# COMP 360 - Fall 2012 - Assignment 1

Due 6:00 pm Sept 28, 2012

**General rules:** In solving these questions you may consult books but you may not consult with each other. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 3rd floor left of the elevators.

1. (40 Points) Consider a flow network $(G, s, t, \{c_e\})$.
   (a) Prove or Disprove: A maximum flow $f$ might assign non-integer flows to some edges?
   (b) Prove or Disprove: Multiplying all the capacities by 2 will multiply the value of the maximum flow by 2.
   (c) Prove or Disprove: If $e$ has the lowest capacity among all edges of $G$, then $e$ is in a minimum cut.
   (d) Prove or Disprove: If the capacity of $e$ is strictly greater than the capacity of every other edge of $G$, then $e$ cannot be in a minimum cut.
2. (10 Points) Prove that for every flow network with at most $m$ edges, there always exists a sequence of at most $m$ augmentations that leads to a maximum flow?
3. (10 Points) Disprove: If $G$ has at least two different maximum flows $f$ and $f'$ , then it has at least two different minimum cuts $(A, B)$ and $(A', B')$.
4. (20 Points) Consider the following variant of bipartite matching problem: As an input we are given a bipartite graph $G$ with parts $X$ and $Y$, and a set $S \subseteq X$. We want to find the largest matching $M$ in $G$ with the property that every vertex in $S$ belongs to an edge in $M$. Solve this problem using the original maximum flow problem.

5. (20 Points) Consider the variant of the maximum flow problem where every node $v$ also has an integer capacity $c_v \geq 0$. We are interested in finding the maximum flow as before, but now with the extra restriction that $f^{\text{in}}(v) \leq c_v$ for every node $v$. Solve this problem using the original maximum flow problem.