CMPT 440 – Spring 2019: Quantum Finite Automata

Evan McElheny
Due Date: 1/5/2019

Preliminaries

We assume a general understanding on linear algebra and howquantum states are represented in bra-ket notation. We also assume a general understanding of DFA's.

1 Formal Definition

We will focous on measure once QFA's as they are easier to understand then other types of QFA's. Measure once Quantum Finite Automata (MO-QFA's) are defined in the following 5 tuple: $M = (Q, \Theta, |q0\rangle, Q_a)$ Zheng S. (2012). Where Q is the set of quantum orthogonal states, Σ is the language, $|q0\rangle$ is the starting state, $Q_a \subset Q$ is the accepting states, and Θ is a transition function defined as for each $\sigma \in \Sigma$ there is a unitary transformation Θ_σ on the Hilbert space spanned by the states in Q. From this definition we can construct QFA's that can outperform standard DFA's in both size and in possible error (compared to PFA's). For the following examples we will define a language Σ as $\{a\}$ and unitary matrix for such an input as:

$$U_\sigma |q0\rangle \rightarrow cos\theta |q0\rangle + sin\theta |q1\rangle \tag{1}$$
$$U_\sigma |q1\rangle \rightarrow -sin\theta |q0\rangle + cos\theta |q1\rangle \tag{2}$$

2 Smaller Size

As an example we will use promise problems. Promise problems are ones where there are disjoint sets P_yes, P_no where the QFA always accepts all states in Q_yes and always rejects all states in Q_no. For any $k > 0$ we can construct a problem $EVENODD^k$. We will define $P^k_{yes} = \{a^{j2^k} : j = 2m\}$ and $P^k_{no} = \{a^{j2^k} : j = 2m + 1\}$. This example was from Say and Yakarylmaz (2014). With the unitary matrix defined above and $\theta = \frac{\pi}{2^k+1}$ we only need two states $|q0\rangle$, and $|q1\rangle$. On the other hand according to Ambainis and Yakarylmaz we would need 2^{k+1} states in order to solve the same problem on a DFA.

3 Bounded Error

Consider the problem MOD^p for some prime number p. Any PFA that has a bounded error needs at least p states. If we pick $\theta = \frac{2\pi}{p}$ we accept each correct state exactly and accept all other states with a probability of less then one. This false positive acceptance probability is bound by

$$cos^2(\frac{\pi}{p}) = 1 - sin^2(\frac{\pi}{p})$$

Which clearly grows to one as p gets larger. However, you can get a O(log$p$) state machine for a bounded error by composing it of several smaller machines. Therefore it is much faster then a PFA Say and Yakarylmaz (2014).

References

A. Ambainis and A. Yakarylmaz. Superiority of exact quantum automata for promise problems. In *Information Processing Letters, 112(7)*, pages 289–291.

A. C. C. Say and A. Yakarylmaz. Quantum finite automata: A modern introduction. In *Computing with New Resources. Lecture Notes in Computer Science, vol 8808.*, 2014.

L. L. G. J. Zheng S., Qiu D. One-way finite automata with quantum and classical states. In *Languages Alive. Lecture Notes in Computer Science, vol 7300.*, 2012.