A Model of Double-Entry Bookkeeping Systems

1 Double-Entry Bookkeeping Space

A double-entry bookkeeping information system can be viewed as a three dimensional space, where each dimension represents:

- $a \in A$, where A is a finite ordered subset of a predefined set of accounts.
- $d \in D$, where D is a finite ordered subset of all possible dates in the Gregorian calendar.
- $m \in M$, where M is a finite ordered subset of all possible discrete points in time, given a precision.

Figure 1 shows a *double-entry bookkeeping space* sample, where each cube represents a integer value different from zero. Consider the cubes $(4, 1, 1)$ and $(6, 1, 1)$. They share the same moment and date—the only difference are the accounts. We can interpret this pair of cubes as a transaction recorded in the system, with one cube representing the debit and another representing the credit.

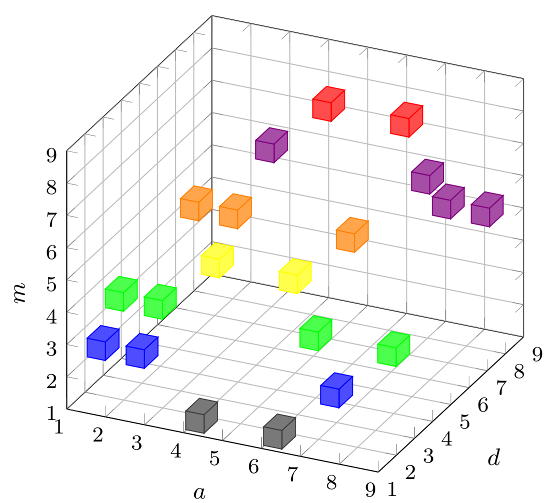

Figure 1: A *double-entry bookkeeping space* sample.

As, by definition, a double-entry transaction must have debits' sum equals to credits' sum, we can deduce that the values for both cubes are the same, in this example.

More specifically, we can have a *double-entry bookkeeping space* represented by a three dimensional array S:

$$S = (s_{ijk}), \ 1 \leq i \leq |A|, \ 1 \leq j \leq |D|, \ 1 \leq k \leq |M|, \ s_{ijk} \in \mathbb{Z} \tag{1}$$

Satisfying the following property:

$$\forall j \ \forall k, \sum_{i=1}^{|A|} s_{ijk} = 0 \tag{2}$$

The equations (1) and (2) assume that debits are represented by positive integers and credits by negative integers.

2 Operations

In the following description of the operations, the notation $\langle x, y \rangle$ means a range specification, and $R^{\langle n \rangle}$ means a set of n range specifications.

A *double-entry bookkeeping space* supports the following operations:

Append(S, S') append the space $S' = \left(s'_{ijk} \right)$ to the space S, such that, exists a set M which is associated with both space S and space S'. Let k_1 be the maximum value of k, such that $s_{ijk} \neq 0$, and let k_2 be the minimum value of k, such that $s'_{ijk} \neq 0$. Given that $k_1 < k_2$, then:

$$s_{ijk} = s'_{ijk}, \ \forall i \ \forall j, \ k_2 \leq k \leq |M|.$$

Projection$(S, D^{\langle n \rangle}, M^{\langle m \rangle})$ returns a potentially smaller space $S' = \left(s'_{pqr} \right)$, where $D^{\langle n \rangle} = \{\langle x_i, y_i \rangle\}$, and $M^{\langle m \rangle} = \{\langle u_i, w_i \rangle\}$, such that:

$$\begin{aligned} s'_{pqr} &= \sum_{j=x_q}^{y_q} \sum_{k=u_r}^{w_r} s_{pjk} \\ 1 &\leq p \leq |A| \\ 1 &\leq q \leq n \\ 1 &\leq r \leq m. \end{aligned}$$

Slice$(S, A', D^{\langle n \rangle}, M^{\langle m \rangle})$ returns another space $S' = \left(s'_{ijk} \right)$, where $D^{\langle n \rangle} = \{\langle x_p, y_p \rangle\}$, and $M^{\langle m \rangle} = \{\langle u_q, w_q \rangle\}$, such that:

$$s'_{ijk} = \begin{cases} s_{ijk}, & \text{if } f(j,k) \cap A' \neq \emptyset \wedge D_j \in \bigcup_{i=1}^{n} x_i, \ldots, y_i \wedge M_k \in \bigcup_{i=1}^{m} u_i, \ldots, v_i \\ 0, & \text{otherwise} \end{cases}$$

where:

$$f(j,k) = \{A_i : s_{ijk} \neq 0 \wedge 1 \leq i \leq |A|\}$$

3 Alloy Model