We defined the category Mota(k) for any adequate equivalence relation - and any field k (also along w/ a field of weff F = 0) Mot(k): Obj: (X, p, n) w/ XE Sch/h P idempotent correspo n EZ (# of pate morphisms:

Hom_{Mot} ((X,p,n),(Y,q,m)) =go CH dimX-P+F(XxY) op This solves the problem of universality for Weil Cohomology theories and this viewpoint helped solve the Weil Conjectures. Motivating Question: What about other imminants of
Schemes? To they have a
Corresponding motive that controls
Vanous classes of invariants? e.g. additive or localizing imanants.

Some additive invariants:
 Ø alg. K-theory (w. Z/e^v coeffs)

 Ø Kanoubi-Villamayor K-Theory

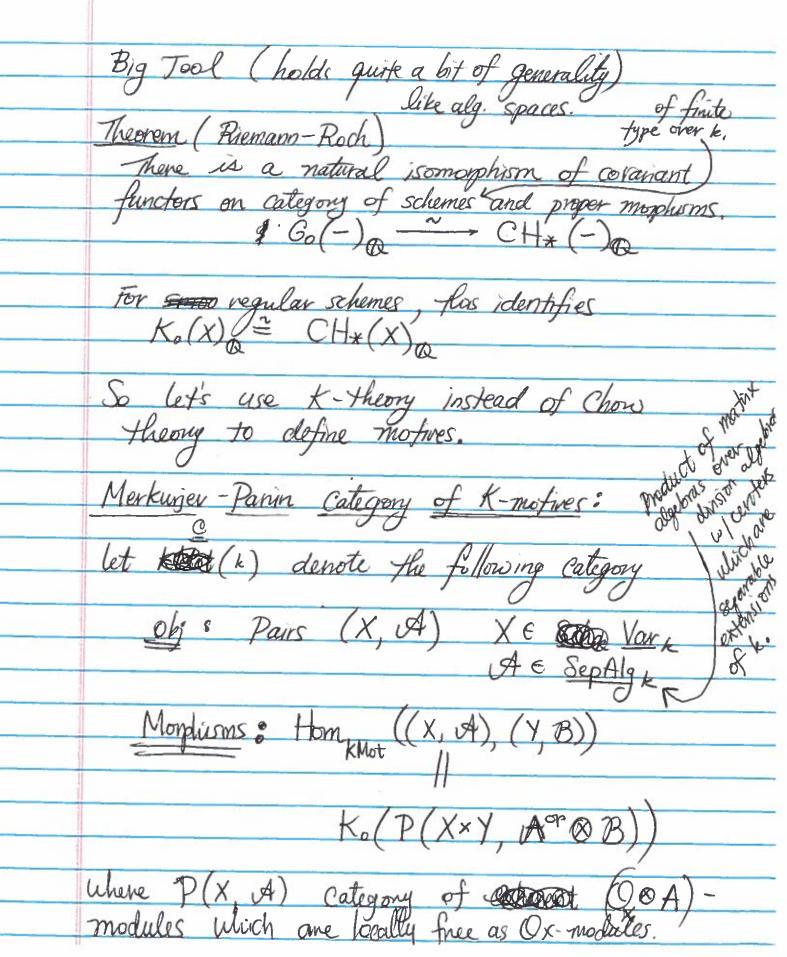
 Ø Nonconnective algebraic K-theory
 @ Homotopy K- theory & Etale K- theory & Mixed complex @ Cyclic Homology @ negative cyclic homology Periodic cyclic homology

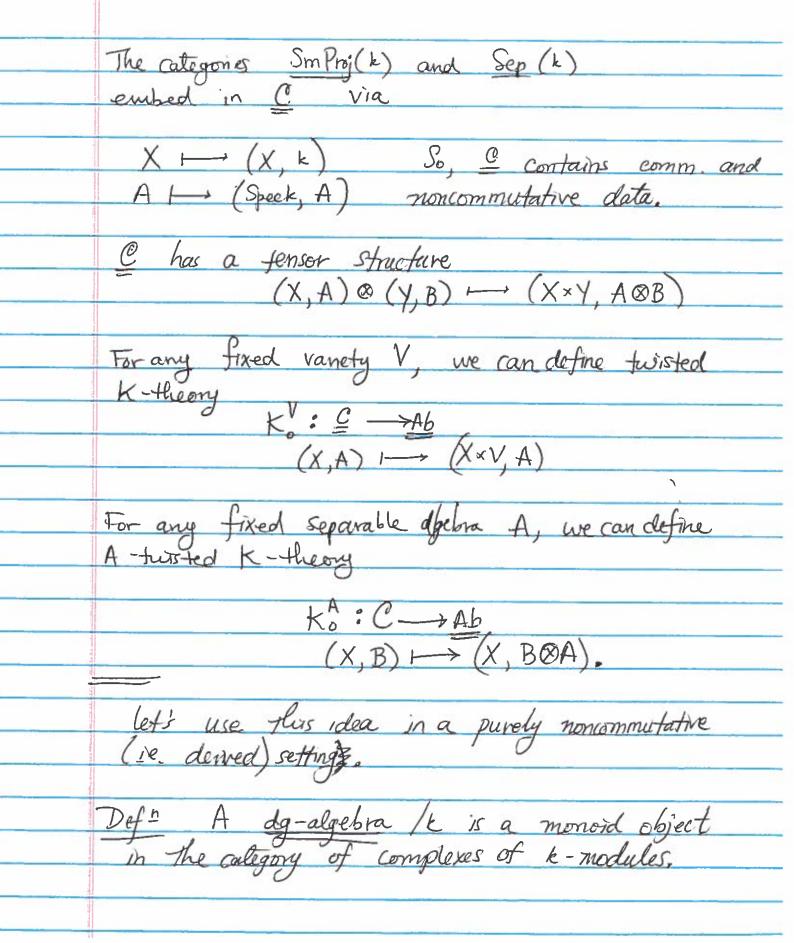
Topological Homology

Topological Cyclic homology. Tabuada Constructs a "universal additive marrant" mapping to allo This provides a francework for studying categories of mothes which are noncommutative analogues of those already defined. Accordinates An additive invainant is one dg-ennached) derved structure, so naturally is viewed from a noncommunitative perspective. inverts Monta equivalences and takes SOD to direct sums)

Preliminaries: K-theory of Schemes. Let P(x) denote the category of V.b. on X (locally free sheaves = locally free Ox-modules) let $K_0(X) = iso(P(X)) / for each SES$ $0 \rightarrow f \rightarrow cy \rightarrow H \rightarrow 0$ [Y] = [f] + [H]. together w/ direct sum, this forms a group w/ Dox forms a ringe Notation: let (x) = category of Cohevent

Ox-modules on X. Same def " as above is denoted Go(X) $\mathcal{P}(X)$ Coho (X) $G_o(x)$ Ko(X) $K^{\circ}(X)$ $K_{\circ}(X)$ $K^{\bullet}(X)$ $K_{\bullet}(X)$ Fact: These are the same if X is so regular. (Resolution theorem: every Coherent sheaf has a resolution by locally frees. For X = Spec R, P(X) f.g. projective modules M(X) f.g. projective modules.





(A. & B.)
$$n = \bigoplus_{i \neq j=n}^{\infty} \bigoplus_{i \neq j=n}^{\infty} (A_i \otimes B_j)$$

That is, it's a perpet A. A. A. A. What is, it satisfies leibniz Rule:

Alab) = $d(a)b + (-1)deg a.d(b)$

Def. A dg-category /k is a category

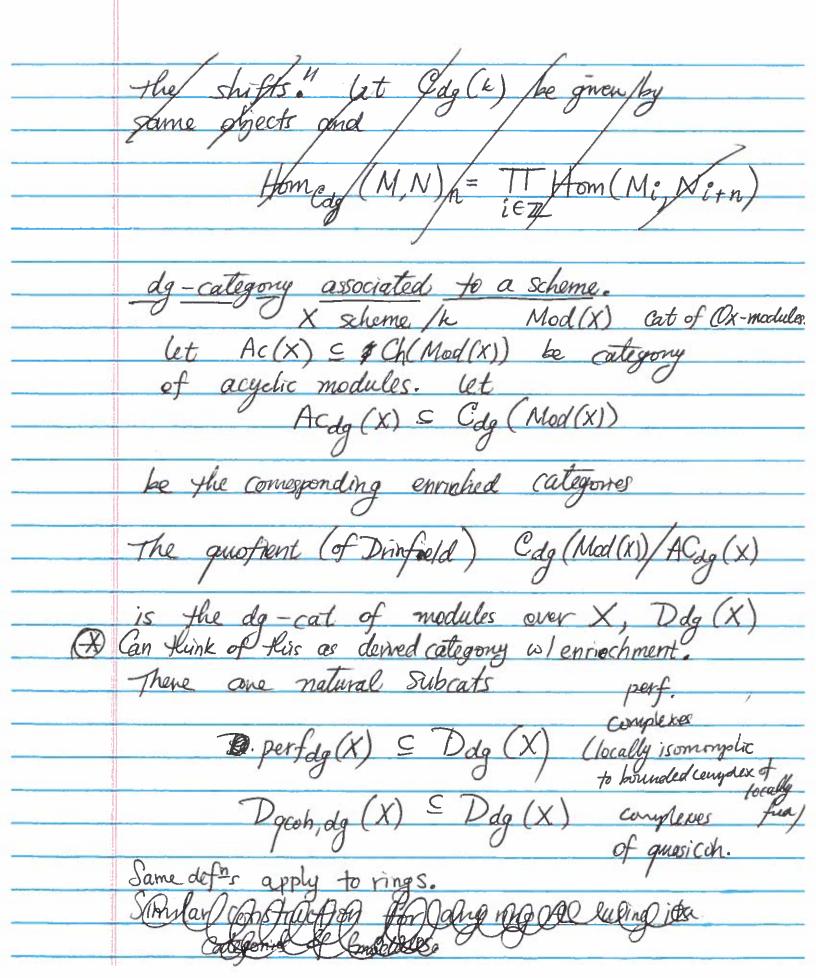
enriched over dg-algebras ever k

(ie. all hom sets are dg-algebras/k).

Ex: Any dg-algebra is a dg-category w/ one obj.

We can enrich this category by remembering the shifts. Let Cdg(k) have same objects and Hom Cdg (M,N) $n = 11$ Hom (Ni, Ni+k)

B If A, B are dg-category of the category of the category



The category NChow(k) is the idempotent completion of the category consisting of obj: nice (ie. smooth/proper) de categories /k. e.g. Dag (X) for X smooth proper scheme/k. maps: Homerchow (A,B) = Ko (APBB). on schemes Homnichow (perfol), perfol) = Ko (XXY) Notice: Kaggings Ko doesn't neflect dimension in the same way as ECH' = CH+, so we can't expect to have an object which shifts dimension, i.e. Tate tursts. Theorem (tabuada) There is an embedding 1 Sm Proj(k) op ____ Chow(k) -> Chow(k)/- OT NChow(k) as long as we timalize the motive, Chow motives ended into noncommutative