COMP 360 - Winter 2016 - Assignment 4

Due: 6pm Mar 16th.

General rules: In solving these questions you may consult books but you may not consult with each other. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

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1. (20 points) Write the duals of the following Linear Program: An independent set in G is a set of vertices, no two of which are adjacent. Let $\mathcal{I}$ denote the set of all independent sets in G.

$$\begin{array}{lll} \min & \sum_{S \in \mathcal{I}} x_S & \\ \text{s.t.} & \sum_{S : u \in S} x_S & \geq 1 \quad \forall u \in V \\ & x_S & \geq 0 \quad S \in \mathcal{I} \end{array}$$

2. (5 points) Show that the solution to the linear program in Question 1 is a lower-bound for the minimum number of colours required to colour the vertices of G so that no two adjacent vertices receive the same colour.

3. (25 points) Prove that if P = NP, then P = NP = CoNP.

4. Recall that in 3COL, given an undirected graph G, we want to know whether it is possible to colour its vertices with 3 colors so that adjacent vertices receive different colours. Let X be the following problem: Given a graph G, we want to know whether there is an edge e in G such that $G - e$ is 3-colourable.

   (a) (15 points) Show that "$X \leq_p$ 3COL".

   (b) (15 points) Show that "3COL $\leq_p X$".

5. (20 points) Let $\overline{\text{3SAT}}$ denote the non-satisfiability problem for 3CNF's. Show that $\overline{\text{3SAT}} \leq_p$ UNQ where in UNQ, given a CNF ϕ we want to know whether there is a unique satisfying assignment for ϕ. (Hint: for every 3CNF ψ construct a CNF ϕ (in polytime) such that ψ is a NO input for 3SAT if and only if ϕ is a YES input for UNQ.)