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AN EXTENDED PROOF OF THE RAMO–SHOCKLEY THEOREM

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Abstract—An introduction of a new functional form of Green's theorem for inhomogeneous media has enabled us to show that the Ramo–Shockley theorem derived for inhomogeneous media with electrodes maintained at constant potentials is still valid even when the electrode potentials are time-varying. Our proof, which is based on this modified Green's theorem, is an extension of Ramo's original proof and is different from the energy balance method which is the existing method of proof of the theorem for inhomogeneous media at constant electrode potentials. Our proof provides a basis for calculations of the instantaneous currents in semiconductor devices using particle simulations such as the Monte-Carlo method since it differentiates clearly the current induced in the electrodes due to the moving charges from the current caused by the time-varying potentials of the electrodes through capacitive couplings among the electrodes.

1. INTRODUCTION

The Ramo-Shockley theorem[1, 2] was originally devised to compute the instantaneous currents induced in neighboring conductors by a given specified electron motion in vacuum tubes. This theorem has been applied for studying the properties of the current pulses generated in semiconductor radiation detectors[3–5]. It has also been applied for calculating the generation–recombination noise in *pn* junctions[6] and the hot carrier noise in semiconductors[7–9]. But questions have been raised on applicability of the theorem to a case where there exist fixed space charges in addition to the moving charges inside a device[10–13]. Recently these questions have been answered by De Visschere[14] who has shown that Ramo's original theorem is generally valid even when fixed space charges exist inside a device. In his derivation of the theorem, De Visschere has considered only the case where a number of electrodes are located in a medium with a uniform dielectric constant and are maintained at constant potentials. Unfortunately his derivation cannot be extended to the case of an inhomogeneous medium because eqns (5) and (28) of Ref.[14] do not hold for inhomogeneous media. To show that the Ramo–Shockley theorem (eqn (11) of Ref.[14] or eqn (17) of this paper) is valid also for an inhomogeneous medium, De Visschere simply quoted the result of Cavalleri *et al.*[5] (eqn (11) of Ref.[5] or eqn (18) of this paper), which has been derived under the assumption that electrodes are maintained at constant potentials. Furthermore, De Visschere also assumed constant electrode potentials. Since most semiconductor devices are composed of several different materials with different dielectric constants and are operating under time-varying electrode potentials, it is useful to show that the Ramo–Shockley theorem is also applicable to such cases.

In this paper we will prove that the Ramo–Shockley theorem (eqn (11) of Ref.[14]) derived for homogeneous media in the presence of space charges is valid even for an inhomogeneous dielectric medium with an arbitrary fixed charge distribution under time-varying potentials of the electrodes. Since De Visschere's derivation of the theorem [14] cannot be easily extended to the case when the potentials of the electrodes are time-varying, our derivation is different from both that of De Visschere and the energy balance method[5]. An introduction of a new functional form of Green's theorem for inhomogeneous media makes our proof simple and applicable to general cases. We will also show that the result obtained by Cavalleri *et al.*[5] (eqn (11) of Ref.[5] or eqn (30) of Ref.[14]) can be further generalized to have a form given by eqn (26) of this paper.

2. DERIVATION OF THE RAMO–SHOCKLEY THEOREM

We consider a system (see Fig. 1) where M arbitrarily shaped electrodes are located in an inhomogeneous medium whose dielectric constant may vary in space. There are also N charged particles in the medium. The M electrodes are maintained at potentials $\phi_1(t)$, $\phi_1(t), \ldots, \phi_M(t)$ by external power supplies. Let q_i, $\underline{r}_i(t)$ and $\underline{v}_i(t)$ be the charge, position and velocity of the ith charged particle at time t,

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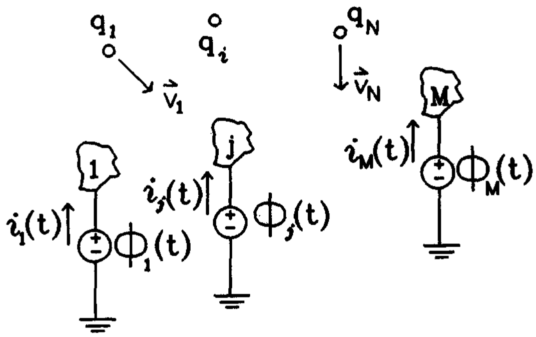

Fig. 1. Schematic representation of electrodes, charges and currents.

respectively, and $\rho(\underline{r}, t)$ be the charge density expressed as:

$$\rho(\underline{r}, t) = \sum_{i=1}^{N} q_i \delta(\underline{r}-\underline{r}_i), \tag{1}$$

where $\delta(\underline{r}-\underline{r}_i)$ is the Dirac-delta function. Here, $\rho(\underline{r}, t)$ includes both free charges and fixed space charges whose velocities are zero. We also assume throughout this paper that the trajectories and velocities of the particles are given. The electric permittivity of the medium is denoted by $\epsilon(\underline{r})$ and is assumed to be linear so that the dielectric constant is independent of both time and field intensity.

Using the superposition principle, we can decompose the potential of our system $V(\underline{r}, t)$ into two parts, $V'(\underline{r}, t)$ and $V''(\underline{r}, t)$, where $V'(\underline{r}, t)$ is the potential due to ρ with all M electrodes grounded, and $V''(\underline{r}, t)$ is the potential due to externally applied voltages with all charges removed from the system, i.e.

$$V(\underline{r}, t) = V'(\underline{r}, t) + V''(\underline{r}, t). \tag{2}$$

Let V_j' be the value of $V'(\underline{r}, t)$ on the jth electrode and V_j'' be the value of $V''(\underline{r}, t)$ on the jth electrode, then $V'(\underline{r}, t)$ and $V''(\underline{r}, t)$ will satisfy the following equations:

$$-\nabla\cdot[\epsilon(\underline{r})\nabla V'(\underline{r}, t)] = \rho(\underline{r}, t), \tag{3}$$

with $V_1' = V_2' = \cdots = V_M' = 0$ and

$$-\nabla\cdot[\epsilon(\underline{r})\nabla V''(\underline{r}, t)] = 0, \tag{4}$$

with $V_1'' = \phi_1(t)$, $V_2'' = \phi_2(t), \ldots, V_M'' = \phi_M(t)$.
In eqns (3) and (4), we have assumed the electrostatic approximation.

Now we introduce a new functional form of Green's theorem for inhomogeneous media:

$$\begin{aligned}\int_V \{&V''(\underline{r}, t)\nabla\cdot[\epsilon(\underline{r})\nabla V'(\underline{r}, t)] \\ &- V'(\underline{r}, t)\nabla\cdot[\epsilon(\underline{r})\nabla V''(\underline{r}, t)]\}\,\mathrm{d}v \\ = \int_S &[V''(\underline{r}_S, t)\epsilon(\underline{r}_S)\nabla V'(\underline{r}_S, t) \\ &- V'(\underline{r}_S, t)\epsilon(\underline{r}_S)\nabla V''(\underline{r}_S, t)]\cdot \mathrm{d}\underline{a},\end{aligned} \tag{5}$$

where V denotes the whole volume of our system except for M electrodes, S is the surface of the M electrodes and $\underline{r}_S$ is a position vector on S. This modified Green theorem makes our proof simple and applicable to general cases.

Substituting eqns (3) and (4) into eqn (5), we readily obtain:

$$-\int_V V''(\underline{r}, t)\rho(\underline{r}, t)\,\mathrm{d}v = \int_S V''(\underline{r}_S, t)\epsilon(\underline{r}_S)\nabla V'(\underline{r}_S, t)\cdot \mathrm{d}\underline{a},$$

which after substitution of eqn (1) on the left-hand-side becomes:

$$-\sum_{i=1}^{N} q_i V''(\underline{r}_i, t) = \sum_{j=1}^{M} \phi_j(t) Q_j'(t), \tag{6}$$

where $Q_j'(t)$ is the charge induced on the jth electrode by the N charged particles and is given by:

$$Q_j'(t) = \int_{S_j} \epsilon(\underline{r}_S)\nabla V'(\underline{r}_S, t)\cdot \mathrm{d}\underline{a}. \tag{7}$$

Now let $f_j(\underline{r})$ denote the electric potential at position $\underline{r}$ when the jth electrode is kept at unit potential while all other electrodes are grounded and all the mobile and fixed charges are removed from our system. Then:

$$V''(\underline{r}, t) = \sum_{j=1}^{M} \phi_j(t) f_j(\underline{r}). \tag{8}$$

We note that $f_j(\underline{r})$ is purely a geometry factor which is independent of t and the charge distribution. Substitution of eqn (8) into eqn (6) yields:

$$\sum_{j=1}^{M} \phi_j(t)[Q_j'(t) + \sum_{i=1}^{N} q_i f_j(\underline{r}_i)] = 0, \tag{9}$$

where $f_j(\underline{r}_i)$ is the value of $f_j(\underline{r})$ at $\underline{r} = \underline{r}_i(t)$. Since

$$\left[Q_j'(t) + \sum_{i=1}^{N} q_i f_j(\underline{r}_i)\right]$$

is independent of the externally applied voltages once $\underline{r}_i(t)$s and $\underline{v}_i(t)$s are given, for eqn (9) to hold for any given values of $\phi_1(t)$, $\phi_2(t), \ldots, \phi_M(t)$,

$$\left[Q_j'(t) + \sum_{i=1}^{N} q_i f_j(\underline{r}_i)\right]$$

should vanish for all j, i.e.

$$Q_j'(t) = -\sum_{i=1}^{N} q_i f_j(\underline{r}_i). \tag{10}$$

Actually, the total charge on the jth electrode denoted by $Q_j(t)$ consists of two components, $Q_j'(t)$ and $Q_j''(t)$:

$$Q_j(t) = Q_j'(t) + Q_j''(t), \tag{11}$$

where $Q_j'(t)$ is given by eqn (7) and $Q_j''(t)$ is given by:

$$Q_j''(t) = \int_{S_j} \epsilon(\underline{r}_S)\nabla V''(\underline{r}_S, t)\cdot \mathrm{d}\underline{a}. \tag{12}$$

$Q_j''(t)$ represents the charge induced on the jth electrode by $\phi_1(t), \phi_2(t), \ldots, \phi_M(t)$ due to capacitive effects. To find the total current flowing in the jth electrode denoted by $i_j(t)$ (see Fig. 1), we differentiate eqn (11) with respect to t:

$$i_j(t) = i_j'(t) + i_j''(t), \tag{13}$$

where

$$i_j(t) = \frac{\mathrm{d}Q_j(t)}{\mathrm{d}t}, \tag{14}$$

$$i_j'(t) = \frac{\mathrm{d}Q_j'(t)}{\mathrm{d}t}, \tag{15}$$

$$i_j''(t) = \frac{\mathrm{d}Q_j''(t)}{\mathrm{d}t}. \tag{16}$$

The current $i_j'(t)$ is solely contributed by the movements of the N charged particles with fixed potentials at electrodes, and the $i_j''(t)$ is the current induced due to the time-varying potentials of the electrodes through capacitive couplings between electrodes. Since the Ramo–Shockley theorem is a theorem which gives the instantaneous current induced in any electrode solely by the specified motions of mobile charges, the theorem is for computing $i_j'(t)$ only. Combining eqns (10) and (15), we obtain the final form of the theorem:

$$i_j'(t) = -\sum_i^N q_i \underline{v}_i(t) \cdot \nabla f_j(\underline{r}_i). \tag{17}$$

This equation has also been derived by De Visschere[14] under the assumptions that the electrodes are kept at constant potentials and dielectric media are uniform.

From eqn (17), on the surface it appears that the fixed space charges and the externally imposed electrode potentials $\phi_1(t), \ldots, \phi_M(t)$ have no direct effect on $i_j'(t)$ once the velocity of the mobile charged particles are given. Actually, the fixed space charges and the electrode potentials contribute to $i_j'(t)$ indirectly by affecting the velocities of the mobile charges.

3. DISCUSSIONS AND CONCLUSIONS

Using the energy balance method, Cavalleri *et al.* [5] have derived the following relation for the case of an inhomogeneous medium (see eqn (11) of Ref. [5]):

$$\sum_{k=1}^M \phi_{ko} i_k'(t) = \sum_{i=1}^N q_i \underline{v}_i(t) \cdot \underline{E}_o''(r_i), \tag{18}$$

where $\underline{E}_o''(\underline{r}_i)$ is the electric field intensity at $\underline{r}_i(t)$ with the M electrodes maintained at constant potentials $\phi_{1o}, \phi_{2o}, \ldots, \phi_{Mo}$ while both the mobile and fixed charges are removed from the system. They have claimed that eqn (18) is an extension of the Ramo–Shockley theorem, but we believe eqn (17) should be the extension of the theorem as pointed out by De Visschere [14].

In order to obtain a more general form of eqn (18) valid for time-varying $\phi_k(t)$s, we simply multiply eqn (17) by $\phi_k(t)$ and sum the resulting equation over k. Since we have a relation

$$\underline{E}''(\underline{r}_i, t) = -\sum_{j=1}^M \phi_j(t) \nabla f_j(r_i),$$

where $\underline{E}''(\underline{r}_i, t)$ is the electric field intensity at $r_i(t)$ due to time-varying potentials $\phi_1(t), \phi_2(t), \ldots, \phi_M(t)$ at the electrodes, we can readily write:

$$\sum_{j=1}^M \phi_j(t) i_j'(t) = \sum_{i=1}^N q_i \underline{v}_i(t) \cdot \underline{E}''(\underline{r}_i, t), \tag{19}$$

which is the generalized form of eqn (18). As indicated in Ref. [14], eqn (17) can be obtained from eqn (19).

As a conclusion, we have shown that the Ramo–Shockley theorem given by eqn (17) is valid for both homogeneous and inhomogeneous media with the electrodes maintained at time-varying or constant potentials. Since the geometry factor $f_j(\underline{r})$ and its gradient in eqn (17) can be obtained for any semiconductor device from the solution of Poisson's equation with all the charged particles removed, and $\underline{v}_i(t)$ can also be obtained from the particle simulation methods such as the Monte-Carlo method, eqn (17) can be used to calculate the short-circuit noise currents in any semiconductor devices under any d.c. operating conditon [15].

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