

A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

A double-entry bookkeeping information system can be viewed as a three dimensional space, where each dimension represents:

- $a \in A$, where A is a finite ordered subset of a predefined set of accounts.
- $d \in D$, where D is a finite ordered subset of all possible dates in the Gregorian calendar.
- $m \in M$, where M is a finite ordered subset of all possible discrete moments in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

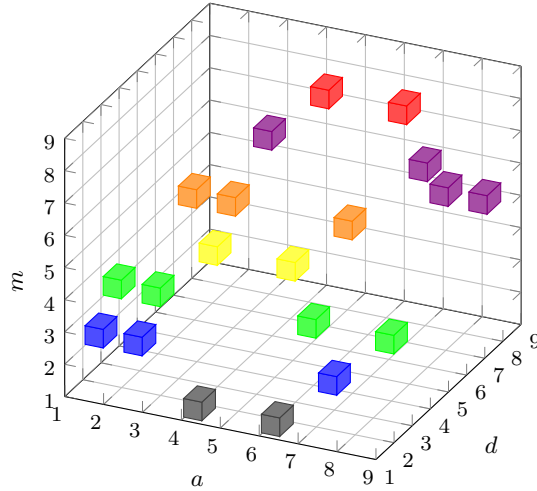


Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes $(4, 1, 1)$ and $(6, 1, 1)$ are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S :

$$S = (s_{ijk}), \text{ where } 1 \leq i \leq |A|, 1 \leq j \leq |D|, 1 \leq k \leq |M|, s_{ijk} \in \mathbb{Z} \quad (1)$$

Satisfying the following property:

$$\forall j \forall k : \sum_{i=1}^{|A|} s_{ijk} = 0 \quad (2)$$

The equations (1) and (2) assume that debits are represented by positive integers and credits by negative integers.

2 Operations

In the following description of the operations, the notation $\langle x, y \rangle$ means a range specification, and $R^{(n)}$ means a set of n range specifications based on the ordered set R .

A *double-entry bookkeeping space* supports the following operations:

APPEND(S, S') append the space $S' = (s'_{ijk})$ to the space S , such that, exists a set M which is associated with both space S and space S' . Let k_1 be the maximum value of k , such that $s_{ijk} \neq 0$, and let k_2 be the minimum value of k , such that $s'_{ijk} \neq 0$. Given that $k_1 < k_2$, then:

$$s_{ijk} = s'_{ijk}, \forall i \forall j, \text{ where } k_2 \leq k \leq |M|.$$

PROJECTION($S, A', D^{(n)}, M^{(m)}$) returns another space $S' = (s'_{ijk})$, where

$A' \subseteq A$, and $D^{(n)} = \{\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle\}$, and $M^{(m)} = \{\langle v_1, w_1 \rangle, \dots, \langle v_m, w_m \rangle\}$, and for all distinct $\{\langle x_i, y_i \rangle\}$ and $\{\langle x_j, y_j \rangle\}$, $\{x_i \dots y_i\} \cap \{x_j \dots y_j\} = \emptyset$, and for all distinct $\{\langle v_i, w_i \rangle\}$ and $\{\langle v_j, w_j \rangle\}$, $\{v_i \dots w_i\} \cap \{v_j \dots w_j\} = \emptyset$. Then:

$$s'_{ijk} = \begin{cases} s_{ijk}, & \text{if } f(j, k) \neq \emptyset \wedge D_j \in X \wedge M_k \in V \\ 0, & \text{otherwise,} \end{cases}$$

where:

$$\begin{aligned}
\sigma_{ijk} &= \sum_{p=j}^{j+|x_r, \dots, y_r|-1} \sum_{q=k}^{k+|v_s, \dots, w_s|-1} s_{ipq}, \\
X &= \{x_1, \dots, x_n\}, \\
V &= \{v_1, \dots, v_m\}, \\
x_r &= D_j, \\
v_s &= M_k, \\
f(j, k) &= \{A_i : \sigma_{ijk} \neq 0\} \cap A', \quad 1 \leq i \leq |A|.
\end{aligned}$$

SLICE($S, A', D^{(n)}, M^{(m)}$) returns another space $S' = (s'_{ijk})$, where $A' \subseteq A$, and $D^{(n)} = \{\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle\}$, and $M^{(m)} = \{\langle v_1, w_1 \rangle, \dots, \langle v_m, w_m \rangle\}$, such that:

$$s'_{ijk} = \begin{cases} s_{ijk}, & \text{if } f(j, k) \neq \emptyset \wedge D_j \in \bigcup_{p=1}^n \{x_p, \dots, y_p\} \wedge M_k \in \bigcup_{q=1}^m \{v_q, \dots, w_q\} \\ 0, & \text{otherwise,} \end{cases}$$

where:

$$f(j, k) = \{A_i : s_{ijk} \neq 0\} \cap A', \quad 1 \leq i \leq |A|.$$

3 Alloy Model