COMP 360 - Winter 2016 - Midterm

Name:

1. (20 Points) Consider the variant of the maximum flow problem where every node v also has an integer capacity $c_v \geq 0$. We are interested in finding the maximum flow as before, but now with the extra restriction that $f^{\text{in}}(v) = f^{\text{out}}(v) \leq c_v$ for every internal node v. Solve this problem using the original maximum flow problem.

   Solution (sketch): Split every vertex v into two vertices v^{in} and v^{out} with an edge of capacity c_v directed from v^{in} to v^{out}. Now all the incoming edges to v will be joined to v^{in}, and all the outgoing edges from v will leave v^{out}.

2. (20 points) Find an efficient algorithm for the following problem: Given a flow network G and an edge e, we want to find out whether e is in every minimum cut. (Hint: How does changing the capacity of such an edge affect the value of the maximum flow?).

   Solution: Compute the MAX-Flow using scaling Ford-Fulkerson, or any other efficient algorithm. Increase the capacity of e and compute it again. If the max-flow increases then it means that e belonged to all the min-cuts as increasing its capacity increased the capacity of all the min-cuts. (Note that decreasing the capacity does not work, since if e belongs to only one min-cut still decreasing its capacity will decrease the value of the min-cut).

3. (20 points) A paint manufacturer uses three minerals to provide four chemicals required in its paint. The composition of the paint must be at least 4% of chemical A, 3% of chemical B, 30% of chemical C, and 16% of chemical D. The following table gives the relevant compositions of the minerals and the unit costs. Because mineral 2 causes an undesirable color when used in excess, no more than 1% of the total mineral content of the paint can be mineral 2. Write a linear program whose solution will provide the necessary composition of chemicals at the least cost.

Mineral	A	B	C	D	cost ($ / lb)
mineral 1	3 %	5 %	35 %	24 %	3.50
mineral 2	7 %	8 %	32 %	12 %	2.50
mineral 3	9 %	1 %	27 %	15 %	3.00

   Solution:

$$\begin{array}{ll}
\max & 3.5x_1 + 2.5x_2 + 3x_3 \\
\text{s.t.} & x_1 + x_2 + x_3 = 100 \\
& 3x_1 + 7x_2 + 9x_3 \geq 4 \\
& 5x_1 + 8x_2 + x_3 \geq 3 \\
& 35x_1 + 32x_2 + 12x_3 \geq 30 \\
& 24x_1 + 12x_2 + 15x_3 \geq 16 \\
& x_2 \leq 1 \\
& x_1, x_2, x_3 \geq 0
\end{array}$$

4. (20 points) Draw the feasible region to the following linear program.

$$\begin{array}{ll} \max & x_1 + x_2 \\ \text{s.t.} & x_1 + 3x_2 \leq 3 \\ & 4x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{array}$$

Solution: Easy! See the similar question in the assignment.

5. (20 points) Write the dual of the following linear program

$$\begin{array}{ll} \max & x_1 + x_2 + 4x_3 \\ \text{s.t.} & x_1 + x_2 + x_3 \leq 8 \\ & x_1 + 2x_2 \leq 4 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Solution:

$$\begin{array}{ll} \min & 8y_1 + 4y_2 \\ \text{s.t.} & y_1 + y_2 \geq 1 \\ & y_1 + 2y_2 \geq 1 \\ & y_1 \geq 4 \\ & y_1, y_2 \geq 0 \end{array}$$