A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

A double-entry bookkeeping information system can be viewed as a three dimensional space, where each dimension represents:

- $a \in A$, where A is a subset of a predefined set of accounts.
- $d \in D$, where D is a subset of all possible dates in the Gregorian calendar.
- $m \in M$, where M is a subset of all possible discrete points in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

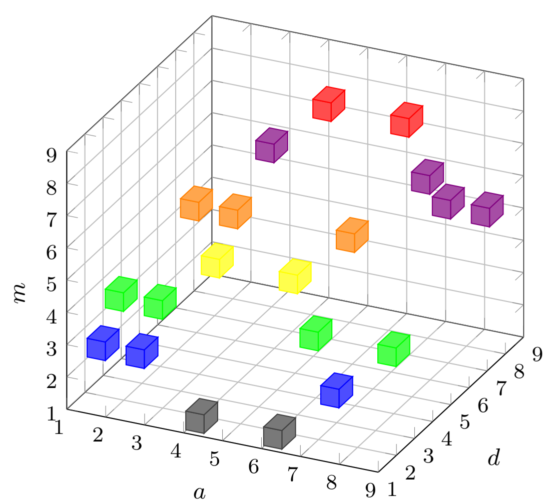

Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S:

$$S = (s_{ijk}),\ 1 \le i \le |A|,\ 1 \le j \le |D|,\ 1 \le k \le |M|,\ s_{ijk} \in \mathbb{Z} \tag{1}$$

Satisfying the following property:

$$\forall j\ \forall k, \sum_{i=1}^{|A|} s_{ijk} = 0 \tag{2}$$

The equations (1) and (2) assume that debits are represented by positive integers and credits by negative integers.

2 Operations

In the following description of the operations, the notation $\langle x, y\rangle$ means a range specification, and $R^{\langle n\rangle}$ means a set of n range specifications.

A *double-entry bookkeeping space* supports the following operations:

Append(S, S') append the space $S' = \left(s'_{ijk}\right)$ to the space S, such that, given the set M associated with the space S and the set M' associated with the space $S' : M \cap M' = \emptyset$, and

$$s_{ij(|M|+k)} = s'_{ijk},\ 1 \le k \le |M'|.$$

The new set $M \cup M'$ will be the new set associated with S instead of M.

Projection$(S, D^{\langle n\rangle}, M^{\langle m\rangle})$ returns a potentially smaller space $S' = \left(s'_{pqr}\right)$, where $D^{\langle n\rangle} = \{\langle x_i, y_i\rangle\}$, and $M^{\langle m\rangle} = \{\langle u_i, w_i\rangle\}$, such that:

$$\begin{aligned} s'_{pqr} &= \sum_{j=x_q}^{y_q} \sum_{k=u_r}^{w_r} s_{pjk} \\ 1 &\le p \le |A| \\ 1 &\le q \le n \\ 1 &\le r \le m. \end{aligned}$$

Slice$(S, A', D^{\langle n\rangle}, M^{\langle m\rangle})$ returns a potentially smaller space $S' = \left(s'_{pqr}\right)$, where

$D^{\langle n \rangle} = \{\langle x_i, y_i \rangle\}$, and $M^{\langle m \rangle} = \{\langle u_i, w_i \rangle\}$, such that:

$$
\begin{aligned}
&s'_{pqr} = s_{pjk} \\
&1 \le p \le |A| \\
&1 \le q \le \sum_{i=1}^{n} y_i - x_i + 1 \\
&1 \le r \le \sum_{i=1}^{m} w_i - u_i + 1 \\
&x_i \le j \le y_i, \forall i \in \{1, \cdots, n\} \\
&u_i \le k \le w_i, \forall i \in \{1, \cdots, m\} \\
\forall A_p \in A' \quad & s'_{pqr} \neq 0.
\end{aligned}
$$