Low-Rank 3D Tensor Completion

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Overview

1. Motivations
2. Overview of algebraic tools
3. Description of the problem and algorithms
   - ALS
   - GeomCG
4. Examples

Motivations

- Movies
- Signal processing
- Multiple parameters approximation

Matricisation

How to matricise

Tucker Format

Tucker

Problem as in ALS

MB and stuff
Inpaiting
Two Algorithms

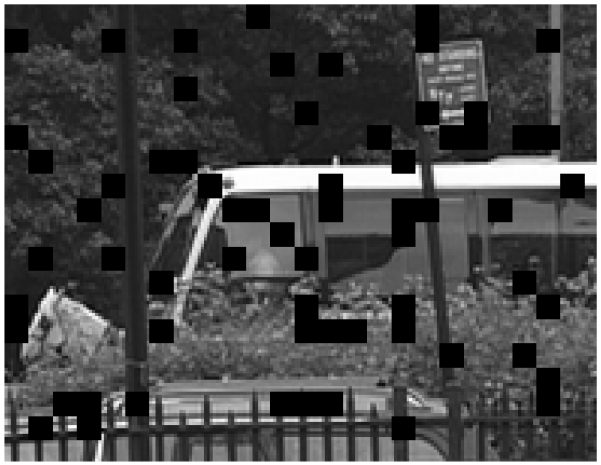
Alternating Least Squares Algorithm

1. Form $\boldsymbol{\mathcal{X}}$ from $\boldsymbol{P}^{0,\dots,K-1}$ using Block Matching criteria
2. $(\boldsymbol{\mathcal{X}}(:,:,1))_{\overline{\Omega}} = \left(\frac{1}{K-1}\sum_{i=1}^{K-1}\boldsymbol{P}^i\right)_{\overline{\Omega}}$
3. Choose mode ranks $\{\mathrm{R}_1, \mathrm{R}_2, \mathrm{R}_3\}$, tolerance σ;
   Initialize $\boldsymbol{A}^{(1)}, \boldsymbol{A}^{(2)}, \boldsymbol{A}^{(3)}$
4. $\boldsymbol{A}^{(3)}(:,1) = [1,\dots,1]^T/K$
5. for n = 1,2,3
   $\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{X}} \times_1 \dots \times_{n-1} \boldsymbol{A}^{(n-1)T} \times_{n+1} \boldsymbol{A}^{(n+1)T} \dots$
   $\boldsymbol{Y}_n \leftarrow$ unfold $\boldsymbol{\mathcal{Y}}$ in mode n
   $\boldsymbol{A}^{(n)} \leftarrow$ first R_n principal component of $\boldsymbol{Y}_n$
   end
6. $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{X}} \times_1 \boldsymbol{A}^{(1)T} \times_2 \boldsymbol{A}^{(2)T} \times_3 \boldsymbol{A}^{(3)T}$
7. $\boldsymbol{\mathcal{X}}_l = \boldsymbol{\mathcal{G}} \times_1 \boldsymbol{A}^{(1)} \times_2 \boldsymbol{A}^{(2)} \times_3 \boldsymbol{A}^{(3)}$
8. If $\|\boldsymbol{\mathcal{X}}_l - \boldsymbol{\mathcal{X}}\|_F \leq \sigma$ STOP, otherwise return to Step 4.
9. Recover missing area in $\boldsymbol{P}^0$: $\left(\boldsymbol{P}^0\right)_{\overline{\Omega}} = (\boldsymbol{\mathcal{X}}_l(:,:,1))_{\overline{\Omega}}$

GeomCG

Optimization on the manifold

Movie reconstruction



Error Plot

Error plot, convergence in one iteration

Importance of initial guess

Plot initial guess

Inpainting






Comparison with GeomCG

Compare results for rank 1

D.T. Nguyen, M.D. Dao, T.D. Tran, *The John Hopkins University*, 2011

Error Concealment Via 3-Mode Tensor Approximation

18th IEEE Conference on Image Processing

D.Kressner, M. Steinlechner, B.Vandereycken, *École Polytehcnique Fédérale de Lausanne*, 2013

Low-Rank Tensor Completion by Riemannian Optimization

T.G. Kolda, B.W. Bader, *Sandia National Laboratories*, 2009

Tensor Decomposition and Applications

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