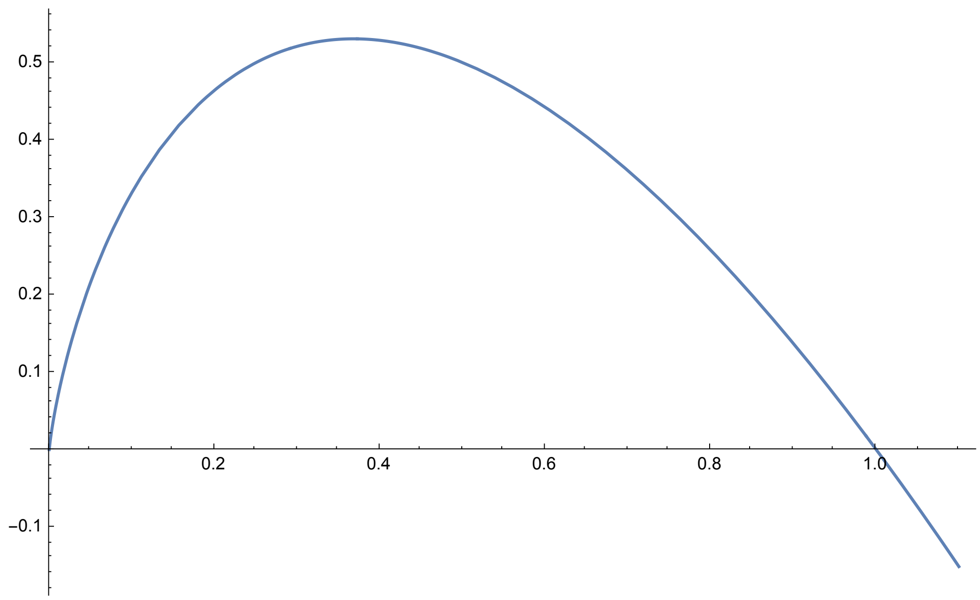
Homework 1

Problem 2.3

Graph

In[68]:=
```
Plot[{-x Log[2, x]}, {x, 0, 1.1}]
```

Out[68]=



Problem 2.10 (a)

Definitions

Define a simplification function based on $0 \le \alpha \le 1$

In[79]:=
```
FS10a[aa_] := FullSimplify[aa, Assumptions → {α ∈ Reals, 0 ≤ α ≤ 1}]
```

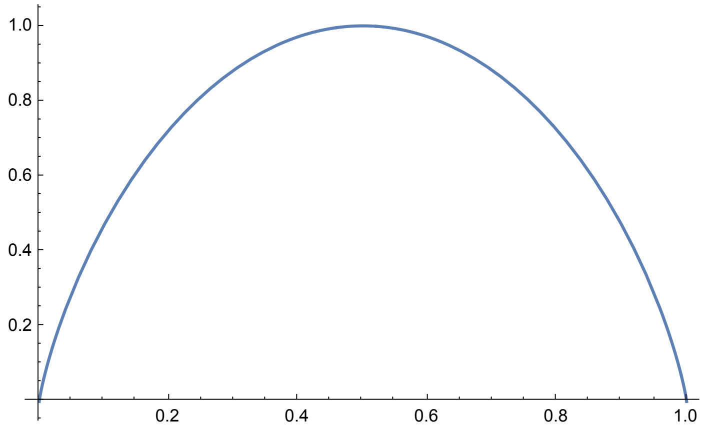
Define $H(\alpha)$ in terms of the binary probability function

In[94]:=
```
Hα[pp_] := -pp Log[2, pp] - (1 - pp) Log[2, 1 - pp]
```

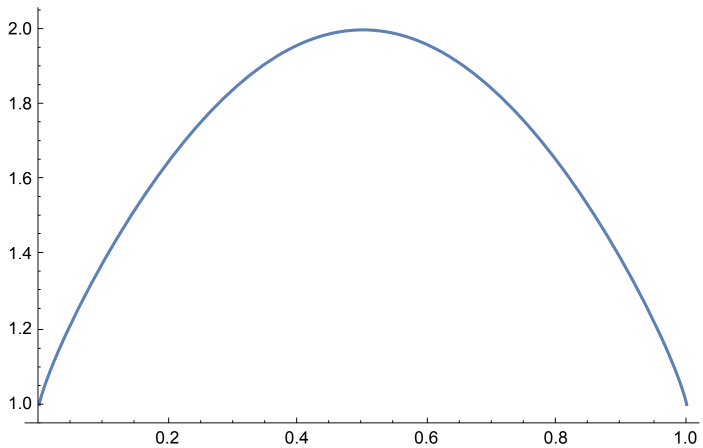
Problem 2.10 (b)

Mininize

In[97]:=
```
Plot[(Hα[x]), {x, 0, 1}]
```

Out[97]=


In[98]:=
```
Plot[2^(Hα[x]), {x, 0, 1}]
```

Out[98]=


In[99]:=
```
D[α X1 + (1 - α) X2, α] // FS10a
```

Out[99]= X1 – X2

In[100]:=
```
Solve[D[Hα[α] + α X1 + (1 - α) X2, α] == 0, α] // FS10a
```

Out[100]= $\left\{\left\{\alpha \to \frac{1}{1 + 2^{-X1+X2}}\right\}\right\}$

In[105]:=
```
(Hα[α] + (α) (X1 + X2)) /. Solve[D[Hα[α] + α X1 + (1 - α) X2, α] == 0, α][[1]] // FS10a
(Hα[α] + (1) (X1 + X2)) /. Solve[D[Hα[α] + α X1 + (1 - α) X2, α] == 0, α][[1]] // FS10a
(Hα[α] + (1 + α) (X1 + X2)) /. Solve[D[Hα[α] + α X1 + (1 - α) X2, α] == 0, α][[1]] // FS10a
```

Out[105]=
$$\frac{-2^{X2}\,\mathrm{Log}\left[\frac{1}{1+2^{X1-X2}}\right]+2^{X1}\left((X1+X2)\,\mathrm{Log}[2]-\mathrm{Log}\left[\frac{1}{1+2^{-X1+X2}}\right]\right)}{\left(2^{X1}+2^{X2}\right)\mathrm{Log}[2]}$$

Out[106]=
$$\frac{\left(2^{X1}+2^{X2}\right)(X1+X2)\,\mathrm{Log}[2]-2^{X2}\,\mathrm{Log}\left[\frac{1}{1+2^{X1-X2}}\right]-2^{X1}\,\mathrm{Log}\left[\frac{1}{1+2^{-X1+X2}}\right]}{\left(2^{X1}+2^{X2}\right)\mathrm{Log}[2]}$$

Out[107]=
$$\frac{2^{X2}\left((X1+X2)\,\mathrm{Log}[2]-\mathrm{Log}\left[\frac{1}{1+2^{X1-X2}}\right]\right)+2^{X1}\left((X1+X2)\,\mathrm{Log}[4]-\mathrm{Log}\left[\frac{1}{1+2^{-X1+X2}}\right]\right)}{\left(2^{X1}+2^{X2}\right)\mathrm{Log}[2]}$$

In[101]:=
```
(Hα[α] + α X1 + (1 - α) X2) /. Solve[D[Hα[α] + α X1 + (1 - α) X2, α] == 0, α][[1]] // FS10a
```

Out[101]=
$$\frac{2^{X2}\left(X2\,\mathrm{Log}[2]-\mathrm{Log}\left[\frac{1}{1+2^{X1-X2}}\right]\right)+2^{X1}\left(X1\,\mathrm{Log}[2]-\mathrm{Log}\left[\frac{1}{1+2^{-X1+X2}}\right]\right)}{\left(2^{X1}+2^{X2}\right)\mathrm{Log}[2]}$$

Problem 2.24 (b)

Definitions

Define a simplification function based on $0 \le p \le 1$

In[69]:=
```
FS24b[aa_] := FullSimplify[aa, Assumptions → {p ∈ Reals, 0 ≤ p ≤ 1}]
```

Define the binary entropy function in terms of base 2 logarithms

In[70]:=
```
H[pp_] := -pp Log[2, pp] - (1 - pp) Log[2, (1 - pp)]
H[p] // FS24b
```

Out[71]=
$$\frac{(-1+p)\,\mathrm{Log}[1-p]-p\,\mathrm{Log}[p]}{\mathrm{Log}[2]}$$

Define the binary entropy function in terms of natural logarithms

In[72]:=
```
He[pp_] := -pp Log[pp] - (1 - pp) Log[1 - pp]
He[p] // FS24b
```

Out[73]= $(-1+p)\,\mathrm{Log}[1-p]-p\,\mathrm{Log}[p]$

Integrate

Using base 2 logarithms

In[74]:=
```
Integrate[H[p], {p, 0, 1}] // FS24b
Integrate[H[p], {p, 0, 1}] // FS24b // N
```

Out[74]= $\frac{1}{\text{Log}[4]}$

Out[75]= 0.721348

Using natural logarithms

In[76]:=
```
Integrate[He[p], {p, 0, 1}] // FS24b
Integrate[He[p], {p, 0, 1}] // FS24b // N
```

Out[76]= $\frac{1}{2}$

Out[77]= 0.5

Checking that changing from nats to bit yields the same result.

In[78]:=
```
(1 / 2) Log[2, e] // N
```

Out[78]= 0.721348