ENME403 CONTROLS LAB

OVERVIEW

Figure 1. Your boss.

Congratulations! You have landed a Graduate Controls Engineer position at Chase Controls Inc. It's your first day on the job, and to make sure you learned something in ENME403, your boss (Figure 1) has tasked you with investigating potential control design methods for a top-secret device. To analyse these methods, you must design controllers for **two** systems: an inverted pendulum on a cart, and three spring-connected carts.

TASK

For **both** systems:

1. Model the system in MATLAB using state space equations (see 'Tips' at end).
2. Design control gains $\mathbf{K}$ and tracking gain N to meet design goals, using a) pole placement and b) LQR.
3. Choose 1-2 sets of gains for each method to bring to lab and test on the physical systems.
4. Compare your simulated results for each method to the real results.
5. Iterate on your design (if desired), and test in the lab once more.

REPORT

Your boss has requested a **concise**, **individually-written** technical report on your work. Your report should be 5 pages **maximum** with 12pt font and single spacing with "standard margins", and ***must*** consist of the following sections:

1. **Recommendations** (~ 0.25 pages)
   - A *very* short summary of your recommended gains for each system to meet design goals as well as possible.
2. **Results** (~ 2 - 2.5 pages)
   - Plot simulation and lab results with carefully created plots showing key results and comparisons of different control approaches, gains, methods, etc.
   - Use only modest amounts of written text to describe results and what they mean.
   - Present performance with respect to design goals using tables and brief written descriptions.
3. **Discussion** (~ 1.5 pages)
   - Compare and contrast your control design methods. Which yielded better results and why?
   - Did your lab results match simulation? If not, why?
   - Justify your gain/method recommendations.
   - Is there anything that should have been differently, or in addition?
   - In all cases, refer explicitly to your figures and tables in the Results. Always!
4. **Method** (~ 0.5 - 1 page)
   - What did you to do get your recommended gains? (Do not re-explain the system model!)
   - How did you determine these were optimal, or your best set? (see 'Tips')

This report is ***not*** expected to be a formal lab writeup. Explaining the systems in detail or deriving their EOMs is not required (nor desired). Your audience (i.e. Geoff) knows the theory, so focus on presenting your process and analysis – what **you** did, how **you** did it, and **why** you got the results you did.

SYSTEM 1: INVERTED PENDULUM

Design Goal

The inverted pendulum shown in Figure 2 is unstable in open loop and will fall over if the cart is pushed. Design a full-state feedback controller to keep the pendulum upright and stable while the cart accurately follows step inputs of magnitude $r(t)$ = 0.1 m back and forth. The controller must balance the requirements of a) getting the cart to its destination quickly, and b) ensuring the pendulum does not fall. Ideal performance will have a rise time (time to reach target position) of under 1-3 seconds for x, and settling time (time to return to stationary position) of under 5-7 seconds for x and θ.

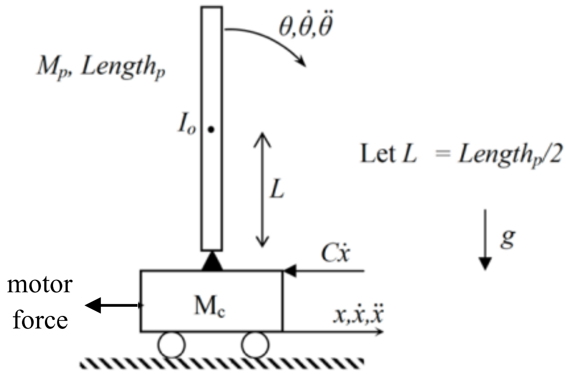

Figure 2. Diagram of inverted pendulum system. A motor mounted on the cart provides force in response to in response to voltage input V. Feedback control is required to stabilise the pendulum while the cart moves to commanded positions.

The motor driving the cart has physical limits on its voltage input (V):

- Voltage limit of $|V| < 10$ V. This is a hardware limitation, commanding a greater voltage will result in saturation.
- Slew rate limit of $\left|\frac{dV}{dt}\right| < 30$ V/s. If your controller commands changes in voltage faster than this rate, the motor driver will not be able to keep up.

State Space Equations

The state space equations for this system (assuming small values of θ) are as follows, in the standard form $\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B_1}u, \;\; \boldsymbol{y} = \mathbf{C_1}\boldsymbol{x}$. Parameter values are provided in Table 1.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-M_p^2 L^2 g}{(M_c + M_p)I_0 + M_c M_p L^2} & \dfrac{(I_0 + M_p L^2)(CRr^2 + k_m^2 k_g^2)}{[(M_c + M_p)I_0 + M_c M_p L^2]Rr^2} & 0 \\ 0 & \dfrac{(M_c + M_p)M_p L g}{(M_c + M_p)I_0 + M_c M_p L^2} & \dfrac{-M_p L(CRr^2 + k_m^2 k_g^2)}{[(M_c + M_p)I_0 + M_c M_p L^2]Rr^2} & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{-(I_0 + M_p L^2)k_m k_g}{[(M_c + M_p)I_0 + M_c M_p L^2]Rr} \\ \dfrac{M_p L k_m k_g}{[(M_c + M_p)I_0 + M_c M_p L^2]Rr} \end{bmatrix} u$$

$$\boldsymbol{y} = \mathbf{I_{4\times 4}}\,\boldsymbol{x}$$

with state vector $\boldsymbol{x} = \begin{pmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{pmatrix} \begin{matrix} \text{[m]} \\ \text{[rad]} \\ \text{[m/s]} \\ \text{[rad/s]} \end{matrix}$ and u = motor input voltage V [Volts].

Table 1. Parameters for inverted pendulum system.

Symbol	Description	Value	Unit
M_p	Pendulum mass	0.215	kg
M_c	Cart mass	1.608	kg
L	Effective pendulum half-length	0.314	m
I_0	Pendulum moment of inertia about centre of mass	7.06×10^{-3}	kg·m^2
R	Motor terminal resistance	0.16	Ω
r	Pinion radius	0.0184	m
k_g	Gearing ratio	3.71	-
k_m	Motor back EMF constant	0.0168	V·s / rad
C	Damping on cart	0*	N·s / m

*The cart damping C is not measured. You may assume the system is undamped… or otherwise investigate the impact of increasing C on your outcomes. We suggest you consider both!

To Bring to Lab Session

- Four gains, $\mathbf{K} = [K_1 \quad K_2 \quad K_3 \quad K_4]$ for contributions of x, θ, $\dot{x}$, and $\dot{\theta}$ respectively.
- Tracking gain N for tracking step inputs of magnitude $r(t)$ = 0.1 m.
    - Note: The state which needs to track the step input is cart position x, so for the purposes of calculating N, use $\mathbf{C} = [1 \quad 0 \quad 0 \quad 0]$.

SYSTEM 2: THREE-CART CONTROLLER

Design Goal

The MDOF spring-mass-damper system can be used to model many different, complex real-world systems such as robotic motion control. An example of this is the linear 3-cart system shown in Figure 3. For this system, your task is to design a full-state feedback controller to move Cart 3 accurately to commanded positions, while achieving fast rise times (time to reach target) and settling times (time for system to return to stationary).

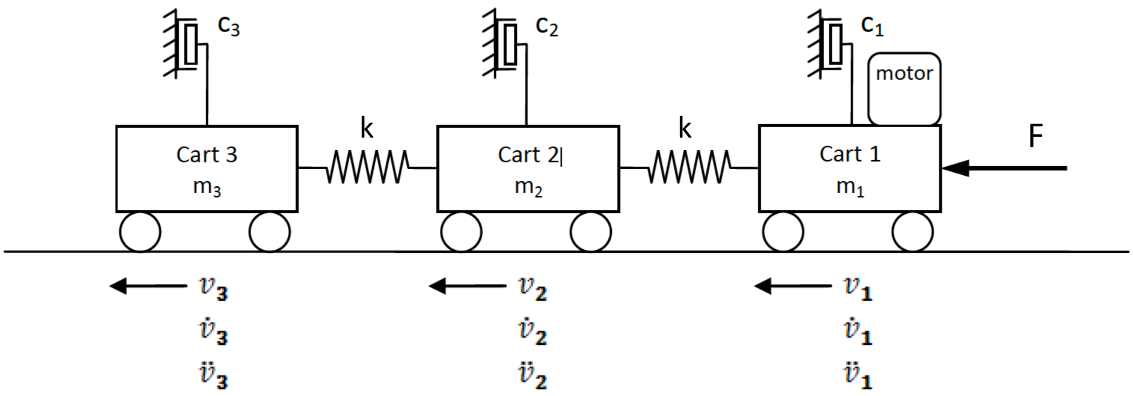

Figure 3. Diagram of 3-cart SMD system. A motor on Cart 1 provides force F in response to voltage input V. Feedback control is required to control the position of Cart 3.

In the lab, repeated step commands are delivered to Cart 1 as a *pulse train* of fixed period T. The commanded step size of any given pulse will be $r(t)$ = 250 mm or 500 mm (±125 mm or ±250 mm about zero position), as shown in Figure 4.

The aim is to **accurately** move Cart 3 back and forth for as **fast** a pulse train as possible. You will need to specify the pulse train period, T, as well as your seven controller gains. The ideal controller and pulse train will allow Cart 3 to settle within 10 mm of the commanded step position r before the next pulse occurs. Additionally, there are additional 500 g weights which can be added to Carts 2 and 3, increasing their mass. Your controller should be robust enough to handle any configuration of masses.

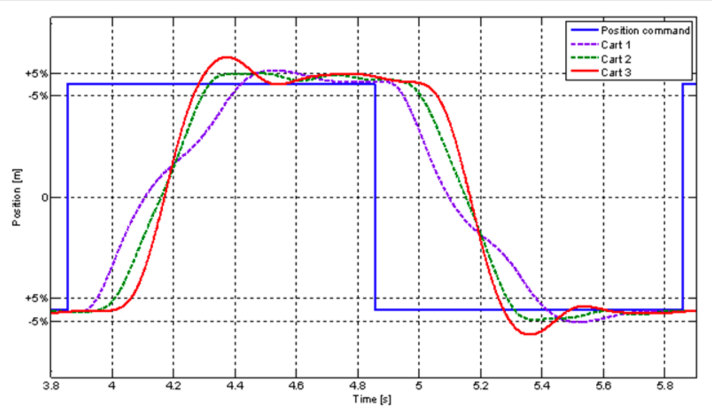

Figure 4. Example of commanded step input (square wave) and cart locations.

The motor driving Cart 1 has physical limits on its voltage input (V). You should consider these in your MATLAB simulation:

- Voltage limit of $|V|$ < 12 V. This is a hardware limitation, commanding a greater voltage will result in saturation.
- Slew rate limit of $\left|\frac{dV}{dt}\right|$ < 30 V/s. If your controller commands changes in voltage faster than this rate, the motor driver will not be able to keep up.

State Space Equations

The state space equations for this system need to be derived from the diagram above and parameter values listed in Table 2. You have full state feedback available, i.e. $C_1 = I_{6\times6}$. Use a state vector $\boldsymbol{x}$ in the format:

$$\boldsymbol{x} = (\mathrm{v}_1 \quad \mathrm{v}_2 \quad \mathrm{v}_3 \quad \dot{\mathrm{v}}_1 \quad \dot{\mathrm{v}}_2 \quad \dot{\mathrm{v}}_3)^{\mathrm{T}}$$

The input force of the motor is:

$$F = \alpha \frac{k_m k_g}{Rr} V - \frac{k_m^2 k_g^2}{Rr^2} \dot{\mathrm{v}}_1 \quad \text{[N]}$$

When deriving the state space equations, you need to arrange the velocity-dependent term of the motor force into the damping matrix, resulting in control input u = motor input voltage V. This derivation is similar to that provided (in a separate document) for the inverted pendulum system.

Table 2. Parameters for three-cart system.

Symbol	Description		Value	Unit
m_1	Cart 1 mass		1.608	kg
m_2, m_3	Cart 2/3 mass	(unloaded)	0.75	kg
		(with extra mass)	1.25	kg
k	Spring constant		175	N/m
C_1	Cart 1 damping		0	N·s / m
C_2, C_3	Cart 2/3 damping		3.68	N·s / m
α	Experimentally-derived “fiddle factor”		12.45	-
R	Motor terminal resistance		1.4	Ω
r	Pinion radius		0.0184	m
k_g	Gearing ratio		3.71	-
k_m	Motor back EMF constant		0.00176	V·s / rad

To Bring to Lab Session

- Six gains, $\mathbf{K} = [K_1 \quad K_2 \quad K_3 \quad K_4 \quad K_5 \quad K_6]$ for the contributions *of* v_1, v_2, v_3, $\dot{v}_1$, $\dot{v}_2$, and $\dot{v}_3$ respectively.
- Tracking gain N for tracking step inputs of magnitude $r(t)$ = 250 mm or 500 mm.
  - Note: The state which needs to track the step input is Cart 3's position v_3, so for the purposes of calculating N, use $\mathbf{C} = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$.
- A pulse train period T to trial. This period can be adjusted in the lab. Remember: we want this as low as possible within design criteria.

TIPS

- Useful MATLAB commands include: `ss, place, lqr, acker, step, lsim`
  - Google "MATLAB [command]" or type "help [command]" into MATLAB to see what these functions do and how to use them.
- For tracking, recall $u = -\mathbf{K}\boldsymbol{x} + Nr(t)$. You will need to adjust your B_1 matrix in MATLAB appropriately.
  - Some MATLAB functions only return response to a **unit** step (magnitude $H(t) = 1.0$). In this case, you will need to use $\widehat{\mathbf{B}}_\mathbf{1} = \mathbf{B_1}Nr$ in place of just $\mathbf{B_1}$ when creating your system in MATLAB to account for the step magnitude (*see page 96 of notes*).
- The physical plant parameters may not exactly match those of the modelled system - make sure your controllers are not too sensitive to small changes in parameter values. Do this by designing your gains, and then simulating with a modified $\mathbf{A}$ matrix. Find gains that work well with all possible $\mathbf{A}$ matrices (and thus all different parameter values)
- Be careful that all your units match – an important controls lesson!
- Make your report readable. Basic effort with white space and good layout of plots, tables, text, etc. all counts.
- Spend time on charts and figures as very good ones create the clearest explanation. More plots is not always better!