Homework #1

Jonathan McFadden

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Information Theory

Problem 2.3): Let $\mathbb{P}^n$ be the set of all n-dimensional probability vectors. Futhermore, let $\vec{p}$ be any element of $\mathbb{P}^n$ ($\vec{p} \in \mathbb{P}^n$) and define $\vec{p}$ as $\vec{p} = (p_1, p_2, \ldots, p_i, \ldots, p_n)$, where $i \in \mathbb{Z}^+ \ni i \leq n$. By the definition of a probability space, we must have

$$\vec{p} \cdot \vec{1} = \sum_{i=1}^{n} \{p_i\} = 1 \ , \ \ \forall \vec{p} \in \mathbb{P}^n, \tag{2.3-1}$$

where $\vec{1} = (1, 1, \ldots, 1, \ldots, 1)$ is the n-dimensional vector having the value 1 for each of its components. Additionally, we may equivalently state, that for $i \in \mathbb{Z}^+ \ni i \leq n$, the vector $\vec{1}$ can be defined as $\vec{1} = (q_1, q_2, \ldots, q_i, \ldots, q_n)$ where $q_i = 1, \forall i \in [1, n] \subseteq \mathbb{Z}^+$.

Now, $\forall i \in \mathbb{Z}^+ \ni i \leq n$, it is clear that the relation $p_i \log_2 [p_i] \geq 0$, holds. Moreover, it is also apparent that the relation $p_i \log_2 [p_i] \geq 0$ simplifies to equality ($p_i \log_2 [p_i] = 0$) for the cases where either $p_i = 0$ or $p_i = 1$. Now, from the result in **??**, it is clear that *only* **<u>ONE</u>** p_i in each $\vec{p} \in \mathbb{P}^n$ may have the value $p_i = 1$. Moreover, for the case where one element p_i of $\vec{p}$ has the value $p_i = 1$, the expression in **??** also requires that $p_j = 0, \forall j \in [1, n] \subseteq \mathbb{Z}^+ \ni j \neq i$. That is to say, of one element p_i of $\vec{p}$ is $p_i = 1$, then all other elements of $\vec{p}$,