Solutions for assignment 2 of COMP 360

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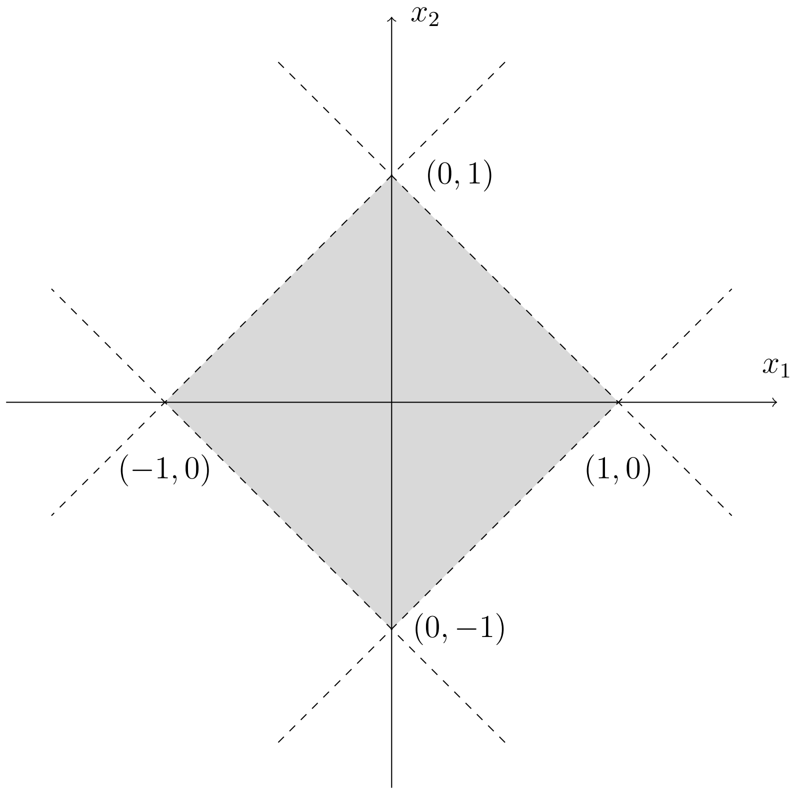
1. The problem can be written as:

$$\begin{array}{lll}
\min & 0.1x_1 + 0.07x_2 + 0.06x_3 & \\
\text{s.t} & 4x_1 + 2x_2 + 5x_3 & \geq 65 \\
& 7x_1 + 3x_2 + 5x_3 & \geq 82 \\
& 3x_1 + 4x_2 + 3x_3 & \geq 70 \\
& x_1, x_2, x_3 & \geq 0.
\end{array}$$

2. The feasible region:



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3. Let $G = (V, E)$ be the graph, where V is the vertex set and E is the edge set. For an edge e we denote its head vertex as e^+ and tail vertex as e^-. Let c_e be the

capacity of edge e. Let c_v be the capacity of vertex v. Let s be the source and t be the destination. Let x_e be the flow on edge e The problem can be written as:

$$\begin{aligned}
max \quad & \sum_{e \in E: e^- = s} x_e \\
s.t \quad & \sum_{e \in E: e^- = v} x_e - \sum_{e \in E: e^+ = v} x_e = 0 && (v \in V \setminus \{s,t\}) \\
& \sum_{e \in E: e^- = v} x_e \leq c_v && (v \in V \setminus \{s,t\}) \\
& x_e \leq c_e && (e \in E) \\
& x_e \geq 0 && (e \in E)
\end{aligned}$$

4.(a) Let $G = (V, E)$ be the graph, where V is the vertex set and E is the edge set. The problem can be written as

$$\begin{aligned}
max \quad & \sum_{(u,v) \in E} x_{u,v} \\
s.t \quad & \sum_{v \in V: (u,v) \in E} x_{u,v} \leq 1 && (u \in V) \\
& x_{u,v} \geq 0 && ((u,v) \in E)
\end{aligned}$$

4.(b) It is equivalent to looking for the maximum matching in the graph.

5.(a) As $x_1, \dots, x_n$ and $x'_1, \dots, x'_n$ are two feasible solutions, for an arbitrary constraint, we have

$$\sum_{i=1}^{n} a_i x_1 \leq b$$

and

$$\sum_{i=1}^{n} a_i x'_1 \leq b.$$

Therefore, for all real numbers $\alpha \in [0, 1]$, we have

$$\alpha \sum_{i=1}^{n} a_i x_i \leq \alpha b$$

and

$$(1 - \alpha) \sum_{i=1}^{n} a_i x'_i \leq (1 - \alpha) b.$$

Summing up the two inequalities, we have

$$\alpha \sum_{i=1}^{n} a_i[\alpha x_i + (1-\alpha)x_i] \le b.$$

In other words $(\alpha x_i + (1-\alpha)x_i)_{1\le i\le n}$ is also a feasible solution.

5.(b) No. If points on the edges of a triangle are feasible solutions, then the interior must also belong to the feasible solutions.

5.(c) No, for the same reason of (b).

5.(d) Yes. For example:

$$\begin{aligned} \max \quad & x_1 + x_2 + x_3 \\ \text{s.t} \quad & \qquad\qquad x_3 = 0. \end{aligned}$$

6. The dual problem is:

$$\begin{aligned} \min \quad & 2y_2 + 5y_3 + 5y_4 \\ \text{s.t} \quad & 2y_1 - y_2 + y_3 + 10y_4 = 1 \\ & -5y_1 - y_2 + y_4 = 2 \\ & y_1 + 3y_3 - y_4 = -1 \\ & y_1, y_2, y_3, y_4 \ge 0. \end{aligned}$$

7. Let $x_3 = x_3' - x_3''$ and $x_2' = -x_2$. Then we can convert the original problem to:

$$\begin{aligned} \max \quad & x_1 - 2x_2' - x_3' + x_3'' \\ \text{s.t} \quad & -2x_1 - 5x_2' - x_3' + x_3'' \le 0 \\ & 2x_1 + 5x_2' + x_3' - x_3'' \le 0 \\ & x_1 - x_2' \le 0 \\ & -x_1 + x_2' \le 0 \\ & -x_1 - x_2' - 3x_3' + 3x_3'' \le -5 \\ & 10x_1 - x_2' - 3x_3' + 3x_3'' \le 5 \\ & x_1, x_2', x_3', x_3'' \ge 0 \end{aligned}$$