COMP 360 - Winter 2016 - Assignment 5

Due: 6pm Mar 30th.

General rules: In solving these questions you may consult books but you may not consult with each other. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

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1. For each one of the following problems either prove that they are NP-complete or prove that they belong to P.

   (a) (15 Points)
   - Input: A CNF ϕ with 10 clauses (and n variables).
   - Question: Is there a truth assignment that satisfies ϕ?

   (b) (15 Points)
   - Input: A CNF ϕ on $2n$ variables.
   - Question: Is there a truth assignment that satisfies ϕ and assigns True to exactly n variables?

   (c) (15 Points)
   - Input: A 3CNF ϕ.
   - Question: Is there a truth assignment that satisfies exactly 10 clauses in ϕ?

   (d) (15 Points)
   - Input: Positive integers $a_1, \ldots, a_n$ and a positive integer M.
   - Question: Is there a subset $S \subseteq \{1, \ldots, n\}$ such that $\prod_{i \in S} a_i = M$?

   (e) (15 Points)
   - Input: A graph G and a positive integer k.
   - Question: Is there a set $S \subseteq V(G)$ of size k such that every vertex of G either belongs to S or has at least one neighbour in S?

2. (25 Points) Consider the following variation of the load balancing problem. Suppose you have a system that consists of m slow machines and k fast machines. The fast machine can perform twice as much work per unit of time as the slow machines. Now you are given a set of n jobs. Job i takes time t_i to process on a slow machine and time $\frac{1}{2}t_i$ on a fast machine. You want to assign each job to a machine, and as before, the goal is to minimize the makespan - that is the maximum, over all machines, of the total processing time of jobs assigned to that machine.

   Give a polynomial time 3-factor approximation algorithm for this problem.