MATH 308 **Group Project**

Work with your group to complete this project. A careful writeup of your results will be submitted in late November. Details about the due date and the format of the writeup will be announced in class and on Blackboard.

A positive integer is a *lucky number* if it can be written as the sum of positive integers whose reciprocals sum to 1. A positive integer that is not lucky is an *unlucky number*. A positive integer is a *happy number* if it can be written as the product of positive integers whose reciprocals sum to 1, while a positive integer that is not happy is an *unhappy number*.

Note: Do not attempt to look up answers anywhere. These terms are not standard, so Google and Wikipedia are your enemies on this assignment. You may apply any mathematical or logical techniques you like to the problem, and programming skills of any members of your group may be utilized. I will give hints if asked. Some hints are free, but bigger hints will cost your group some points.

Consider the three following claims.

Claim 1 *Suppose n is a happy number that is written as the product of k lucky numbers. Then n can be written as the sum of j positive integers whose reciprocals sum to k.*

Claim 2 *If n is lucky, then n is happy.*

Claim 3 *If n is happy, then n is lucky.*

Answer the following questions.

1. Prove or disprove each of the three claims and salvage if possible.

2. How many unlucky numbers are there? Prove it.

3. How many unhappy numbers are there? Prove it.