

The Indian Buffet Process

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February 10, 2014

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Introduction

Indian Buffet Process:

- 1 N customers enter (in sequence) a buffet restaurant with an infinite number of dishes
- 2 First customer fills her plate with $\text{Poisson}(\alpha)$ number of dishes
- 3 i^{th} customer samples dishes in proportion to their popularity, with probability $\frac{m_k}{i}$, where m_k is the number of previous customers who sampled dish k
- 4 i^{th} customer then samples a $\text{Poisson}(\frac{\alpha}{i})$ number of new dishes

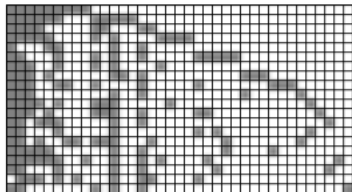


Figure: Griffiths and Ghahramani (2011) Figure 5

Background: Dirichlet Process

Finite version (Dirichlet distribution):

- Assignment of an object to a class is independent of all other assignments: $P(c|\theta) = \prod_{i=1}^N P(c_i|\theta) = \prod_{i=1}^N \theta_{c_i}$
- $\theta|\alpha \sim \text{Dirichlet}(\frac{\alpha}{K}, \dots, \frac{\alpha}{K})$ (if symmetric)
- $c_i|\theta \sim \text{Discrete}(\theta)$, where Discrete : Bernoulli :: Multinomial : Binomial

Integrating out θ : $P(c) = \frac{\prod_{k=1}^K \Gamma(m_k + \frac{\alpha}{K})}{\Gamma(\frac{\alpha}{K})^K} \frac{\Gamma(\alpha)}{\Gamma(N + \alpha)}$

What happens as $K \rightarrow \infty$? We get the Dirichlet process.

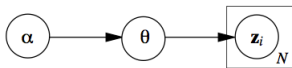


Figure: Griffiths and Ghahramani (2011) Figure 1

Background: Chinese Restaurant Process

- 1 N customers enter (in sequence) a restaurant with an infinite number of tables, each with infinite seating
- 2 First customer sits at first table with probability $\frac{\alpha}{\alpha} = 1$
- 3 i^{th} customer sits at the k^{th} table with probability $\frac{m_k}{i+\alpha-1}$, where m_k is the number of previous customers who sat at table k

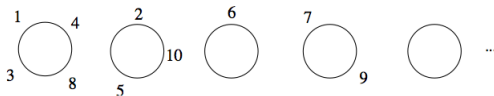


Figure: Griffiths and Ghahramani (2011) Figure 2

Limitation: each object (customer) can only belong to one class (table).

Beta Process

Indian Buffet Process

Properties of the Resulting Distribution

- The “effective” dimension $K_+ \sim \text{Poisson}(\alpha H_N)$
- The number of dishes on each customer’s plate is distributed $\text{Poisson}(\alpha)$ (by exchangeability)
- \mathbf{Z} remains sparse as $K \rightarrow \infty$: effective dimensions of \mathbf{Z} are $N \times K_+$, and the expected number of entries is $N\alpha$

Inference by Gibbs Sampling

Application 1: Choice Behavior

Application 2: Collaborative Filtering

Extension 1: Topic Modeling

Extension 2: Collaborative Filtering

Extensions