COMP 360 - Fall 2012 - Assignment 3

Due 6:00 pm Nov 2nd, 2012

General rules: In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 2nd floor.

1. Given an example of a primal problem that has no feasible solution and whose dual also has no feasible solution.

2. Let $G = (V, E, \{c_e\})$ be a graph where every edge e has a cost c_e. Write the duals of the following Linear Programs:

   (a)
$$\begin{array}{lll} \max & \sum_{uv \in E} c_{uv} x_{uv} & \\ \text{s.t.} & \sum_{u_0 v \in E} x_{u_0 v} & \leq 1 \quad \forall u_0 \in V \\ & x_{uv} & \geq 0 \quad uv \in E \end{array}$$

   (b)
$$\begin{array}{lll} \min & \sum_{uv \in E} c_{uv} x_{uv} & \\ \text{s.t.} & \sum_{uv \in C} x_{uv} & \geq 1 \quad \text{for every cycle } C \text{ in } G \\ & x_{uv} & \geq 0 \quad uv \in E \end{array}$$

   (c)
$$\begin{array}{llll} \max & \sum_{uv \in E} c_{uv}(x_u + x_v) & & \\ \text{s.t.} & \sum_{u \in V} x_u & \leq 100 & \\ & x_u & \leq 1 & u \in V \\ & x_u & \geq 0 & u \in V \end{array}$$

   (d) An independent set in G is a set of vertices, no two of which are adjacent. Let $\mathcal{I}$ denote the set of all independent sets in G.
$$\begin{array}{lll} \min & \sum_{S \in \mathcal{I}} x_S & \\ \text{s.t.} & \sum_{S : u \in S} x_S & \geq 1 \quad \forall u \in V \\ & x_S & \geq 0 \quad S \in \mathcal{I} \end{array}$$

3. Show that the solution to the linear program in Question 2.d is a lower-bound for the minimum number of colours required to colour the vertices of G so that no two adjacent vertices receive the same colour.

4. Formulate the following problem as a linear program: Given $a_1, \ldots, a_n \geq 0$, and an $n \times n$ matrix A, we want to find $x_1, \ldots, x_n \geq 0$ (and not all of them zero) such that

$$\frac{a_1 x_1 + a_2 x_2 + \ldots, a_n x_n}{x_1 + x_2 + \ldots + x_n}$$

is maximized and

$$A \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \leq \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}.$$