COMP 360 - Winter 2016 - Assignment 1

Due: 6pm Jan 28th.

General rules: In solving these questions you may consult books but you may not consult with each other. There are in total 110 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

"Oh no, not homework again."

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1. (50 Points) Recall that for $f, g : \mathbb{N} \to \mathbb{R}^+$, we say $f = O(g)$ if and only if

$$\exists c, n_0 > 0 \; \forall n > n_0 \qquad f(n) \le cg(n).$$

We say $f = \Omega(g)$ if and only if $g = O(f)$. We say $f = \Theta(g)$ if and only if $f = O(g)$ and $f = \Omega(g)$. We say $f = o(g)$ if and only if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0.$$

And finally $f = \omega(g)$ if and only if $g = o(f)$.

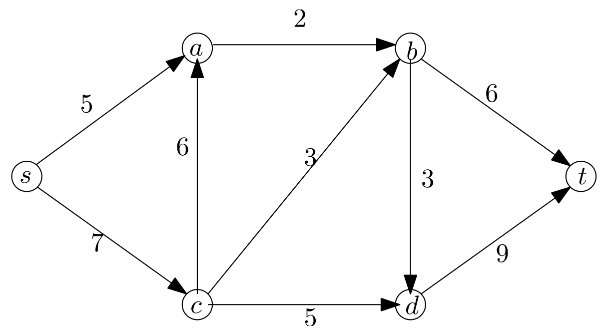
Give a formal proof for the following statements:

   (a) $\sqrt{n} + n\sqrt{n} = O(n^2)$.
   (b) $(n + \log_2 n)^5 = \Theta(n^5)$.
   (c) $n! = 1 \times 2 \times \ldots \times n = o(n^n)$.
   (d) $\log_2 n = o(n^{1/100})$.
   (e) $2^n = \omega(n^{\sqrt{n}})$.

2. (40 Points) True of False? (Prove or Disprove).

   (a) $2^{2^{n+1}} = O(2^{2^n})$.
   (b) $(\log_2 n)^5 = O\left(\log_2 n^5\right)$.
   (c) $n^{1/n} = \Theta(1)$.

3. (20 Points) Run the Ford-Fulkerson algorithm on the following network.



- For each step of the algorithm, give the augmenting path by listing the vertices on the path and state the amount by which the path increases the current value of the flow.
- Label each edge of the network with its flow at the end of the execution of the algorithm. State the value of the maximum flow.