Overview

A feedback control scheme for the attitude stabilization of the AUV will be presented. The goal is to to control the AUV without going through Euler Angles, avoiding singularities at extreme attitudes. To be able to propose a proper control scheme, the inputs and outputs of the controller must be defined:

Inputs:

1. Current measured quaternion orientation, denoted as $q_{(vi)1}$.
2. Measured angular velocity vector in the body frame, denoted as $\omega_m = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$
3. Desired quaternion orientation, denoted as $q_{(vi)2}$.

Outputs:

1. Control torque needed in body frame to track the desired orientation, denoted as $\tau = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$

Assumptions:

1. We choose the origin of the body frame to coincide with the vehicle's center of mass.

Methodology:

All calculations use the left-hand notation, with a quaternion being defined as follows:

$$q = (w, \mathbf{v}) = (w, x, y, z) = \left(\cos\frac{\theta}{2},\quad \hat{\mathbf{u}}_x \sin\frac{\theta}{2},\quad \hat{\mathbf{u}}_y \sin\frac{\theta}{2},\quad \hat{\mathbf{u}}_z \sin\frac{\theta}{2}\right)$$

- A **unit axis** of rotation, $\hat{\mathbf{u}}$
- A **rotation angle** θ, about $\hat{\mathbf{u}}$

We first calculate a quaternion error, q_e, that satisfies the equation:

$$q_{(vi)2} = q_e \otimes q_{(vi)1}$$

Isolating for q_e:

$$q_e = q_{(vi)2} \otimes q^*_{(vi)1}$$

Where q^*_m is the conjugate of q_m. From the definition of the quaternion, we notice that the vector part of q_e is proportional to the **sine of the error** about each of the principal axes. Thus, the simplest we can transform our quaternion error in 4D to a 3D error we can use to calculate a control torque is:

$$\boldsymbol{e} = \mathrm{vec}(q_e) = \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix}$$

Since our AUV can be well-approximated as rectangular prism, its Inertia tensor is diagonal with $I_{xx} \neq I_{yy} \neq I_{zz}$. To account for the difference in moments of inertia about each axis, we can scale our error vector $\boldsymbol{e}$ using a diagonal matrix such that:

$$\tau = \boldsymbol{P_e}\,\boldsymbol{e} = \begin{bmatrix} P_{e_x} & 0 & 0 \\ 0 & P_{e_y} & 0 \\ 0 & 0 & P_{e_z} \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix}$$

We now have a P-controller with our control torque directly proportional to the sine of the error angle. To add damping, we add another term proportional to the measured angular velocity. So now,

$$\tau = \boldsymbol{P_e}\,\boldsymbol{e} - \boldsymbol{P_\omega}\,\boldsymbol{\omega} = \begin{bmatrix} P_{e_x} & 0 & 0 \\ 0 & P_{e_y} & 0 \\ 0 & 0 & P_{e_z} \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} - \begin{bmatrix} P_{\omega_x} & 0 & 0 \\ 0 & P_{\omega_y} & 0 \\ 0 & 0 & P_{\omega_z} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

We now have a P^2 controller with the error terms providing the bulk of our control torque with the velocity term dampening the response. The velocity term serves a similar function as the D term is in a PID controller. The main advantage this has over differentiating the quaternion error is that $\boldsymbol{\omega}$ can be easily acquired from our IMU without worrying about what is the correct way of numerically computing the quaternion derivative.

Finally, our AUV has a passive restoring torque due to its center of buoyancy being above its center of mass. This restoring torque increases in magnitude as the AUV rotates further away from the neutral position. This restoring torque can be treated as a disturbance and we could tune our P matrices to compensate for it, although it is unclear how effective this would be. Since we already know the location of the AUV's center of buoyancy from our CAD model, we can add a feedforward term to actively compensate for this restoring torque.

The restoring torque, in the body frame is calculated as:

$$\boldsymbol{\tau_r} = \boldsymbol{r_v^{bv}} \times \left(\boldsymbol{R}\big(q_{(vi)1}\big) \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} \right)$$

Where $\boldsymbol{R}(q_{vi})$ is the rotation matrix that takes the buoyancy force vector, $\begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}$, from the world frame to the body frame. $\boldsymbol{r_v^{bv}}$ is the translation vector from the vehicle frame to the center of buoyancy, expressed in the vehicle frame. Note that the $\boldsymbol{R}(q_{vi})$ can be directly calculated from q_{vi}.

Finally,

$$\tau = \boldsymbol{P_e}\, \boldsymbol{e} - \boldsymbol{P_\omega}\, \boldsymbol{\omega} - \boldsymbol{\tau_r}$$