A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

We can view a double-entry bookkeeping information system as a three dimensional space $S = \{(a, d, m) \mid a \in A, d \in D, m \in M\}$, where,

- A is a finite ordered subset of a predefined set of accounts.
- D is a finite ordered subset of all possible dates in the Gregorian calendar.
- M is a finite ordered subset of all possible discrete moments in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a tuple of the form (a, d, m) and is associated with a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

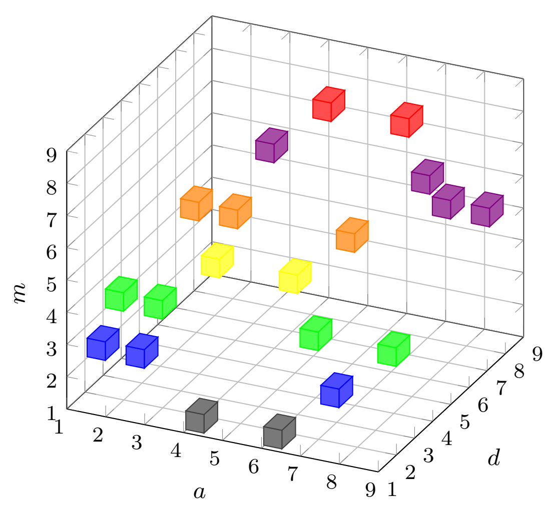

Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes $(4, 1, 1)$ and $(6, 1, 1)$ are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S:

$$S = (s_{ijk})\,, where\ 1 \leq i \leq |A|,\ 1 \leq j \leq |D|,\ 1 \leq k \leq |M|,\ s_{ijk} \in \mathbb{Z} \tag{1}$$

Satisfying the following property:

$$\forall j, k \left(\sum_{i=1}^{|A|} s_{ijk} = 0 \right) \tag{2}$$

In equations (1) and (2) we assume that debits are represented by positive integers and credits by negative integers.

2 Operations

In the following description of the operations, the notation $\langle x, y \rangle$ means a range specification, and $R^{\langle n \rangle}$ means a set of n range specifications based on the ordered set R.

A *double-entry bookkeeping space* supports the following operations:

Append(S, S') append the space $S' = \left(s'_{ijk}\right)$ to the space S, such that, exists a set M which is associated with both space S and space S'. Let k_1 be the maximum value of k, such that $s_{ijk} \neq 0$, and let k_2 be the minimum value of k, such that $s'_{ijk} \neq 0$. Given that $k_1 < k_2$, then:

$$\forall i, j \left(s_{ijk} = s'_{ijk}\right), where\ k_2 \leq k \leq |M|.$$

Projection$(S, A', D^{\langle n \rangle}, M^{\langle m \rangle})$ returns another space $S' = \left(s'_{ijk}\right)$, where $A' \subseteq A$, and $D^{\langle n \rangle} = \{\langle x_1, y_1 \rangle, \ldots, \langle x_n, y_n \rangle\}$, and $M^{\langle m \rangle} = \{\langle v_1, w_1 \rangle, \ldots, \langle v_m, w_m \rangle\}$, and for all distinct $\{\langle x_i, y_i \rangle\}$ and $\{\langle x_j, y_j \rangle\}$, $\{x_i \ldots y_i\} \cap \{x_j \ldots y_j\} = \emptyset$, and for all distinct $\{\langle v_i, w_i \rangle\}$ and $\{\langle v_j, w_j \rangle\}$, $\{v_i \ldots w_i\} \cap \{v_j \ldots w_j\} = \emptyset$. Then:

$$s'_{ijk} = \begin{cases} \sigma_{ijk}, & \text{if } f(j,k) \neq \emptyset \wedge D_j \in X \wedge M_k \in V \\ 0, & \text{otherwise,} \end{cases}$$

where:

$$\begin{aligned}
\sigma_{ijk} &= \sum_{p=j}^{j+|x_r,\ldots,y_r|-1} \sum_{q=k}^{k+|v_s,\ldots,w_s|-1} s_{ipq}, \\
X &= \{x_1, \ldots, x_n\}, \\
V &= \{v_1, \ldots, v_m\}, \\
x_r &= D_j, \\
v_s &= M_k, \\
f(j,k) &= \{A_i : \sigma_{ijk} \neq 0\} \cap A',\ 1 \leq i \leq |A|.
\end{aligned}$$

Slice$(S, A', D^{\langle n\rangle}, M^{\langle m\rangle})$ returns another space $S' = \left(s'_{ijk}\right)$, where $A' \subseteq A$, and $D^{\langle n\rangle} = \{\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle\}$, and $M^{\langle m\rangle} = \{\langle v_1, w_1\rangle, \ldots, \langle v_m, w_m\rangle\}$, such that:

$$s'_{ijk} = \begin{cases} s_{ijk}, & \text{if } f(j,k) \neq \emptyset \wedge D_j \in \bigcup_{p=1}^{n} \{x_p, \ldots, y_p\} \wedge M_k \in \bigcup_{q=1}^{m} \{v_q, \ldots, w_q\} \\ 0, & \text{otherwise,} \end{cases}$$

where:

$$f(j,k) = \{A_i : s_{ijk} \neq 0\} \cap A',\ 1 \leq i \leq |A|.$$

3 Alloy Model