COMP 360 - Winter 2014 - Assignment 4

Due: 6pm March 24th.

General rules: In solving these questions can use that SAT, 3SAT, Max Independent Set, Max Clique, Min Vertex Cover, 3-Colourability, Subset Sum are NP-complete.

There are in total 110 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

1. (15 points) Use the complementary slackness to show that $x_1^* = x_3^* = 0.5$, $x_2^* = x_4^* = 0$, $x_5^* = 2$ is an optimal solution for the following Linear Program:

$$\begin{array}{ll} \max & 3.1x_1 + 10x_2 + 8x_3 - 45.2x_4 + 18x_5 \\ \text{s.t.} & x_1 + x_2 + x_3 - x_4 + 2x_5 \leq 5 \\ & 2x_1 - 4x_2 + 1.2x_3 + 2x_4 + 7x_5 \leq 16 \\ & x_1 + x_2 - 3x_3 - x_4 - 10x_5 \leq -20 \\ & 3x_1 + x_2 + 3x_3 + \frac{3}{2}x_4 + \frac{7}{3}x_5 \leq 10 \\ & x_2 + x_3 + 6x_4 + 2x_5 \leq 4.5 \\ & 2x_2 - x_4 + x_5 \leq 2 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

2. (20 points) Either prove that the following problem is NP-complete, or show that it belongs to P.

   - Input: A number k, and a formula ϕ in conjunctive normal form.
   - Output: Is there a truth assignment that satisfies ϕ and assigns False to exactly k variables?

   What happens if in the above problem we replace k with the fixed number 100?

3. (15 points) Recall the project selection problem: There is a set of projects P, and every $i \in P$ has a revenue p_i which can be negative or

positive, and certain projects are prerequisites for others. We learned how to use circulation with demands to find a subset $A \subseteq P$ such that the prerequisites of every project in A also belong to A, and its profit $\sum_{i \in A} p_i$ is maximized. Either prove that the following variant of this problem is NP-complete, or show that it belongs to P.

- Input: A positive integer k, and as in the original problem a set of projects P, with their revenues and lists of prerequisites.
- Output: Is there a subset $A \subseteq P$ that respects the prerequisite condition and has profit *exactly* k:

$$\sum_{i \in A} p_i = k.$$

4. (20 points) Either prove that the following problem is NP-complete, or show that it belongs to P.

   - Input: A positive integer k, and $(G, \{c_e\}_{e \in E}, \{b_v\}_{v \in V})$ where $G = (V, E)$ is a graph and to every edge e a positive cost $c_e > 0$ is assigned, and to every vertex v a positive benefit $b_v > 0$ is assigned.
   - Output: Is there a subset S of the vertices such that the total benefit of the vertices in S minus the total cost of the edges in S is at least k? That is

$$\sum_{v \in S} b_v - \sum_{\substack{e = uv \in E \\ u, v \in S}} c_e \geq k.$$

5. (20 points) Show that if in linear programming we allow constraints of the form $\sum_{i=1}^{n} a_i |x_i| \geq b$ for integers b and a_i, then the problem becomes NP-complete.

6. (20 points) Show that the following problem is NP-complete:

   - Input: A formula ϕ in conjunctive normal form such that each clause in ϕ either involves only positive literals (i.e., variables), or it involves only negative literals (i.e., negated variables).
   - Output: Is there a truth assignment that satisfies ϕ?