COMP 360 - Fall 2012 - Assignment 5

Due 6:00 pm Dec 5th, 2012

General rules: In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 2nd floor. There will be a penalty of 10% if you hand your assignment on Dec 6th.

1. Consider the load balancing problem with k machines. Show that the following simple greedy algorithm is a $\frac{2k-1}{k}$-factor approximation algorithm.

   - Start with no jobs assigned
   - For $i = 1, \ldots, n$ do
   - Assign the ith job to the machine with the smallest current load.

   (Note that the algorithm does not sort the jobs.)

2. We are given a set P of n points on the plane, and a positive integer k. We want to partition these points into k sets such that the largest distance between any two points which belong to the same part is minimized. Show that the following is a 2-factor approximation algorithm.

   - Set $S = \emptyset$.
   - For $i = 1, \ldots, k$ do
   - Select the farthest point from S and add it to S.
   - EndFor
   - Put each point $p \in P$ in the same part as the closest point in S to p.

   (The distance from a point to S is the distance of the point to the closest point in S.)

3. (Bonus question) Suppose that we are given as an input a *directed* graph $G = (V, E)$. We want to partition the vertices of G into two parts A and $B = V - A$ so as to maximize the total number of the edges going from A to B.

   (a) Give a simple randomized $\frac{1}{4}$-factor approximation algorithm for this problem.

   (b) Show that the following integer program models the problem:

$$\begin{array}{lll} \max & \sum_{(u,v)\in E} z_{uv} & \\ \text{s.t.} & z_{uv} \le x_u & \forall (u,v) \in E \\ & z_{uv} \le 1 - x_v & \forall (u,v) \in E \\ & x_u \in \{0,1\} & u \in V \\ & 0 \le z_{uv} \le 1 & \forall (u,v) \in E \end{array}$$

   (c) Consider the following randomized rounding algorithm that solves the linear programming relaxation of the above integer program:

$$\begin{array}{lll} \max & \sum_{(u,v)\in E} z_{uv} & \\ \text{s.t.} & z_{uv} \le x_u & \forall (u,v) \in E \\ & z_{uv} \le 1 - x_v & \forall (u,v) \in E \\ & x_u \in \{0,1\} & u \in V \\ & 0 \le z_{uv} \le 1 & \forall (u,v) \in E \end{array}$$

   and then puts $u \in A$ with probability $\frac{1}{4} + \frac{x_u}{2}$. Show that this gives a randomized $\frac{1}{2}$-factor approximation algorithm for the described problem.