

A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

A double-entry bookkeeping information system can be viewed as a three dimensional space, where each dimension represents:

- $a \in A$, where A is a finite ordered subset of a predefined set of accounts.
- $d \in D$, where D is a finite ordered subset of all possible dates in the Gregorian calendar.
- $m \in M$, where M is a finite ordered subset of all possible discrete moments in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

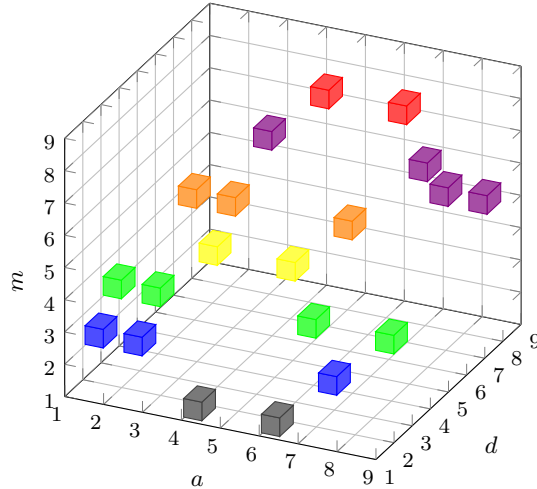


Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes (4, 1, 1) and (6, 1, 1) are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S :

$$S = (s_{ijk}), \ 1 \leq i \leq |A|, \ 1 \leq j \leq |D|, \ 1 \leq k \leq |M|, \ s_{ijk} \in \mathbb{Z} \quad (1)$$

Satisfying the following property:

$$\forall j \ \forall k, \sum_{i=1}^{|A|} s_{ijk} = 0 \quad (2)$$

The equations (1) and (2) assume that debits are represented by positive integers and credits by negative integers.

2 Operations

In the following description of the operations, the notation $\langle x, y \rangle$ means a range specification, and $R^{(n)}$ means a set of n range specifications based on the ordered set R .

A *double-entry bookkeeping space* supports the following operations:

APPEND(S, S') append the space $S' = (s'_{ijk})$ to the space S , such that, exists a set M which is associated with both space S and space S' . Let k_1 be the maximum value of k , such that $s_{ijk} \neq 0$, and let k_2 be the minimum value of k , such that $s'_{ijk} \neq 0$. Given that $k_1 < k_2$, then:

$$s_{ijk} = s'_{ijk}, \ \forall i \ \forall j, \ k_2 \leq k \leq |M|.$$

PROJECTION($S, A', D^{(n)}, M^{(m)}$) returns another space $S' = (s'_{pqr})$, where $A' \subseteq A$, and $D^{(n)} = \{\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle\}$, and $M^{(m)} = \{\langle v_1, w_1 \rangle, \dots, \langle v_m, w_m \rangle\}$, and for all distinct $\{\langle x_i, y_i \rangle\}$ and $\{\langle x_j, y_j \rangle\}$, $\{x_i \dots y_i\} \cap \{x_j \dots y_j\} = \emptyset$, and for all distinct $\{\langle v_i, w_i \rangle\}$ and $\{\langle v_j, w_j \rangle\}$, $\{v_i \dots w_i\} \cap \{v_j \dots w_j\} = \emptyset$. Then:

$$s'_{ijk} = \begin{cases} s_{ijk}, & \text{if } f(j, k) \neq \emptyset \wedge D_j \in X \wedge M_k \in V \\ 0, & \text{otherwise,} \end{cases}$$

Where:

$$\begin{aligned}
\sigma_{ijk} &= \sum_{p=j}^{j+|x_r, \dots, y_r|-1} \sum_{q=k}^{k+|v_s, \dots, w_s|-1} s_{ipq}, \\
X &= \{x_1, \dots, x_n\}, \\
V &= \{v_1, \dots, v_m\}, \\
x_r &= D_j, \\
v_s &= M_k, \\
f(j, k) &= \{A_i : \sigma_{ijk} \neq 0\} \cap A', \quad 1 \leq i \leq |A|.
\end{aligned}$$

SLICE($S, A', D^{(n)}, M^{(m)}$) returns another space $S' = (s'_{ijk})$, where $A' \subseteq A$, and $D^{(n)} = \{\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle\}$, and $M^{(m)} = \{\langle v_1, w_1 \rangle, \dots, \langle v_m, w_m \rangle\}$, such that:

$$s'_{ijk} = \begin{cases} s_{ijk}, & \text{if } f(j, k) \neq \emptyset \wedge D_j \in \bigcup_{p=1}^n \{x_p, \dots, y_p\} \wedge M_k \in \bigcup_{q=1}^m \{v_q, \dots, w_q\} \\ 0, & \text{otherwise,} \end{cases}$$

where:

$$f(j, k) = \{A_i : s_{ijk} \neq 0\} \cap A', \quad 1 \leq i \leq |A|.$$

3 Alloy Model