COMP 360 - Fall 2012 - Assignment 4

Due 6:00 pm Nov 16th, 2012

General rules: In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 2nd floor. There will be a penalty of 10% if you hand your assignment on Saturday.

1. Prove that the following problems are NP-complete.

   (a)

   $$X = \{\langle G, k\rangle : G \text{ is a graph that has a cycle of length } k\}.$$

   (b)

   $$X = \{\langle G, v\rangle : G \text{ has Hamiltonian path that starts from the vertex } v\}.$$

   (c) Given positive integers $a_1, \ldots, a_n$, we want to know whether there exists $\epsilon_1, \ldots, \epsilon_n \in \{-1, 1\}$ such that $\epsilon_1 a_1 + \ldots + \epsilon_n a_n = 0$.

   (d) Given an undirected graph G with vertex set $\{1, \ldots, n\}$ and edge set E, we want to know whether there exists numbers

   $$x_1, \ldots, x_n, y_1, \ldots, y_n \in \{1, 2, 3\}$$

   such that

   $$\prod_{ij \in E(G)} |x_i - x_j| \neq \prod_{ij \in E(G)} |y_i - y_j|.$$

   (e) Show that if in linear programming we allow constraints of the form $\sum_{i,j=1}^{n} a_{ij} x_i x_j = b$ for integers b and a_{ij}, then the problem becomes NP-complete.

   (f) Show that if in linear programming we allow constraints of the form $|\sum_{i=1}^{n} a_i x_i| \geq b$ for integers b and a_i, then the problem becomes NP-complete.