COMP 362 - Winter 2017 - Assignment 3

Due: 6pm March 7th

General rules: In solving these questions you may consult your book; you can discuss high level ideas with each other. But each student must find and write his/her own solution. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 2nd floor.

1. (15 points) Use the complementary slackness to show that $x_1^* = x_3^* = 0.5$, $x_2^* = x_4^* = 0$, $x_5^* = 2$ is an optimal solution for the following Linear Program:

$$
\begin{array}{ll}
\max & 3.1x_1 + 10x_2 + 8x_3 - 45.2x_4 + 18x_5 \\
\text{s.t.} & x_1 + x_2 + x_3 - x_4 + 2x_5 \leq 5 \\
 & 2x_1 - 4x_2 + 1.2x_3 + 2x_4 + 7x_5 \leq 16 \\
 & x_1 + x_2 - 3x_3 - x_4 - 10x_5 \leq -20 \\
 & 3x_1 + x_2 + 3x_3 + \frac{3}{2}x_4 + \frac{7}{3}x_5 \leq 10 \\
 & x_2 + x_3 + 6x_4 + 2x_5 \leq 4.5 \\
 & 2x_2 - x_4 + x_5 \leq 2 \\
 & x_1, x_2, x_3, x_4, x_5 \geq 0
\end{array}
$$

2. (10 Points) Show that if P = NP, then P = NP = CoNP.

3. (20 points) Prove that the following problem is NP-complete:

   - Input: A number k, and a formula ϕ in conjunctive normal form.
   - Output: Is there a truth assignment that satisfies ϕ and assigns False to exactly k variables?

   What happens if in the above problem we replace k with the fixed number 100?

4. (15 points) Prove that the following problem is NP-complete.

   - Input: A graph G and a vertex v of G.

- Output: Does G have a Hamiltonian path that starts from the vertex v?

5. (20 points) Show that if in the decision version of linear programming we allow constraints of the form $|\sum_{i=1}^{n} a_i x_i| \geq b$ for integers b and a_i, then the problem becomes NP-complete.

6. (20 points) Show that the following problem is NP-complete:

   - Input: A formula ϕ in conjunctive normal form such that each clause in ϕ either involves only positive terms (i.e., variables), or it involves only negative terms (i.e., negated variables).
   - Output: Is there a truth assignment that satisfies ϕ?