A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

A double-entry bookkeeping information system can be viewed as a three dimensional space, where each dimension represents:

- $a \in A$, where A is a subset of a predefined set of accounts.
- $d \in D$, where D is a subset of all possible dates in the Gregorian calendar.
- $m \in M$, where M is a subset of all possible discrete points in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

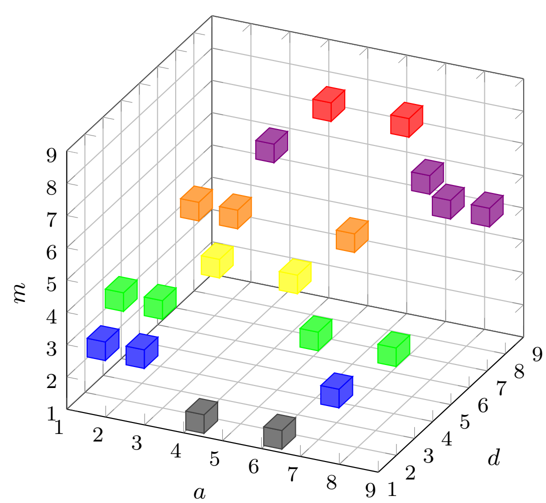

Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S:

$$S = (s_{ijk})\,,\ 1 \leq i \leq |A|,\ 1 \leq j \leq |D|,\ 1 \leq k \leq |M|,\ s_{ijk} \in \mathbb{Z} \tag{1}$$

Satisfying the following property:

$$\forall j\ \forall k, \sum_{i=1}^{|A|} s_{ijk} = 0 \tag{2}$$

The equations (1) and (2) assume that debits are represented by positive integers and credits by negative integers.

2 Operations

A *double-entry bookkeeping space* supports the following operations:

Append(S, S') append the space $S' = \left(s'_{ijk}\right)$ to the space S, such that, given the set M associated with the space S and the set M' associated with the space $S' : M \cap M' = \emptyset$, and

$$s_{ijk} = s'_{ij},\ |M| + 1 \leq k \leq |M| + |M'|.$$

The new set $M \cup M'$ will be the new set associated with S instead of M.

Projection$(S, \langle b_1, e_1 \rangle, \cdots, \langle b_n, e_n \rangle)$ returns a two dimensional array $P = (p_{qr})$ such that:

$$p_{qr} = \sum_{j=b_r}^{e_r} \sum_{k=1}^{|M|} S_{qjk},\ 1 \leq q \leq |A|,\ 1 \leq r \leq n$$

Slice$(S, A', \langle d_b, d_e \rangle, \langle m_b, m_e \rangle)$ returns a potentially smaller space $S' = \left(s'_{pqr}\right)$, such that:

$$\begin{array}{l} s'_{pqr} = s_{pjk}, \\ 1 \leq p \leq |A|,\ 1 \leq q \leq d_e - d_b + 1,\ 1 \leq r \leq m_e - m_b + 1, \\ d_b \leq j \leq d_e,\ m_b \leq k \leq m_e, and \\ \forall A_p \in A' \quad s'_{pqr} > 0. \end{array}$$