COMP 360 - Winter 2016 - Assignment 6

Due: 6pm April 13th.

General rules: In solving these questions you may consult books but you may not consult with each other. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

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1. (25 Points) We are given a set P of n points on the plane, and a positive integer k. We want to partition these points into k sets such that the largest distance between any two points which belong to the same part is minimized. Show that the following is a 2-factor approximation algorithm.

   - Set $S = \emptyset$.
   - For $i = 1, \ldots, k$ do
   - Select the farthest point from S and add it to S.
   - EndFor
   - Put each point $p \in P$ in the same part as the closest point in S to p.

   (The distance from a point to S is the distance of the point to the closest point in S.)

2. (25 Points) Problem 10 of Chapter 11: Suppose you are given an $n \times n$ grid graph G. Associated with each node v is an integer weight $w(v) \geq 0$. You may assume that all the weights are distinct. Your goal is to choose an independent set S of nodes of the grid, so that the sum of the weights of the nodes in S is as large as possible. (The sum of the weights of the nodes in S will be called its total weight.) Consider the following greedy algorithm for this problem.

   - Start with $S := \emptyset$.
   - While some node remains in G:
     - Pick a node v of maximum weight.
     - Add v to S.
     - Delete v and its neighbors from G
   - Endwhile.

   Show that this algorithm returns an independent set of total weight at least $\frac{1}{4}$ times the maximum total weight of any independent set in the grid graph G.

3. (25 points) Consider the triangle elimination problem. We are given a graph $G = (V, E)$, and want to find the smallest possible set of vertices $U \subseteq V$ such that deleting these vertices removes all the triangles (i.e. cycles of length 3) from the graph. Prove that the following algorithm is a 3-factor approximation algorithm for this problem:

   - While there is still a triangle C left in G:
   - Delete all the three vertices of C from G

- EndWhile
- Output the set of the deleted vertices

4. (25 Points) Given a set P of n points on the plane, consider the problem of finding the smallest r such that there exist 10 circles of radius r such that together they contain all the points in P. Design a PTAS algorithm for this problem. In other words, given any fixed $\epsilon > 0$, design an algorithm whose running time is polynomial in n, and its output is at most $1 + \epsilon$ times the optimal output.