STUDENT NAME: _____
STUDENT ID: _____

McGill University
Faculty of Science
School of Computer Science
Final exam

COMP-424A
Artificial Intelligence I

December 14, 2005
2:00-5:00

Examiner: Prof. Joelle Pineau

Associate Examiner: Prof. Doina Precup

This examination is closed-book, closed-notes.
There are 16 pages, including the title page.
Start by writing down your name. Answer the questions directly on the question booklet. Additional pages are provided if necessary. Do not forget to write your name on the additional pages too!
Read the whole exam before starting to work on it.
There are 10 questions, all of which require written answers. Values for each question are shown in parenthesis. As usual, partial credit will be given for incomplete or partially correct answers.

Good luck!

Question 1. Search algorithms [11 points]
Consider the following boolean expression: **(D v B)** & **(¬ C v A)** & **(¬ B v ¬ C)**.
Your goal is to use search techniques to find a variable assignment (0 or 1 for each variable) such that the expression is true. This is an instance of a constraint satisfaction problem.
Your set of operators are: *ChangeVariableA, ChangeVariableB, ChangeVariableC, ChangeVariableD.*
The effect of each operator is as you would expect *If A=1, then ChangeVariableA => A=0;*
If A=0, then ChangeVariableA => A=1; and similarly for the other variables.

(a) [2 points] Describe a suitable state representation for this search problem. Define the initial state.

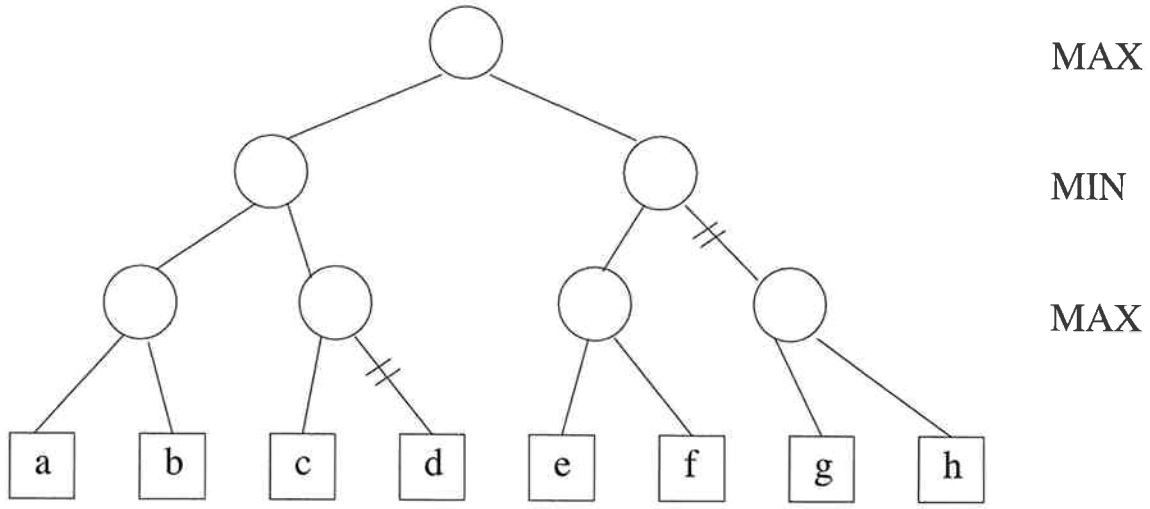
(b) [3 points] List the states in the order in which they will be expanded if you were to search according to Depth-Limited Search (DFS with max depth of 4). Assume operators are applied in the order in which they are listed above. Stop the search when you find a state that satisfies the boolean expression.

(c) [3 points] Consider the following heuristic: h = *number of satisfied disjunctions.* Is this heuristic admissible for this search problem? Justify your answer.

(d) [3 points] Search for a solution to our boolean expression by applying Greedy Best-First Search using the heuristic h = *number of satisfied disjunctions.* Provide the open-list, the heuristic estimate for each node, and the order in which it is expanded. Stop the search when you find a state that satisfies the boolean expression.

Question 2. Alpha-beta prunning [5 points]

Consider the following game search tree:



(a) [3 points] Give values for a, b and c such that d will be pruned during alpha-beta searching.

(b) [2 points] Given your values for a, b and c above, give values for e and f such that both g and h will be pruned during alpha-beta searching.

Question 3. Probabilities [9 points]

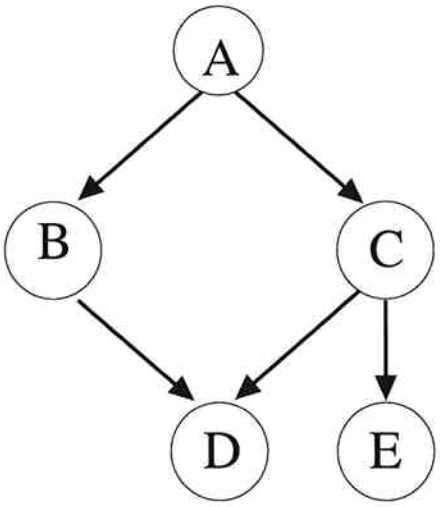
(a) [4 points] After John's annual checkup, the doctor calls to inform him that he tested positive for a serious disease, and that the test is 99% accurate (i.e. probability of testing possitive, given that someone has the disease is 0.99, probability of testing negative given that someone doesn't have the disease is also 0.99). The good news is that this is a rare disease, affecting only one in 1,000 people. What are the chances that John actually has the disease? (You can leave your answer in fractional form.)

(b) [3 points] The doctor suggests that John takes the test again. After he does so, the doctor reports that the second test was negative. What can you conclude (mathematically) about the chances that John has the disease? You can assume that the outcome of the two tests are independent, given that someone has (or not) disease. (You can leave your answer in fractional form.)

(c) [2 points] Is it reasonable to conclude that John was cured? State your assumptions and justify your conclusion.

Question 4. Bayes Nets [11 points]

(a) [5 points] Propose a real-world meaning for each of the binary random variables A, B, C, D, E that would justify the following Bayes Net structure. Justify your answer (in words).



(b) [6 points] For the same graph, give a *Yes/No* answer to each of the following questions, and justify your answer (e.g. list which case of conditional independence applies, or describe application of the Bayes ball algorithm).

(i) Are B and E conditionally independent, assuming that I don't know the value of the other variables?

(ii) Are B and E conditionally independent, assuming that I know the value of variable A, but not the value of the other variables?

(iii) Are B and E conditionally independent, assuming that I know the value D, but not the value of the other variables?

Question 5. Machine learning [15 points]

(a) [4 points] Consider a linear perceptron with one input unit, one output unit, and one weight (no hidden units). Assume you are given training data of the form $(x_1, y_1), (x_2, y_2), ..., (x_N, y_N)$. Now assume that the error criterion we wish to minimize is:

$$E(w) = w^2 + \sum_{i=1}^{N} (y_i - wx_i)^2 \quad (1)$$

Give the update rule for training w.

(b) [4 points] Every day at 8am you must make a prediction as to whether the stock market will go up or down that day. To allow yourself to sleep in longer, you decide to use a Neural Net software package to make this decision for you. The input to the Neural Net will be yesterday's closing prices and other financial indicators. There are four software packages available to help you. When you try them out, you get the following results:

Software package	Time to Train (hours)	Time to make 1 prediction (secs)	Training Set Error (% wrong predictions)	Test Set Validation Error (% wrong predictions)
ANN1	15	17	0	3.6
ANN2	17	12	5	3.2
ANN3	10	28	1	1.2
ANN4	8	26	3	25

Based on this, which Neural Net package would you choose for the job? Justify your answer (in words).

(c) [7 points] You are given data about 10 people's yearly consumption of wine and chocolate. You know that all the subjects are either French, Swiss or Belgian. Given this data, you would like to find the Maximum Likelihood hypothesis of each person's nationality using the K-means algorithm.

Subject	Wine (L)	Chocolate (kg)
1	90	1
2	80	3
3	70	1
4	60	3
5	50	2
6	20	2
7	0	6
8	30	6
9	50	9
10	70	7

You are told that the biggest wine consumer is French, the biggest chocolate consumer is Swiss, and the smallest consumer (combined wine and chocolate) is Belgian. [Hint: Use this information in the first E-step of K-means. For the M-step, assume hard EM, not the soft version.].

(i) Apply the first iteration of the K-means algorithm:

E-step:

M-step:

(ii) Apply the second iteration of the K-means algorithm:

E-step:

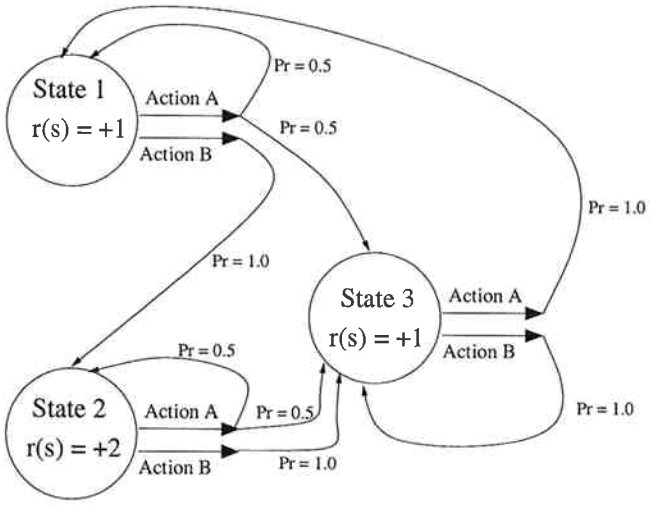
M-step:

Question 6. Decision theory [6 points]
Fred's new course schedule leaves him free everyday between 7am and noon. He finds a new job on the south shore, which pays $20 an hour. To get to this job, Fred must drive from campus to work (always leaving at 7am). He has a choice of two routes R_1 and R_2. Route R_1 typically takes 15 minutes, however if there is an accident on the bridge (20% of days) then the commute is 1 hour. Route R_2 is less travelled, and always takes 30 minutes. The return home is always done via route R_1 as there is no traffic at that hour. Fred has asked you to help him decide which route to take in the morning.

(a) [3 points] Define an appropriate utility function for this problem and apply the principles of maximum expected utility to decide which route Fred should choose in the morning.

(b) [3 points] Suppose Fred can call a traffic report service that will tell him accurately whether or not there is an accident on route R_1 that day. How much should Fred be willing to pay for such a service?

Question 7. MDPs and reinforcement learning [12 points]
The following Markov Decision Process has 3 states. Each state has two actions: A and B. The transitions and rewards are as shown in the diagram. Assume we have a discount factor $\gamma = 0.9$.



(a) [3 points] Consider the fixed policy of always choosing Action A, no matter what state we are in. Write down three linear equations relating the values V(S1), V(S2) and V(S3).

(b) [3 points] Assuming we want to maximize the expected discounted sum of future rewards, always applying action A is not the optimal policy. What is the optimal policy?

(c) [6 points] Consider the case where you don't know the transition and reward parameters. Estimate your value function using online TD-learning (with learning rate $\alpha = 0.1$). Give the TD-update rule and fill in the table below with state values after every transition is made.

TD-update: $V(s_t) =$

State/action pair	V(S1)	V(S2)	V(S3)
Initial value	0	0	0
$< s_{t=0} = S1, a_{t=0} = A, r_{t=0} = 1, s_{t=1} = S3 >$			
$< s_{t=1} = S3, a_{t=1} = B, r_{t=1} = 1, s_{t=2} = S3 >$			
$< s_{t=2} = S3, a_{t=2} = A, r_{t=2} = 1, s_{t=2} = S1 >$			
$< s_{t=3} = S1, a_{t=3} = B, r_{t=3} = 1, s_{t=3} = S2 >$			
$< s_{t=4} = S2, a_{t=4} = A, r_{t=4} = 2, s_{t=4} = S2 >$			

Question 8. Game theory [8 points]

(a) [2 points] In the following 2 player game, circle any pure strategy Nash equilibria.

Player 1 payoff:

	A	B
A	3	−10
B	10	−1

Player 2 payoff:

	A	B
A	3	10
B	−10	−1

(b) [4 point] Consider the following cooperative game:

Player 1 payoff:

	A	B
A	5	0
B	0	1

Player 2 payoff:

	A	B
A	1	0
B	0	5

(i) Which of the four following joint strategies is a correlated equilibrium (circle all that apply)?

	A	B
A	1/2	0
B	0	1/2

σ_1

	A	B
A	1	0
B	0	0

σ_2

	A	B
A	0	0
B	0	1

σ_3

	A	B
A	1/4	1/4
B	1/4	1/4

σ_4

(ii) What is the maximum expected payoff for player A of playing a correlated equilibrium strategy?

(c) [2 points] In the following 2x2 game, fill in the payoff matrices such that there exists a single dominant strategy for both players. Circle the dominant strategy.

Player 1 payoff:

	A	B	C
A	5	2	3
B	2	5	4
C	2	1	

Player 2 payoff:

	A	B	C
A	2	5	4
B		0	6
C	7	0	1

Question 9. Applying AI methods [8 points]

(a) [4 points] At the first meeting of the AI Debating Club, someone proposes that gradient descent is the only search technique needed in AI. Argue against this claim by describing two types of problems where gradient descent is not appropriate and suggest a better search technique for each case.

(b) [4 points] At the second meeting of the AI Debating Club, someone proposes that people should universally use Neural Nets with Backpropagation for all learning tasks. Argue against this claim by giving two weaknesses (or limitations) of neural nets with backpropagation. In each case, suggest a learning algorithms that overcomes this limitation.

Question 10. True or False [15 points]

(a) Bayes nets are too computationally expensive for practical applications.

(b) If A and B are independent then $\mathrm{P}(\neg A|\neg B) + \mathrm{P}(A|B) = 1$.

(c) A multi-layer neural net with 10 inputs, 10 hidden units, and 10 outputs has fewer than 50 weights.

(d) The primary advantage of the C-space (configuration space) transform, is that it transforms an n-Degree-of-Freedom motion planning problems into a 2-d problem.

(e) The XOR function can be learned by a single perceptron unit with 2 inputs and 1 output.

(f) The following translation from English to 1st-order logic is correct.
"There are no born leaders" <=> $\forall x, \neg BornLeader(x)$

(g) In statistical language modelling, it takes more data to train a trigram model than to train a bigram model.

(h) When optimizing the recall score for an information retrieval system, one will automatically improve the precision rate also.

(i) For any finite state Markov Decision Processes, there is always an optimal deterministic policy.

(j) In problems where the reward function is always negative, Monte-Carlo learning and TD-learning converge to the same value estimates.

(k) Any two-player zero-sum game is guaranteed to have a pure strategy Nash equilibrium.

(l) Genetic algorithms could be used to train a neural net, instead of backpropagation.

(m) Robot path planning with variable resolution cell decomposition yields a longer path than robot path planning using visibility graphs.

(n) Maximizing expected utility is equivalent to maximizing expected reward.

(o) John Nash's theorem states that the expected value of the correlated equilibrium solution is always lower (or equal) to the expected value of the Nash equilibrium solution.