

A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

A double-entry bookkeeping information system can be viewed as a three dimensional space, where each dimension represents:

- $a \in A$, where A is a subset of a predefined set of accounts.
- $d \in D$, where D is a subset of all possible dates in the Gregorian calendar.
- $m \in M$, where M is a subset of all possible discrete points in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

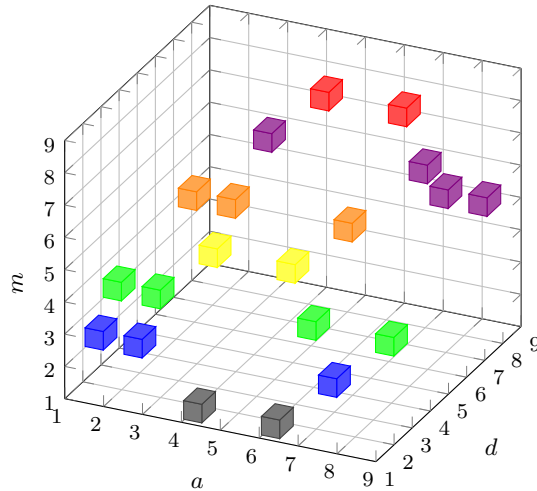


Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S :

$$S = (s_{ijk}), \ 1 \leq i \leq |A|, \ 1 \leq j \leq |D|, \ 1 \leq k \leq |M|, \ s_{ijk} \in \mathbb{Z} \quad (1)$$

Satisfying the following property:

$$\forall j \ \forall k, \sum_{i=1}^{|A|} s_{ijk} = 0 \quad (2)$$

The equations (1) and (2) assume that debits are represented by positive integers and credits by negative integers.

2 Operations

In the following description of the operations, the notation $\langle x, y \rangle$ means a range specification, and $R^{(n)}$ means a set of n range specifications.

A *double-entry bookkeeping space* supports the following operations:

APPEND(S, S') append the space $S' = (s'_{ijk})$ to the space S , such that, given the set M associated with the space S and the set M' associated with the space $S' : M \cap M' = \emptyset$, and

$$s_{ij(|M|+k)} = s'_{ijk}, \ 1 \leq k \leq |M'|.$$

The new set $M \cup M'$ will be the new set associated with S instead of M .

PROJECTION($S, D^{(n)}, M^{(m)}$) returns a potentially smaller space $S' = (s'_{pqr})$, where $D^{(n)} = \{\langle x_i, y_i \rangle\}$, and $M^{(m)} = \{\langle u_i, w_i \rangle\}$, such that:

$$\begin{aligned} s'_{pqr} &= \sum_{j=x_q}^{y_q} \sum_{k=u_r}^{w_r} s_{pjk} \\ 1 &\leq p \leq |A| \\ 1 &\leq q \leq n \\ 1 &\leq r \leq m. \end{aligned}$$

SLICE($S, A', D^{(n)}, M^{(m)}$) returns a potentially smaller space $S' = (s'_{pqr})$, where

$D^{(n)} = \{\langle x_i, y_i \rangle\}$, and $M^{(m)} = \{\langle u_i, w_i \rangle\}$, such that:

$$\begin{aligned}
s'_{pqr} &= s_{pjk} \\
1 &\leq p \leq |A| \\
1 &\leq q \leq \sum_{i=1}^n y_i - x_i + 1 \\
1 &\leq r \leq \sum_{i=1}^m w_i - u_i + 1 \\
x_i &\leq j \leq y_i, \forall i \in \{1, \dots, n\} \\
u_i &\leq k \leq w_i, \forall i \in \{1, \dots, m\} \\
\forall A_p \in A' \quad s'_{pqr} &\neq 0.
\end{aligned}$$

3 Alloy Model

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module model

sig A {}
sig D {}
sig M {}

sig Debit {}
sig Credit {}
sig V in Debit+Credit {}

sig DI {
  start: D,
  end: D
}
sig MI {
  start: M,
  end: M
}

sig S { arr: MI -> DI -> A -> V } {
  all m: MI, d: DI | #arr[m,d].V > 1
  all m: MI, d: DI, a: A | arr[m,d,a] in Debit or arr[m,d,a] in Credit
  all m: MI, d: DI | #arr[m,d,A] & Debit = #arr[m,d,A] & Credit
}

fact DebitAndCreditAreUnitaryUnitsOfAccount {
  all x : V | lone arr.x
}

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assert theSumOfDebitsMustBeEqualsTheSumOfCreditsForTheWholeSpace {
    all s: S | #s.arr[MI,DI,A] & Debit = #s.arr[MI,DI,A] & Credit
}
check theSumOfDebitsMustBeEqualsTheSumOfCreditsForTheWholeSpace
    for 5 but 1 S, 3 A, 5 Int

pred show(s : S) {
    some m: MI, d: DI |
        #s.arr[m,d].V > 2 and
        #s.arr.V.A[m] > 1 and
        #s.arr.V.A.DI > 1
}

run show for 5 but 1 S, 3 A, 5 Int

```