Alternative analysis of the delayed-choice quantum eraser with two entangled particles.

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According to Quantum Mechanics, the particles can exhibit either particle properties or wave properties depending on the experimental set up (wave-particle dualism). A special behavior occurs for a system of two entangled particles 1 and 2 that propagate along two different directions in the space. In such a case, the entangled particle 2 should exhibit either the particle behavior or the wave behavior depending on the kind of measurement that is performed on particle 1 whatever is the actual distance between the two particles. The apparently surprising fact is that the "choice" of what kind of measurement is performed on particle 1 can be also made when particle 2 has been already detected (delayed-choice quantum eraser). These theoretical predictions have been confirmed by the experiments and seem to suggest that a future measurement can affect a past event. Recently, both the concepts of "delayed choice" and of "quantum erasure" have been criticized by Ellerman and by Kastner. In this paper we propose an alternative analysis of the delayed-choice quantum eraser with two entangled particles and we show that it does not imply an inversion of the natural cause-effect temporal order of the physical events.

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INTRODUCTION: THE QUANTUM ERASER.

According to Quantum Mechanics, particles can exhibit either wave properties or particle properties (wave-particle dualism) depending on the experimental set up[1]. The wave-particle duality leads to many counter-intuitive predictions as well as, for instance the delayed-choice quantum erasure that has been the object of many theoretical and experimental investigations and of an interesting review paper[2]. We remind the reader to this reference for details and for an extended bibliography. Recently Ellerman[3] and Kastner[4] have strongly criticized the concepts of "delayed-choice" and "erasure".

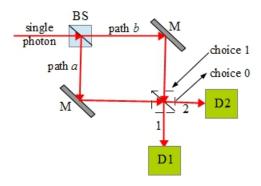


Figure 1. A single photon impinges on the symmetric 50% beam splitter BS, is reflected by mirrors M and is detected by the single photon detectors D1 ans D2. A second beam splitter (dashed in the figure) can be either inserted (choice "1") or removed (choice "0"). If the beam splitter is removed, the clicking of either detector D1 or detector D2 allow to know which path (a or b) has been followed by the photon. In the other case the which-path information is erased and interference occurs.

Here we report an alternative analysis of the delayed choice quantum eraser with two entangled particles and we show that it does not imply the counterintuitive inversion of the natural temporal order of the physical events. A famous delayed-choice gedanken experiment was proposed by Wheeler in 1984[5] and is shown in Figure 1. A single photon impinges on the first beam splitter of a Mach-Zender interferometer. If the second beam splitter is removed (choice "0"), the detection of the photon by either detector D1 or D2 allows to know which is the path followed by the photon (which-path information and particle behavior). In such a case, no interference occurs and the clicks of detectors D1 and D2 are completely indipendent on the optical path difference between the two arms of the interferometer. The insertion of the second beam splitter (choice "1") erases the which-path information and interference occurs at outputs 1 and 2 (wave behavior). In such a case, the clicks of detectors D1 and D2 depend on the difference of optical paths between the two arms of the interferometer. Wheeler emphasized an important feature: if paths a and b are sufficiently long, the choice of either inserting a second beam splitter or not can be made after the photon has already passed through the first beam splitter. Then, by such a delayedchoice experiment, we get the counterintuitive result that the future choice seems to affect the past history of the photon. In particular, if the beam splitter is inserted,

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the which-path information is erased. However, Wheeler himself wrote: "in actuality it is wrong to talk of the route of a photon and it makes no sense to talk of the phenomenon until it has been brought to a close and irreversible act of amplification". Other delayed-choice gedanken experiments have been proposed in the literature and the Quantum Mechanics predictions have been always verified in successive real experiments [2].

Scully et al.[6, 7] investigated an atom-photon entangled state and proposed the so-called delayed-choice quantum eraser[8] that is the object of our successive analysis. After this pioneering work, other configurations of entangled systems have been investigated (see[2]) and, in particular photon-photon entangled configurations. Figure 2 shows the case of two polarization entangled photons, the environment (or idler) photon e and the system (or signal) photon e, that are generated at point e0 by spontaneous parametric down conversion [9] and propagate along different directions in the space. This kind of apparatus has been used, for instance, in reference[10]. Photons e and e0 are in the polarization maximally entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_e |V\rangle_s + |V\rangle_e |H\rangle_s \right), \tag{1}$$

where H and V denote horizontal and vertical polarization, respectively. The entangled state represents a very special state predicted by Quantum Mechanics that has no equivalent in classical and semi-classical physics. In particular, each entangled photon has not a definite polarization (linear, circular or elliptical polarization) but the polarizations of the two entangled photons are strictly correlated (EPR correlations). For instance, if photon e is found in the vertically polarized state $|V\rangle_e$, photon s is found in the horizontally polarized state $|H\rangle_s$. When the system photon s passes through the polarizing beam splitter PBS of the interferometer and gets the inputs of the successive beam splitter BS, the polarization entangled state becomes the polarization-path hybrid state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_e |b\rangle_s + i e^{i\theta} |V\rangle_e |a\rangle\rangle_s \right), \tag{2}$$

where we use the reduced forms $|a\rangle_s$ and $|b\rangle_s$ to denote a one-photon state at the a-input of the beam splitter BS with a vacuum state at the b-input $(|a\rangle_s = |1\rangle_a |0\rangle_b)$ and a one-photon state at the b-input with a vacuum state at the a-input $(|b\rangle_s = |0\rangle_a |1\rangle_b)$, respectively. The phase contributions i and $e^{i\theta}$ appearing in Eq.(2) are due to the PBS transfer operator (see Eq.(1.21) in [11] with $t_H = 0, t_V = 1, r_H = 1$ and $r_V = 0$) [12] and to the dephasing plate operator, respectively. According to Eq.(2), the polarizations H and V of the environment photon are strictly related to the paths a and b followed by the system photon in the interferometer. Then, it is usually concluded that the environment photon e carries the which-path information on the system photon

s although this conclusion has been strongly criticized by R.E.Kastner[4]. Choice "0" corresponds to the case where the electro-optic modulator EOM is "off" and the polarization of the environment photon e is not modified by it. Then, the PBS in front to the EOM in figure 2 sends the vertically and horizontally polarized photons V and H to outputs 1 and 2, respectively. In this case the components V and H of the polarization of the environment photon are measured by detectors D1 and D2, respectively. Choice "1" corresponds to the case where the electro-optic modulator is "on" and its dephasing is set in such a way that right-hand (R) and left-hand (L) circularly polarized photons impinging on the electo-optic modulator EOM are changed by it to linear vertically (V)and horizontally (H) polarized photons, respectively. In such a case the optical system (EOM + PBS) sends the incident right and left hand circular polarizations R and L to outputs 1 and 2 of the polarizing beam splitter, respectively. Then, the system EOM+PBS behaves as a beam splitter for the circular polarizations in this case. In the case of choice "0", if the environment photon is found to be in the state $|V\rangle_e$ (or $|H\rangle_e),$ the system photon must be in the state $|a\rangle_s$ (or $|b\rangle_s$) and, thus, the current interpretation is that the photon has passed only in path a (or b). In this case, no interference can occur and the coincidences between detectors Di (i=1,2) and Dj (j=3,4)are independent of the dephasing θ between the two arms of the interferometer. A completely different behavior is predicted if the right-hand circular polarization (R) and the left-hand circular polarization (L) of the environment photon are measured (choice "1"). Rewriting the entangled state of Eq.(2) in the new circular orthonormal basis $|R\rangle_e = \frac{1}{\sqrt{2}} \left(|H\rangle_e + i \, |V\rangle_e \right) \text{ and } |L\rangle_e = \frac{1}{\sqrt{2}} \left(|H\rangle_e - i \, |V\rangle_e \right)$ and disregarding a common multiplicative coefficient i, one gets the alternative expression

$$|\psi\rangle = \frac{1}{2} \left[|L\rangle_e \left(-e^{i\theta} |a\rangle_s + |b\rangle_s \right) + |R\rangle_e |\left(e^{i\theta} |a\rangle_s + |b\rangle_s \right) \right]. \tag{3}$$

From Eq.(3) we infer that, if the environment photon is found in the state either $|R\rangle_e$ or $|L\rangle_e$, the which-path information is lost and interference occurs. Then, the choice of measuring the circular polarizations R and L of the environment photon (choice "1") seems to produce the erasure of the which-path information. At the final beam splitter BS, $|a\rangle_s \to \frac{1}{\sqrt{2}}\left(|3\rangle_s + i\,|4\rangle_s\right)$ and $|b\rangle_s \to \frac{1}{\sqrt{2}}\left(|4\rangle_s + i\,|3\rangle_s\right)$ where $|3\rangle_s$ and $|4\rangle_s$ shortly denote one-photon states at outputs 3 and 4, respectively. Then, the entangled state in Eq.(3) becomes the hybrid entangled state

$$\begin{split} |\psi'\rangle = & \frac{1}{\sqrt{2}} \left[i \cos \alpha |L\rangle_e |3\rangle_s + \sin \alpha |R\rangle_e |3\rangle_s \\ -i \sin \alpha |L\rangle_e |4\rangle_s + \cos \alpha |R\rangle_e |4\rangle_s \right], \end{split} \tag{4}$$

where α is the phase coefficient

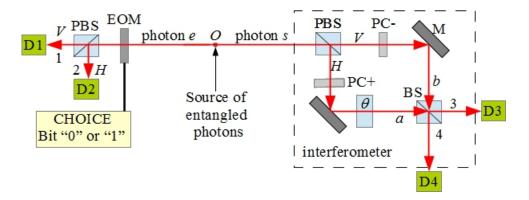


Figure 2. Scheme of a delayed-choice quantum eraser with two entangled photons (the environment photon e and the system photon e). PBS are polarizing beam splitters, M are mirrors, BS is a symmetric 50% beam splitter. PC+ and PC- are polarization controllers that rotate the vertical and horizontal polarizations V and H by 45° in opposite directions to align them along the same diagonal axis to make possible interference. The dashed rectangle delimits a modified Mach-Zehnder interferometer where the usal first beam splitter is replaced by a polarizing beam splitter. θ denotes a dephasing plate that introduces the phase delay θ between the arms e0 and e1 of the interferometer. EOM is an electro-optic modulator. Choice "0" correspond to the case where the EOM is "off", while choice "1" corresponds to the case where the EOM is "on".

$$\alpha = \frac{\theta}{2} + \frac{\pi}{4}.\tag{5}$$

A common inessential multiplicative phase factor $e^{i\alpha}$ was disregarded in Eq.(4). We remind here that, for choice "1", detectors D1 and D2 collect the right hand circularly polarized photons and the left hand circularly polarized photons, respectively. Then, from Eq.(4) we infer that the probabilities $p_{ij}(i=1,2 \text{ and } j=3,4)$ of having correlated clicks of detectors Di and Dj (i=1,2 and j=3,4) are

$$p_{13} = \frac{1}{2}\sin^2\alpha,\tag{6}$$

$$p_{23} = \frac{1}{2}\cos^2\alpha,\tag{7}$$

$$p_{14} = \frac{1}{2}\cos^2\alpha,\tag{8}$$

$$p_{24} = \frac{1}{2}\sin^2\alpha. (9)$$

The dependence of these probabilities on phase α evidences the wave behavior of the photons. It is important to notice that the probabilities that the system photon is detected by either detector D3 or detector D4 are $p_3 = p_{13} + p_{23} = \frac{1}{2}$ and $p_4 = p_{14} + p_{24} = \frac{1}{2}$ that are independent of phase α as in the case of choice "0". Then, the interference contributions in Eqs.(6)-(9) can be only observed if correlated measurements between detectors D1 (or D2) and D3 (or D4) are performed for each couple of entangled photons. The delayed-choice quantum erasure occurs if the paths of the environment photon e to reach the polarizing elements (EOM and PBS) and the successive detectors D1 and D2 are much longer than the paths needed to the system photon s to reach the

interferometer and detectors D3 and D4. In these conditions, the "choice" on what kind of polarization of the environment photon is measured can be made when the system photon has already passed through the interferometer and has been already detected (and registered) by either detector D3 or detector D4. Then, the analysis above leads to the counterintuitive conclusion that the successive choice decides if the system photon behaves as a particle or as a wave after it has been already detected and registered. We note here that the interpretation above suffers of an implicit realism hypothesis while, according to Bohr, " in Quantum Mechanics no elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon". According to the above sentence, the interpretation of the experimental results in terms of erasure of the which-path information can be object of some criticism. However, the experiments with entangled particles seem to suggest that an inversion of the causal temporal order also occurs as far as the correlations between the two physical events (the clicks of the detectors) are concerned. This latter behavior appears to be very counterintuitive. Reference[2] strongly stressed that the choice must be totally random in an ideal experiment and the passage of the system photon through the first beam splitter and the "choice" must be space-like events. These requirements avoid possible causal interpretations of the observed phenomena in terms of subluminal or luminal communications between the events. Experiments satisfying these conditions have been carried out successfully by Xiao-Song Ma et al.[10] and by F. Kaiser et al. 13.

The analysis reported in this introductive Section follows that already given in reference[10]. In Section I we reanalyze the delayed-choice quantum eraser with two entengled photons using a different point of view and we show that the delayed-choice quantum erasure admits an alternative interpretation that does not lead to

the usual counterintuitive conclusion that a future choice affects the result of a past measurement in agreement with the conclusions of Elleman[3] and Kastner[4]. We show that the delayed-choice quantum erasure it strictly related to the non-local character of the Quantum Mechanics that has been evidenced by the EPR paradox[14] (Einstein, Podolsky and Rosen) and confirmed by many successive EPR experiments (see, for instance, the most recent loophole-free experiments[15–17]).

I. THE ALTERNATIVE ANALYSIS OF THE DELAYED-CHOICE QUANTUM ERASER.

In this Section we analyzes the delayed-choice eraser using an alternative approach. For the successive analysis it is convenient to rewrite the entangled state in eq.(1) in terms of the orthonormal basis of elliptically polarized states

$$|E\rangle_{e,s} = \frac{1}{\sqrt{2}} \left[|H\rangle_{e,s} + e^{i\theta} |V\rangle_{e,s} \right],$$
 (10)

$$|E^{\perp}\rangle_{e,s} = \frac{1}{\sqrt{2}} \left[|H\rangle_{e,s} - e^{i\theta} |V\rangle_{e,s} \right], \tag{11}$$

where subscripts e and s refer to the environment photon and the system photon, respectively and θ is the delay contribution on the a-arm of the interferometer in figure 2. The expression of the entangled state in the new elliptical basis is (up to an inessential multiplicative phase factor $e^{-i\theta}$):

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[|E\rangle_e |E\rangle_s - |E^{\perp}\rangle_e |E^{\perp}\rangle_s \right].$$
 (12)

Our successive analysis is strictly related to an important property of the modified Mach Zehnder interferometer of figure 2: this interferometer behaves as a elliptical polarizing beam splitter. The standard linear polarizing beam splitter sends an input photon that is in the linearly polarized state $|V\rangle$ at one of the two outputs of the polarizing beam splitter and an input photon that is in the $|H\rangle$ state at the other output. Analogously, using the procedures of Quantum Optics, it can be easily shown that the interferometer of figure 2 sends the elliptically polarized input state $|E\rangle_s$ at output 3 of the final beam splitter BS and the input state $|E^{\perp}\rangle_s$ at output 4. In particular, we get $|E\rangle_s \longrightarrow i\,|3\rangle_s$ and $|E^{\perp}\rangle_s \longrightarrow -\,|4\rangle_s$ at up an inessential common multiplicative coefficient $e^{i\theta}$. We emphasize here that in both the cases (either incident $|E\rangle_s$ or $|E^{\perp}\rangle_s$) the system photon propagates in both the arms of the interferometer leading to a fully constructive interference at one of the two outputs (either 3 or 4) and a fully destructive interference at the other output. If a photon is detected by detector D3 (or D4) it necessarily means that this photon has passed both the arms of the interferometer leading to a constructive interference

at output 3 (or 4) of the interferometer (wave behavior). As soon as the system photon s exits from the interferometer, the incident polarization entangled state in Eq.(12) becomes the hybrid entangled state:

$$|\psi_C\rangle = \frac{1}{\sqrt{2}} \left[i|E\rangle_e |3\rangle_s + |E^\perp\rangle_e |4\rangle_s \right]. \tag{13}$$

This means that if detector D3 (or D4) clicks, the state of the environment photon collapses to the elliptical state $|E\rangle_e$ (or $|E^{\perp}\rangle_e$) and, thus, it is not surprising that the correspondent successive clicks of detectors D1 and D2 will depend on what kind of polarization of the environment photon is measured (either linear or circular). According to this alternative point of view, it is not the delayed-choice that affects the past history of the system photon but is the detection of the system photon by one of the detectors D3 and D4 that determines the result of the successive polarization measurement performed on the environment photon. No inversion of the temporal order occurs at all and the successive "choice" does not affect the detection of the system photon. If the future choice is "0", the polarizations V and H of the environment photon e are measured by detectors D1 and D2, respectively. From Eq.(13) we infer that the detection of the system photon s by detector either D3 or D4 implies that the corresponding environment photon has collapsed to either the elliptically polarized state $|E\rangle_e=\frac{1}{\sqrt{2}}\left[|H\rangle_e+e^{i\theta}|V\rangle_e\right]$ or $|E^\perp\rangle_e=\frac{1}{\sqrt{2}}\left[|H\rangle_e-e^{i\theta}|V\rangle_e\right]$, respectively. Substituting these latter expressions in Eq.(13) we get an alternative expression of the photon state $|\psi_C\rangle$ in terms of the horizontally and vertically polarized states of the environment photon:

$$|\psi_C\rangle = \frac{1}{2} \left[i |H\rangle_e |3\rangle_s + i e^{i\theta} |V\rangle_e |3\rangle_s + |H\rangle_e |4\rangle_s - e^{i\theta} |V\rangle_e |4\rangle_s \right]. \tag{14}$$

We remind that, in the case of choice "0", the environment photons in the states $|V\rangle_e$ and $|H\rangle_e$ are collected by detectors D1 and D2, respectively while the system photons in the states $|3\rangle_s$ and $|4\rangle_s$ are collected by detectors D3 and D4, respectively. Then, from Eq.(14) we infer that the probabilities of finding "correlated" clicks of detectors Di (i = 1,2) and Dj (j = 3,4) are given by $p_{ij} = \frac{1}{4}$ and are independent of θ . This is just the same result already obtained in the introduction but, now, the physical interpretation of the phenomenon is not the same. In the introduction, the detection of the polarizations Vand H of the environment photon forced the system photon to pass through only one arm of the interferometer (which-path information). This interpretation could be appropriate to describe the case where the first measurement is performed on the envelopment photon althogh some remark is needed (see the note [18]). On the contrary, in the delayed-choice case, the first detection of the system photon by one of detectors D3 and D4 implies that the system photon has passed in both the interferometer arms giving a total constructive interference at

one of the two outputs 3 and 4, respectively. Then, the which-path information is not allowed in this case. In both these cases (delayed choice and not delayed choice), the same final probabilities of "correlated" clicks $\mathbf{p}_{ij}=\frac{1}{4}$ are predicted to occur but the physical interpretation is different. If the choice "1" is made, the circular R and L polarizations of the environment photon e are measured. The elliptical states $|E\rangle_e$ and $|E^\perp\rangle_e$ in terms of the circular basis vectors $|R\rangle_e=\frac{1}{\sqrt{2}}\left[|H\rangle_e+i\,|V\rangle_e\right]$ and $|L\rangle_e=\frac{1}{\sqrt{2}}\left[|H\rangle_e-i\,|V\rangle_e\right]$ are (up to a common inessential phase factor $e^{i\alpha}$)

$$|E\rangle_e = [-i \sin \alpha |R\rangle_e + \cos \alpha |L\rangle_e],$$
 (15)

$$|E^{\perp}\rangle_e = [\cos \alpha |R\rangle_e - i \sin \alpha |L\rangle_e].$$
 (16)

Substituting these expressions in Eq.(13) we get the state $|\psi_C\rangle$ in terms of $|R\rangle_e$ and $|L\rangle_e$:

$$\begin{split} |\psi_C\rangle = &\frac{1}{\sqrt{2}} \left[i \, \cos \alpha |L\rangle_e |3\rangle_s + \sin \alpha |R\rangle_e |3\rangle_s \\ &- i \, \sin \alpha |L\rangle_e |4\rangle_s + \cos \alpha |R\rangle_e |4\rangle_s \right]. \end{split} \tag{17}$$

The latter expression coincides with equation (4) already obtained in the Introduction and, thus, also in this case the probabilities p_{ij} given in eqs.(6)-(9) are immediately recovered. The main difference is that, now, it is the first detection at detectors D3 and D4 that produces the collapse of the entangled state and the successive correlated detections of detectors D1 and D2. Although the calculations made in the introduction and in Section I lead to the same final results for the probabilities p_{ij} , we think that the approach of Section I has to be physically preferred in the case of a delayed-choice experiment with entangled particles. In fact, according to Eq.(14), it is the detection of the system photon by either detector D3 or D4 that induces the collapse of the environment photon to either $|E\rangle_e$ or $|E^{\perp}\rangle_e$, respectively. For instance, according to Eq.(13), if detector D3 clicks it means that the environment photon has collapsed to the elliptical state $|E\rangle_e$. This collapse of the environment photon to the state $|E\rangle_{a}$ represents a well defined physical event that occurs before the detection of the environment photon. In fact, the environment photon will be always found in the polarizion state $|E\rangle_e$ if a successive measurement of the elliptical polarizations E or E^{\perp} is performed. We emphasize here that the interferometer of figure 2 is equivalent to an elliptical polarizing beam splitter and, thus, the experiment shown in figure 2 is analogous to a typical EPR experiment where the polarization correlations between the entangled photons are measured [15–17]. The analysis above support the main conclusion of reference [4]: "the delayed-choice quantum eraser neither erases nor delays". In particular, the successive measurement of the H-polarization of the environment photon does not allow to obtain the which-path information on the system photon. If, for instance, detector D3 clicks it means that

there is a fully costructive interference between the two paths a and b at output 3 and a fully destructive interference at output 4. These interferences can occur only if the system photon has passed through both the arms of the interferometer.

There is another possible scenario that has been suggested by a referee and that was not taken into account in this paper and in the previous ones. In this scenario the environment photon is detected after the system photon has passed through the PBS on the right, but before the system photon reaches the subsequent beam-splitter on its way to detection. In such a case, the interpretations of the quantum erasure appears evident here. However, we note that in this case too, there is no inversion of the natural temporal order between the physical events related to the detections of the entangled photons. It is the first detection of the environment photon that produces a quantum collapse of the entangled state and, thus, determines the result of the next detection of the system photon.

So far we have analyzed the special case of a delayed-choice quantum erasure for a photon-photon entangled state but analogous considerations could be applied to any other entangled state of two sub-systems 1 and 2. Assume $|\psi_{12}\rangle$ is a pure entangled state of two sub-systems 1 and 2 and A and B are two observables for system 1 and 2, respectively. We indicate by $|A_i\rangle$ $(i=1,\infty)$ and $|B_j\rangle$ $(j=1,\infty)$ a complete set of orthonormal eigenstates of the observables A and B, respectively. Quantum Mechanics predicts that the probability that the pure state $|\psi_{12}\rangle$ is found in the state $|A_KB_L\rangle = |A_K\rangle|B_L\rangle$ is

$$p_{KL} = \left| \langle A_K | \langle B_L | \psi_{12} \rangle \right|^2, \tag{18}$$

where state $|\psi_{12}\rangle$ in the basis $|A_i\rangle|B_j\rangle$ writes

$$|\psi_{12}\rangle = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \langle A_i | \langle B_j | \psi_{12} \rangle | A_i \rangle | B_j \rangle.$$
 (19)

The calculation of the probability p_{KL} in Eq.(18) could be performed in two successive steps. Assume, for instance, that the observable A of system 1 is first measured. The probability that system 1 is found in the state $|A_K\rangle$ is:

$$p_K = \sum_{j=1}^{\infty} \left| \langle A_K | \langle B_j | \psi_{12} \rangle \right|^2 \tag{20}$$

According to Eq.(19), the normalized state of system 2 after this measurement is

$$|\psi_2\rangle = \frac{\sum_{j=1}^{\infty} \langle A_K | \langle B_j | \psi_{12} \rangle | B_j \rangle}{\sqrt{p_K}}.$$
 (21)

From Eq.(21) we infer that the conditional probability $p_{K|L}$ that system 2 if found in the state $|B_L\rangle$ after system 1 is found in the state $|A_K\rangle$ is

$$p_{K|L} = \frac{\left| \langle A_K | \langle B_L | \psi_{12} \rangle \right|^2}{p_K}.$$
 (22)

Then, the joint probability p_{KL} to find both the states $|A_K\rangle$ and $|B_L\rangle$ is $p_{KL}=p_K\times p_{K|L}$ that coincides with the initial expression of p_{KL} given in Eq.(18). The same conclusions are reached if we consider the alternative case where observable B is first measured on system 2. Then, the order of the measurements does not affect the value of the joint probability p_{KL} but, according to the discussion above, we think that the natural time ordered sequence has to be physically preferred because it corresponds to the actual order of the collapses of the entangled state.

II. CONCLUSIONS

In conclusion, we have shown that the interpretation of the delayed-choice experiments with entangled systems does not necessarily imply the counterintuitive conclusion that the natural temporal order between cause and effect has been reversed. Although the Quantum Mechanics formalism in Eq.(18) does not account for the temporal order of the measurements, the delayed-choice results should be analyzed tacking into account for the natural temporal order of the measurements. In particular, in the case of the delayed-choice experiment shown in figure 2, it is the first detection of the system photon by detector D3 (or D4) that determines the polarization state of the entangled environment photon and the correspondent successive detections by detectors D1 and D2. Furthermore, no which-path information is allowed in this delayed case. It has to be emphasized that the analysis above greatly depends on the Quantum Mechanics assumption that the collapse of the wave function occurs instantaneously everywhere in the space. In particular, in the previous analysis it was assumed that the environment photon collapses instantaneously to the states $|E\rangle_e$ or $|E^{\perp}\rangle_e$ when the system photon is collected by detectors D3 and D4, respectively. Without the assumption of an instantaneous collapse (or at the least of a sufficiently fast superluminal collapse), the recent experimental results[10, 13] on the delayed-choice quantum erasure performed in conditions of full space-like separation between the investigated events (random choice and detections of the environment photon and of the system photon) would not have been possible. The non-locality of the quantum measurement process is in apparent contrast with the Relativity theory and this is just the more

controversial aspect of Quantum Mechanics that leads to some well known paradoxes and, in particular, to the EPR(Einstein, Podolsky and Rosen) paradox[14]. Some physicists are unsatisfied with the non-locality of Quantum Mechanics that is in contrast with the predictions of any other previous theory as well as the Maxwell electromagnetic theory and the Relativity theory. Alternative local models based on local hidden variables have been proposed in the past but the recent high accuracy EPR experiments[15–17] on the Bell-inequality[19] have definitely invalidated any hidden variables model also closing the main residual loopholes. In more recent years, some physicists [20, 21] proposed alternative local models where the correlations between entangled particles would be established by superluminal signals propagating in a preferred frame. The existence of a preferred frame where the superluminal signals propagate isotropically in the space is needed to avoid the known causal paradoxes[22, 23]. In the limit case of an infinite velocity of the superluminal communications, these local superluminal models lead to the same predictions of Quantum Mechanics and, thus, no experiment can definitely invalidate them. In this limit case, the choice between Quantum Mechanics and superluminal models would only be a matter of taste. On the contrary, it has been shown [24] that there are special experimental configurations where discrepancies could be experimentally evidenced if the velocity of the superluminal communications had a finite value lower than a maximum detectable velocity that is imposed by the features of the experimental apparatus. Some experiments have been performed in recent years [24–26] to evidence these discrepancies but, so far, the predictions of Quantum Mechanics have been always confirmed. As stated above, these negative results did not allow to invalidate the local superluminal models but only allowed to establish lower bounds for the possible values of the superluminal velocities up to a few million times the speed of light.

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- ment photon gives the which-path information on the system photon. However it has to be noted that this information is lost when the system photon is detected by one of detectors D3 and D4 because it has been shown that the detection by one of the two detectors at the outputs of the interferometer can occur only if the photon has passed through both the paths a and b (note that a photon propagating along one arm of the interferometer can be always considered in a suitable superposition of photon states propagating through both the arms and corresponding at the two incident elliptical polarized states $|E\rangle_s$ and $|E^\perp\rangle_s). Then, the detection of the photon by detector D3 (or D4) comes from a costructive interference at outputs 3 (or 4) and evidences the wave behavior of the photon.$
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