STUDENT NAME: _____

STUDENT ID: _____

McGill University
Faculty of Science
School of Computer Science
Final exam

COMP-424A
Artificial Intelligence I

December 8, 2006
2:00-5:00

Examiner: Prof. Doina Precup

Associate Examiner: Prof. Theodore Perkins

This examination is closed-book, closed-notes.

There are 17 pages, including the title page.

Start by writing down your name. Answer the questions directly on the exam booklet. Additional pages are provided if necessary. Do not forget to write your name on the additional pages too.

Read the whole exam before starting to work on it. There are 11 questions, all of which require written answers. Values for each question are shown in brackets. Partial credit will be given for incomplete or partially correct answers.

Good luck!

1. [10 points] **Search algorithms**

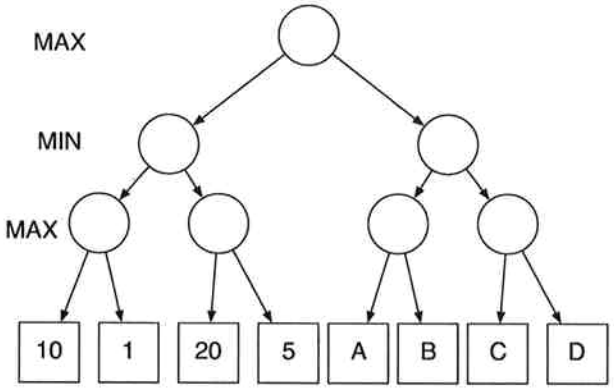
   (a) [4 points] Suppose you have an admissible heuristic h. Show whether $\log(h+1)$ is admissible or not. If it is admissible, explain if it is better to use h or $\log(h+1)$ in an A^* search.

   (b) [1 point] Suppose you were using a genetic algorithm and you have the following two individuals, represented as strings of bits:

   11010011 and 0001010

   Show the result of performing crossover between the 3rd and 4th bit.

(c) [2 points] Consider the game tree in the figure below.



On the left hand side of the tree, mark the arrows corresponding to the moves that would be chosen by Minimax, and shade the nodes on the left that would be pruned by $\alpha\beta$-pruning.

(d) [2 points] In the same figure, indicate one assignment of values for nodes A, B, C, D for which *no pruning* would occur on the right side of the tree. Indicate values for nodes A, B, C, D for which a *maximum number of nodes* would be pruned by $\alpha\beta$-pruning on the right side of the tree.

(e) [1 point]

Suppose you are using minimax for chess. Your old computer let you search ahead 2 ply. Your new machine is more powerful, so you can search ahead 4 ply. Would the values returned by the minimax function be different between the two machines or not? Justify your answer

2. [10 points] **Logic**

(a) [6 points] Translate the following sentences in first-order logic:

i. [2 points] If two people are friends, they always go to the movies together.

ii. [2 points] Friendship is transitive

iii. [1 point] Amy and Bert are friends. Bert and Cleo are friends.

iv. [1 point] Amy is going to Star Wars.

(b) [4 points]
Prove formally that Cleo is going to Star Wars.

3. [10 points] **Neural networks**

   (a) [5 points] In class we showed how to train the weights of a neural network using gradient descent. Could we use simulated annealing for the same purpose? If your answer is yes, explain how this would be implemented, and whether this would work better or worse than plain gradient descent. If your answer is no, explain why not.

   (b) [5 points] You have a Boolean function of 3 inputs, A, B and C, which returns true if and only if exactly two of the inputs are true. Show either the perceptron or the smallest network of perceptrons that correctly represents this function. Show all the necessary weights.

4. [5 points] **Maximum likelihood estimation**

   Let X be a random variable drawn uniformly from and interval $[a, b]$, where a and b are unknown. More precisely,

$$p(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{if } x > b \end{cases}$$

   You observe a sequence of samples $x_1, \ldots x_n$, with $n \geq 2$. Assume that at least two of these samples have different values.

   Compute the maximum likelihood values for a and b.

5. [5 points] **Gradient descent for learning**

   Suppose that you want to train a hypothesis of the form:

   $$h(x) = w_0 + w_1 \sin(w_2 x)$$

   Give update rules for w_0, w_1 and w_2, assuming you do incremental gradient descent using sum-squared error as an error function.

6. [5 points] **Overfitting**

   Suppose that we have a data set in which there are n binary attributes and the desired output is also binary. The examples that you see come from a random function and are perfect (the training data has the correct label). You have a "learning algorithm" that does the following: it memorizes the training data. When asked for the output for a new instance, the algorithm will look up the instance and if it is in memory, it answers with the recorded label. Otherwise, it will answer randomly.

   (a) [1 point] What is the training error of this algorithm?

   (b) [2 points] Suppose the algorithm has seen k distinct training examples. It is tested on a new example drawn randomly from the entire space of possible instances. What is the expected error on this test instance? What happens to the error as k increases?

   (c) [1 points] Suppose that after seeing k training examples, the algorithm is tested on an instance which has not been part of the training data. What is the testing error as a function of k?

   (d) [1 point] Explain the relationship between this example and the issues of overfitting and cross-validation discussed in class.

7. [15 points] **Bayes nets and expectations**

The chef at a French cafe is specialist in making cakes. His customers find his cakes either delicious or just good. The chef has noticed that the amount of sugar and butter that he puts in the cake influence these comments. More precisely, if he puts a *high* amount of both sugar and butter, 80% of customers find the cake delicious. If he puts a *high* amount of one of these two ingredients but a *low* amount of the other, 50% of customers will find the cake delicious. If he puts a *low* amount of both ingredients, 25% of customers find the cake delicious. However, the cook is health-conscious, so he only has a 20% chance of putting a high amount of sugar and a 10% chance of putting a high amount of butter on any given day. Customers also tip the waiter, of course, and the tip can be *low* or *high*. Customers who find the cake delicious have a 50% chance of giving a high tip, while customers who find the cake just good have a 25% chance to give a high tip.

(a) [5 points] Draw a Bayes net describing the problem statement above. Specify both the graph structure and all the parameters.

(b) [3 points] Compute the probability that the waiter will receive a high tip from a random customer. Please show the formula you are using, then substitute the adequate parameters and show the resulting number.

(c) [2 points] Suppose that 20 customers visit the cafe in a given day. What is the expected number of high tips that the waiter will get?

(d) [3 points] The waiter wants to maximize the likelihood of getting high tips, so he sneaks in the kitchen and ensures that both sugar and butter are high. What is the probability of getting a high tip now? Again, show the formula first and then substitute the parameters.

(e) [2 points] Suppose that a high tip is worth \$10 and a low tip is worth \$1. What is the expected utility gain of the waiter from changing the cake composition?

8. [10 points] **Utility theory**

Your ankle hurts. The doctor thinks there is a 40% chance of having a bone chip in there. If it is not removed, you will be in pain for a long time (utility=-100). You can remove it by surgery. After surgery you will be in pain for a few days, regardless whether the bone chip was there or not (utility -10).

(a) [2 points] What is the best action given this information? Give the calculation justifying your answer.

(b) [2 points] What is the expected value of perfect information about whether the bone chip is there or not?

(c) [3 points] The doctor has been practicing a new technique that will given better, if not perfect, information about whether the bone chip is there or not. The technique involves minor discomfort (utility -2). But this is not a perfect technique. Suppose that the sensor has $P(\text{positive}|\text{bone chip}) = x$ and $P(\text{positive}|\neg\text{bone chip}) = y$. Draw a decision graph for this setting of the problem.

(d) [3 points] For what values of x and y is it worthwhile to have the procedure done?

9. [10 points] **Planning**

A 2-year-old is in a room with some candy. The candy is hanging *high* on the Christmas tree. The child is *low*. There is a stool in the room, also at height *low*. If the child climbs on the stool, her height will become *high*. Initially, the child is at location A, the stool at location B and the Christmas tree at location C. The child can *climb* on the stool if they are at the same location. The child can *grab* an object if they are in the same location and at the same height. As a result, the child will hold the object. The child can *release* an object that she is holding. The child can also *go* from one location to another. The goal of the child is to get the candy.

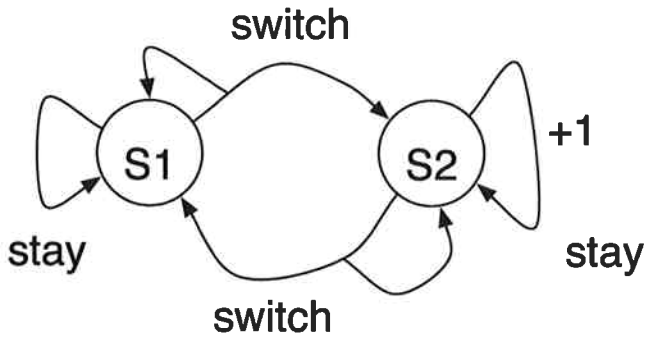
(a) [8 points] Describe all the operators in STRIPS language.

(b) [1 point] Describe the initial state and goal state.

(c) [1 point] Write a plan that solves the problem.

10. [10 points] **Markov Decision Processes**

Consider the MDP in the figure below. There are two states, *S1* and *S2*, and two actions, *switch* and *stay*. The *switch* action takes the agent to the other state with probability 0.8 and stays in the same state with probability 0.2. The *stay* action keeps the agent in the same state with probability 1. The reward for action *stay* in state *S2* is 1. All other rewards are 0. The discount factor is $\gamma = \frac{1}{2}$.



(a) [2 points] What is the optimal policy?

(b) [4 points] Compute the optimal value function by solving the linear system of equations corresponding to the optimal policy.

(c) [2 points] Suppose that you are doing value iteration to compute the optimal value function. You start with all value estimates equal to 0. Show the value estimates after 1 and 2 iterations respectively.

(d) [2 points] Suppose you are doing TD-learning. You start with all value estimates equal to 0, and you observe the following trajectory (sequence of states, actions and rewards):

S1,switch,0,S2,stay,+1,S2

Assuming the learning rate $\alpha = 0.1$, show the TD-updates that are performed.

11. [10 points] **Problem formulation**

(a) [5 points] Suppose you are developing software to track airplanes using blips observed on a radar screen. The radar is a noisy sensing device, which reports radio waves that bounce off of objects in the sky. When objects are present, they usually (but not always) reflect the waves, and a blip shows up on the screen in the approximate location of the object. The blips are updated every second. Suppose you are planning to use a Hidden Markov Model for this problem. What would be the states and what would be the observations? How would you obtain the transition probabilities and emission probabilities?

(b) [5 points] You got a summer job at a sandwich shop downtown. The shop sells different kinds of sandwiches, each consisting of a type of bread, a type of meat (or substitute), a type of cheese (if the customer wants some) and a type of vegetable. Each day, you have to go to the grocery store and buy supplies for the day's sandwiches. Bread has to be discarded at the end of the day. Meat and cheese have to be discarded after a week. Vegetables are discarded after 3 days. You know that customers come in randomly. If a customer cannot have the desired sandwich, she will leave and there is a 0.5 probability that she will never come to the shop again. Of course, customers pay different amounts for different sandwiches, and you have to pay for the groceries. Having taken AI, you want to optimize, in a principled way, the amount of purchases you are making. Describe this problem using any AI technique of your choice. Explain *all* the components of your model.