Problem 1): We know that the expression

$$\Pr\left[M = m \,|\, C = c\right] = \Pr\left[M = m\right], \tag{1.1}$$

which holds for some encryption scheme `(Gen, Enc, Dec)`. We apply Bayes' formula to the left-hand-side of expression 1.1 to yield

$$\Pr\left[M = m \,|\, C = c\right] = \frac{\Pr\left[M = m, C = c\right]}{\Pr\left[C = c\right]} \tag{1.2}$$

Since the $\Pr\left[M = m, C = c\right]$ term in expression 1.2 represents a Joint Probability Distribution, we also have

$$\Pr\left[M = m, C = c\right] = \Pr\left[M = m\right]\,\Pr\left[C = c \,|\, M = m\right]$$

This allows the result in expression 1.2 to be equivalently expressed as

$$\Pr\left[M = m \,|\, C = c\right] = \frac{\Pr\left[M = m\right]\,\Pr\left[C = c \,|\, M = m\right]}{\Pr\left[C = c\right]}$$

which is divided by $\Pr\left[M = m\right]$ to give

$$\frac{\Pr\left[M = m \,|\, C = c\right]}{\Pr\left[M = m\right]} = \frac{\Pr\left[C = c \,|\, M = m\right]}{\Pr\left[C = c\right]} \tag{1.3}$$

By noting that expression 1.1 implies

$$\frac{\Pr\left[M = m \,|\, C = c\right]}{\Pr\left[M = m\right]} = 1,$$

the result in expression 1.3 becomes

$$\frac{\Pr[C = c \mid M = m]}{\Pr[C = c]} = 1$$

Multiplying this result by $\Pr[C = c]$ gives

$$\Pr[C = c \mid M = m] = \Pr[C = c], \tag{1.4}$$

thereby proving that $\Pr[M = m \mid C = c] = \Pr[M = m]$ implies $\Pr[C = c \mid M = m] = \Pr[C = c]$.

$\square$