# COMP 360 - Fall 2012 - Assignment 2

Due 6:00 pm Oct 19, 2012

**General rules:** In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 2nd floor.

1. (10 Points) A farmer can choose from three feeds for his milk cows. The nutritional facts and costs of these feeds are shown in the following table. The minimum daily requirements of nutrients $A$, $B$, and $C$ are 65, 82, 70 units, respectively. Write a linear program to determine the mixture of feeds that will supply the minimum nutritional requirement at least cost.

| Feed | A (units/lb) | B (units/lb) | C (units/lb) | Cost /lb |
|---|---|---|---|---|
| Feed 1 | 4 | 7 | 3 | 0.10 |
| Feed 2 | 2 | 3 | 4 | 0.07 |
| Feed 3 | 5 | 5 | 3 | 0.06 |

2. (10 Points) Draw the feasible region of the following system of linear constraints:

$$\begin{aligned} x_1 + x_2 &\leq 1 \\ x_1 + x_2 &\geq -1 \\ x_1 - x_2 &\leq 1 \\ x_1 - x_2 &\geq -1 \end{aligned}$$

3. (15 Points) Consider the variant of the maximum flow problem where every node $v$ also has an integer capacity $c_v \geq 0$. We are interested in finding the maximum flow as before, but now with the extra restriction that $f^{\text{in}}(v) \leq c_v$ for every node $v$. Write a linear program to solve this problem.

4. (a) (10 Points) Write a linear program to solve the following problem: Given an undirected graph $G$ as an input, we want to assign non-negative numbers to the *edges* of the graph so that the following conditions hold:
      - For every vertex, the sum of the numbers on the edges incident to it is at most 1.
      - The total sum of the numbers on all the edges is maximized.

   (b) (5 Points) If we also require that the numbers assigned to edges are integers, then what does the above problem correspond to?

5. (a) (15 Points) Prove that if $(x_1, \ldots, x_n)$ and $(x'_1, \ldots, x'_n)$ are feasible solutions to a linear program, then so is every point on the line segment between these two points. (Note that the points on the line segment are $(\alpha x_1 + (1-\alpha)x'_1, \ldots, \alpha x_n + (1-\alpha)x'_n)$ where $0 \leq \alpha \leq 1$. Also for simplicity you may assume that all the constraints are of the form $a_1x_1 + \ldots + a_nx_n \leq b$ where $a_1, \ldots, a_n, b \in \mathbb{R}$.)

   (b) (5 Points) Is there a linear program whose feasible region consists of the sides of a triangle?

   (c) (5 Points) Is there a linear program whose feasible region consists of the surface of a 3-dimensional cube?

   (d) (5 Points) Is there a linear program with 3 variables whose feasible region is a 2-dimensional plane?

6. (10 Points) Write the dual of the following linear program:

$$
\begin{array}{lll}
\max & x_1 + 2x_2 - x_3 & \\
\text{s.t.} & 2x_1 - 5x_2 + x_3 & \leq 0 \\
& -x_1 - x_2 & \leq 2 \\
& x_1 + 3x_3 & \leq 5 \\
& 10x_1 + x_2 - x_3 & \leq 5
\end{array}
$$

7. (10 Points) Convert the following linear program to canonical form.

$$
\begin{array}{lll}
\max & x_1 + 2x_2 - x_3 & \\
\text{s.t.} & 2x_1 - 5x_2 + x_3 & = 0 \\
& -x_1 - x_2 & = 2 \\
& x_1 - x_2 + 3x_3 & \geq 5 \\
& 10x_1 + x_2 - x_3 & \leq 5 \\
& x_1 & \geq 0 \\
& x_2 & \leq 0
\end{array}
$$