COMP 360 - Winter 2014 - Assignment 3

Due: 6pm Feb 28th.

General rules: In solving these questions you may consult books but you may not consult with each other. There are in total 110 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

1. (15 Points) Draw the feasible region of the following system of linear constraints:

$$\begin{aligned} 2x_1 + x_2 &\leq 1 \\ 2x_1 + x_2 &\geq -1 \\ 2x_1 - x_2 &\leq 1 \\ 2x_1 - x_2 &\geq -1 \end{aligned}$$

2. We have a supply of m different kinds of material. Specifically, we have b_j units of raw material of kind j. They are n different kinds of products that we can make from these raw material. Each unit of product i takes a_{i1} unit of raw material 1, a_{i2} units of raw material 2 etc. to create, and can be sold at the price c_i dollars. It is possible to sell fractional amounts of any product. Our goal is to produce the most profitable set of products with our available material.

   (a) (15 Points) Formulate the above optimization problem as a linear program.

   (b) (15 Points) Write the dual of this program.

3. Consider an undirected graph $G = (V, E)$, and the following linear program with variables x_{uv} for edges $uv \in E$.

$$\begin{array}{lll} \max & \sum_{uv \in E} x_{uv} & \\ \text{s.t.} & \sum_{v: uv \in E} x_{uv} \leq 1 & \forall u \in V \\ & x_{uv} \geq 0 & uv \in E \end{array}$$

(a) (10 Points) Show that the optimum solution to this linear program is at least the size of the largest matching in G.

(b) (15 Points) Write the dual of this linear program.

4. Consider an undirected graph $G = (V, E)$, and let s and t be two distinct vertices in G. Consider the following linear program with variables x_u for vertices $u \in V$:

$$\begin{array}{lll} \max & x_s - x_t & \\ \text{s.t.} & x_u - x_v & = 0 \quad \forall uv \in E \end{array}$$

   (a) (10 Points) What does the solution to this linear program tell us about s and t?

   (b) (15 Points) Write the dual of the program.

5. (15 Points) Formulate the following problem as a Linear Program:

   - Input: An undirected graph $G = (V, E)$.
   - Question: What is the maximum of

   $$\frac{\sum_{uv \in E} x_{uv}}{\sum_{u,v \in V} x_{uv}},$$

   where x_{uv} are non-negative numbers (and not all zeros) and satisfy the triangle inequality $x_{uv} + x_{vw} \geq x_{uw}$ for all $u, v, w \in V$?