The δ_α basis and covariance $\langle \delta_\alpha \delta_\beta \rangle$

Joseph E. McEwen

May 10, 2016

1 Basis functions

The basis functions we use are defined in terms of the spherical Bessel function and spherical harmonic

$$\psi_\alpha(r,\theta,\phi) = j_{l_\alpha}(k_\alpha r) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\theta,\phi) \ , \tag{1}$$

where $Y^{\mathrm{R}}_{lm}(\theta,\phi)$ is the real spherical harmonic. The radial basis function should be zero at the sector boundary. Thus, fiven the maximum radius of the sector r_{max} the super wave vector k_α is found by

$$j_{l_\alpha}(k_\alpha r_{\mathrm{max}}) = 0. \tag{2}$$

2 Super Survey Matter Field

In our basis the super survey mode is defined as

$$\begin{aligned}
\delta_\alpha(k_\alpha) &= \int_\Omega \delta(\mathbf{r}) j_{l_\alpha}(k_\alpha r) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{r}) d^3\mathbf{r} \\
&= \int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta(\mathbf{k}) \int_0^{r_{\mathrm{max}}} dr^2 j_{l_\alpha}(k_\alpha r) \int d^2\hat{r} e^{ikr\hat{k}\cdot\hat{r}} Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{r}) \\
&= 4\pi i^{l_\alpha} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta(\mathbf{k}) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{k}) \int_0^{r_{\mathrm{max}}} dr^2 j_{l_\alpha}(k_\alpha r) j_{l_\alpha}(kr) \ ,
\end{aligned} \tag{3}$$

where in the second equality $\delta(\mathbf{r}) = (2\pi)^{-3} \int d^3\mathbf{k} \exp(i\mathbf{k}\cdot\mathbf{r})\delta(\mathbf{k})$ was used and in the third equality the following identity was used

$$\int_{S^2} d^2\hat{r} Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{r}) e^{i\mathbf{k}\cdot\mathbf{r}} = 4\pi i^{l_\alpha} j_{l_\alpha}(kr) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{k}) \ . \tag{4}$$

Including a normalization[1] $N_\alpha = \int_\Omega Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{r}) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{r}) d^3\mathbf{r}$ we have

$$\delta_\alpha = \frac{4\pi i^{l_\alpha}}{N_\alpha} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta(\mathbf{k}) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{k}) \int_0^{r_{\max}} dr^2 j_{l_\alpha}(k_\alpha r) j_{l_\alpha}(kr) \ . \tag{5}$$

To get the covariance of the super survey field we take the ensemble average:

$$\begin{aligned}\langle \delta_\alpha \delta_\beta \rangle = & \frac{(4\pi)^2}{N_\alpha N_\beta} \int \frac{d^3\mathbf{k}_1}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3} \langle \delta(\mathbf{k}_1) \delta^*(\mathbf{k}_2) \rangle Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{k_1}) Y^{\mathrm{R}}_{l_\alpha m_\alpha}(\hat{k_2}) \\ & \times \int_0^{r_{\max}} drr^2 j_{l_\alpha}(k_\alpha r) j_{l_\alpha}(k_1 r) \int_0^{r_{\max}} drr^2 j_{l_\alpha}(k_\alpha r) j_{l_\alpha}(k_2 r) \\ & \frac{(4\pi)^2}{N_\alpha N_\beta} \int \frac{d^3\mathbf{k}}{(2\pi)^3} P(k) I_\alpha(k, r_{\max}) \times I_\beta(k, r_{\max}) \ ,\end{aligned} \tag{6}$$

where $(2\pi)^3 \delta^3_{\mathrm{D}}(\mathbf{k} + \mathbf{k}') P(k) = \langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle$ was used along with the definition

$$\begin{aligned}I_\alpha(k, r_{\max}) &= \int_0^{r_{\max}} drr^2 j_{l_\alpha}(k_\alpha r) j_{l_\alpha}(k_1 r) \\ &= \frac{\pi}{2} \sqrt{\frac{1}{k_\alpha k_1}} \int_0^{r_{\max}} drr J_{l_\alpha+1/2}(k_\alpha r) J_{l_\alpha+1/2}(k_1 r) \\ &= \frac{\pi}{2} \frac{r_{\max}}{\sqrt{k_\alpha k_1}} \frac{\left[J_{l_\alpha+1/2}(k_1 r_{\max}) J'_{l_\alpha+1/2}(k_\alpha r_{\max}) - J_{l_\alpha+1/2} k_\alpha r_{\max}) J'_{l_\alpha+1/2}(k_1 r_{\max}) \right]}{k_1^2 - k_\alpha^2} \ .\end{aligned} \tag{7}$$

[1] In our case N_α is 1.