP?

- As a prior on sparse binary matrices with a countably infinite number of columns

Introduction

Indian Buffet Process:

1. N customers enter (in sequence) a buffet restaurant with an infinite number of dishes
2. First customer fills her plate with Poisson(α) number of dishes
3. i^{th} customer samples dishes in proportion to their popularity, with probability $\frac{m_k}{i}$, where m_k is the number of previous customers who sampled dish k
4. i^{th} customer then samples a Poisson($\frac{\alpha}{i}$) number of new dishes

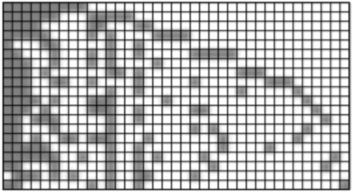


Figure: Griffiths and Ghahramani (2011) Figure 5

Background: Dirichlet Process

Finite version (Dirichlet distribution):

- Assignment of an object to a class is independent of all other assignments: $P(c|\theta) = \prod_{i=1}^{N} P(c_i|\theta) = \prod_{i=1}^{N} \theta_{c_i}$
- $\theta|\alpha \sim \text{Dirichlet}(\frac{\alpha}{K}, \ldots, \frac{\alpha}{K})$ (if symettric)
- $c_i|\theta \sim \text{Discrete}(\theta)$, where Discrete : Bernoulli :: Multinomial : Binomial

Integrating out θ: $P(c) = \frac{\prod_{k=1}^{K} \Gamma(m_k + \frac{\alpha}{K})}{\Gamma(\frac{\alpha}{K})^K} \frac{\Gamma(\alpha)}{\Gamma(N+\alpha)}$

What happens as $K \to \infty$?

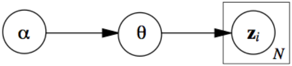


Figure: Griffiths and Ghahramani (2011) Figure 1

Background: Chinese Restaurant Process

1. N customers enter (in sequence) a restaurant with an infinite number of tables, each with infinite seating
2. First customer sits at first table with probability $\frac{\alpha}{\alpha} = 1$
3. i^{th} customer sits at the k^{th} table with probability $\frac{m_k}{i+\alpha-1}$, where m_k is the number of previous customers who sat at table k, or a new table with probability $\frac{\alpha}{i+\alpha-1}$

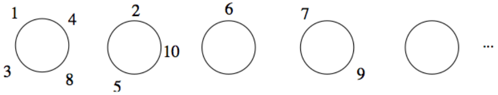


Figure: Griffiths and Ghahramani (2011) Figure 2

Limitation: each object (customer) can only belong to one class (table).

Beta Process

Indian Buffet Process

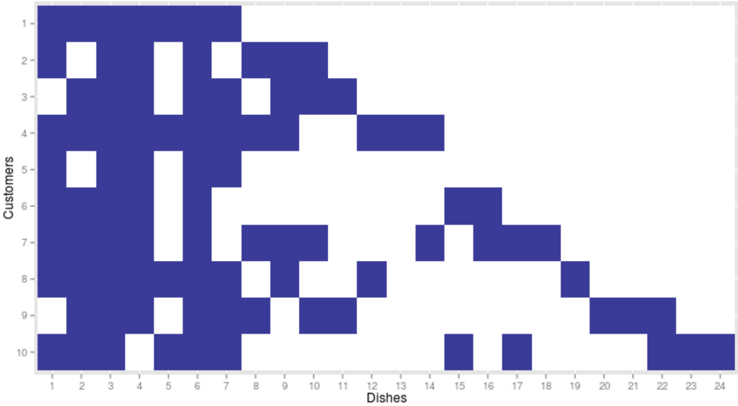
Stick-Breaking Construction of IBP

1. Recursively break (an initially unit-length) stick, breaking off a Beta$(\alpha, 1)$ portion at each step
2. Let each portion of the "stick", π_k represent the probability of each feature (sorted from largest to smallest)

This helps to show the relation between the Dirichlet process and the IBP. The stick-breaking construction is also useful for defining inference algorithms.

Demo

mcdickenson.shinyapps.io/ibp-demo



Properties of the Resulting Distribution

- The "effective" dimension $K_+ \sim \text{Poisson}(\alpha H_N)$
- The number of dishes on each customer's plate is distributed Poisson(α) (by exchangeability)
- $\mathbf{Z}$ remains sparse as $K \to \infty$: effective dimensions of $\mathbf{Z}$ are $N \times K_+$, and the expected number of entries is $N\alpha$

Inference by Gibbs Sampling

Variational Inference

Application 1: Choice Behavior

"A Choice Model with Infinitely Many Latent Features"
(Görür, Jäkel, and Rasmussen, ICML 2006)

- Customers compare items (e.g. cell phones) based on the (binary) features of each; more features are better
- Number of features is potentially infinite and ordering is not important, so IBP is used
- Celebrity example: "With whom would you prefer to spend an hour of conversation?"

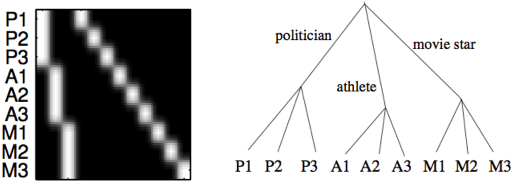


Figure: Görür, Jäkel, and Rasmussen (2006) Figure 3

Application 2: Collaborative Filtering

Extension 1: Topic Modeling

Williamson, Wang, Heller, and Blei (2010)

Stick-breaking construction:
$\mu_k \sim \text{Beta}(\alpha, 1); \pi_k = \prod_{j=1}^{k} \mu_j; b_{m,k} \sim \text{Bernoulli}(\pi_k)$

Focused topic model:

1. for $k = 1, 2, \ldots$
    - Sample stick length π_k
    - Sample relative mass $\phi_k \sim \text{Gamma}(\gamma, 1)$
    - Draw topic distribution over words: $\beta_k \sim \text{Dirichlet}(\eta)$
2. for $m = 1, \ldots, M$
    - Sample binary vector b_m
    - Draw total number of words $n^{(m)} \sim NB(\sum_k b_{m,k}\phi_k, 1/2)$
    - Sample distribution over topics $\theta_m \sim \text{Dirichlet}(b_m \cdot \phi)$
    - For each word $w_{m,i}, i = 1, \ldots, n^{(m)}$
        1. Draw topic index $z_{m,i} \sim \text{Discrete}(\theta_m)$
        2. Draw word $w_{m,i} \sim \text{Discrete}(\beta_{z_{m_i}})$

Extension 2: Collaborative Filtering

Discussion

Limitations of IBP:

1. Coupling of average number of features α and total number of features $N\alpha$ (can be overcome with a two-parameter generalization)
2. Computationally complex, can be time-consuming

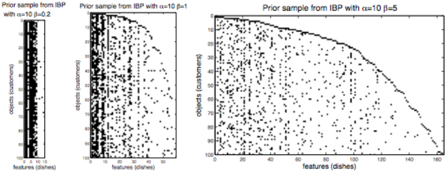


Figure: Griffiths and Ghahramani (2011) Figure 10