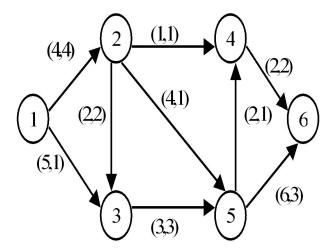
## MP305 Practical 2025/2026 - Network Flows I

- The Python notebook Network\_Flows\_I that contains the maximal network flow algorithm can be accessed via any web browser. See the MP305 CANVAS web page for details and instructions.
- Solutions to all questions with (\*) have to be calculated. You must include text commentary (in Python notebook Markdown cells) to explain your answers to the questions asked.

You can only get full marks if you have the right final answers, have correct intermediate stages (and intermediate answers), show that you understand what is going on by answering any questions the TA asks about the code or the calculations, and have many comments in your PYTHON notebook.

- This practical is worth 5% of your final grade.
- 1. Find the maximal flow for minimal capacity for the network below where the capacity c(i,j) and the flow  $\phi(i,j)$  is shown on each arc (i,j):

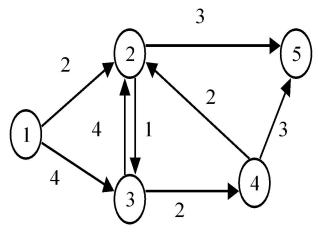


This is the example discussed in the lecture notes. You may read in the data for this example from the Python notebook Network\_Flows\_I. See the MP305 CANVAS web page for details and instructions.

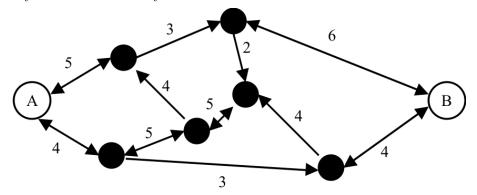
- (a) Find the incremental network and capacities at each iteration of the Ford Fulkerson algorithm.
- (b) In the **last** iteration when the maximal flow is found, identify which arcs are normal and which are inverted in the incremental network.
- (c) Hence find the minimal capacity cut of this network flow model.

In the following two problems, first define the network and its capacities following the template of problem 1 and then run the Python code.

2. (\*) Find the maximal flow through the following network with the given capacities:



- (a) Set the initial flow to 0 at each arc and find the incremental network and capacities at each iteration of the Ford-Fulkerson algorithm.
- (b) In the **last** iteration when the maximal flow is found, identify which arcs are normal and which are inverted in the incremental network.
- (c) Hence find the minimal capacity cut of this network flow model.
- 3. (\*) A road network is shown below with the capacity on each road indicated. Notice that many roads are two way.



- (a) Find the maximal flow through the network from A to B.
- (b) Compare this to maximal flow from B to A.