

**Problem 9):** To begin, consider an arbitrary cyclic group  $\mathbb{G}$  and a generator  $g \in \mathcal{G}$ . Then, given any two group elements  $h_1$  and  $h_2$ , we define the function  $\text{DH}(h_1, h_2)$  as

$$\text{DH}(h_1, h_2) = g^{\log_g h_1 \cdot \log_g h_2} \quad (9.1)$$

The **DDH Assumption** is that the result of  $\text{DH}(h_1, h_2)$ , on any uniform group elements  $h_1$  and  $h_2$ , is indistinguishable from any other uniform element of the group.

Now, we define the **El Gamal encryption** algorithm according to

- Accept input public key  $pk = \langle \mathbb{G}, q, g, h \rangle$ .
- Chose  $y \leftarrow \mathbb{Z}_q$
- Output the cipher text  $c = \langle c_1, c_2 \rangle$  determined according to

$$c = \langle c_1, c_2 \rangle = \langle g^y, h^y \cdot m \rangle$$

We also note that the **El Gamal encryption** algorithm uses the private key  $sk = \langle \mathbb{G}, q, g, x \rangle$ . Where  $(\mathbb{G}, q, g, )$  is obtained from a generator and  $h$  is defined  $h \equiv g^x$ . Therefore, cipher-text may be encrypted according to the relation

For cipher-text  $c = \langle c_1, c_2 \rangle$  decrypt using  $m \equiv \frac{c_2}{c_1^x}$