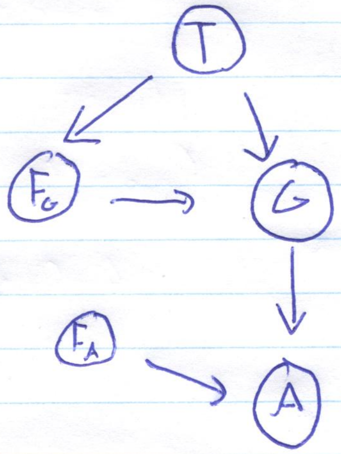
Solution Manual, A3, Comp 424.

1. a)



b) Not a polytree because there exists a cycle (T, F_G, G), if the edges had no directions.

c) T: 1 = high, 0 = normal G: prob of high vs prob of low F_G: 1 = faulty, 0 = normal

T	F_G	$P(G=High \mid T, F_G)$	$P(G=Normal \mid T, F_G)$
1	1	y	$1-y$
1	0	x	$1-x$
0	1	$1-y$	y
0	0	$1-x$	x

d)

G	F_A	$P(A \mid G, F_A)$	$P(\neg A \mid G, F_A)$
High	T	0	1
High	F	1	0
Normal	T	0	1
Normal	F	0	1

e) We are looking for $P(T=H \mid A, \overline{F_G}, \overline{F_A})$

A and F_A are independent of T so:

$$P(T \mid \neg F_G, G) = \frac{P(T, \neg F_G, G)}{\sum_t P(T=t, \neg F_G, G)}$$

$$= \frac{P(T)\, P(\neg F_G \mid T)\, P(G \mid T, \neg F_G)}{P(T)\, (\neg F_G \mid T)\, P(G \mid T, \neg F_G) + P(\neg T)\, P(\neg F_G \mid \neg T)\, P(G \mid \neg T, \neg}$$

$$= \frac{P(T)\, P(\neg F_G \mid T)\, x}{P(T)\, P(\neg F_G \mid T)\, x + P(\neg T)\, P(\neg F_G \mid \neg T)(1-x}$$

from part c)

2. a) $P(s, r) = P(s) \cdot P(r)$ (independent)

$$P(s,r) = \sum_{A,B} P(s \mid A) \cdot P(A \mid B) \cdot P(B) \cdot P(r)$$

$$= 0.15\,(0.2\,(0.8 \cdot 0.6 + 0.4 \cdot 0.2) + 0.8\,(0.4 \cdot 0.8 + 0.6 + 0$$

$$= 0.0696$$

b) $$p(a, \neg t) = \sum_{R,B,S} P(R) \cdot P(B) \cdot P(a \mid B) \cdot P(S \mid a) \cdot P(\neg t, R, B, a)$$

$$= \sum_{R,B,S} P(R, B, a, T = \neg t, s)$$

$$= \sum P(R) \cdot \sum_{B,S} P(B) \cdot P(a \mid B) \cdot P(S \mid a) \cdot P(\neg t \mid R, B, a)$$

$$= \sum_R P(R) \sum_B P(B)\, P(a \mid B)\, P(\neg t \mid R, B, a) \cdot \underbrace{\sum_S P(S \mid a)}_{1}$$

$$= 0.85\,(\cancel{0.2 \cdot 0.6 \cdot} 0.65 + 0.8 \cdot 0.4 \cdot 0.4) + 0.15\,(0.2 \cdot 0.6 \cdot 0.05 + 0.8 \cdot 0.4 \cdot 0.0$$

$$= 0.17984$$

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c) $P(t|s) = \dfrac{P(t,s)}{P(s)}$

$$P(t,s) = \sum_{R,B,A} P(R) \cdot P(B) \cdot P(A|B) \cdot P(s|A) \cdot P(t|R,B,A)$$

Where the sum is given by the following table:

R	B	A	P(R=r)	P(B=b)	P(A=a|B=b)	P(t|R=r, B=b, A=A)	P(s
r	b	a	0.15	0.6	0.45	0.8	0.0
$\bar{r}$	b	a	0.85	0.06	0.35	0.8	00
r	$\bar{b}$	a	0.15	0.4	0.42	0.8	0.0
r	b	$\bar{a}$	0.15	0.4	0.40	0.2	0.0
$\bar{r}$	$\bar{b}$	a	0.85	0.4	0.60	0.8	0.13
$\bar{r}$	b	$\bar{a}$	0.85	0.4	0.4	0.2	0.00
r	$\bar{b}$	$\bar{a}$	0.15	0.6	0.85	0.2	0.01
$\bar{r}$	$\bar{b}$	$\bar{a}$	0.85	0.6	0.05	0.2	0.00

So $P(t,s) = 0.23205$

$P(s) = \dfrac{P(r,s)}{P(r)} = \dfrac{0.0696}{0.15} = 0.464.$

from a)

So $P(t|s) = \dfrac{P(t,s)}{P(s)} = \dfrac{0.23205}{0.464} = 0.5$

3. We need $P(T|a)$

List of factors:

$P(R), P(B), P(A|B), P(T|R,B,A), P(S|A), \delta(A,a)$

Eliminate R.

$m_R(T,B,A) = \sum_r P(r) \cdot P(T|r,B,A)$.

$$\begin{matrix} t,b,a & \neg t,b,a \\ t,b,\neg a & \neg t,b,\neg a \\ t,\neg b,a & \neg t,b,a \\ t,\neg b,\neg a & \neg t,\neg b,\neg a \end{matrix}$$

$$= 0.15 \begin{pmatrix} 0.95 & 0.05 \\ 0.9 & 0.1 \\ 0.92 & 0.08 \\ 0.85 & 0.15 \end{pmatrix} + 0.85 \begin{pmatrix} 0.35 & 0.65 \\ 0.4 & 0.6 \\ 0.6 & 0.4 \\ 0.05 & 0.95 \end{pmatrix}$$

$$m_R(T,B,A) = \begin{pmatrix} 0.44 & 0.56 \\ 0.475 & 0.525 \\ 0.648 & 0.352 \\ 0.17 & 0.83 \end{pmatrix}$$

Eliminate S:

Factors:

$P(R), P(B), P(A|B), P(S|A), \delta(A,a), m_R(T,B,A)$

$m_S(A) = \sum_s P(S|A)$

$$[\, s,a \quad \neg s,a \,]$$

$m_S A = (0.8 \;\; 0.2) + (0.2 \;\; 0.8)$

$m_S A = [\, 1 \;\; 1 \,]$

Factors: $P(B)$, $P(A, B)$, $\delta(A, a)$, $m_R(T, B, A)$, $m_S(A)$

Elim B:

$$m_B(T, A) = \sum_b P(b) \cdot P(A, b) \cdot m_R(T, B, A)$$

$$= P(b) \cdot P(A, b) \cdot \begin{bmatrix} t, b, a & \neg t, b, a \\ t, b, \neg a & \neg t, b, \neg a \end{bmatrix}$$

$$+ P(\neg b) P(A \mid \neg b) \begin{bmatrix} t, \neg b, a & \neg t, \neg b, a \\ t, \neg b, \neg a & \neg t, \neg b, \neg a \end{bmatrix}$$

$$= 0.12 \begin{bmatrix} 0.44 & 0.56 \\ 0.475 & 0.525 \end{bmatrix} + 0.32 \begin{bmatrix} 0.648 & 0.352 \\ 0.17 & 0.83 \end{bmatrix}$$

$$m_B(T, A) = \begin{bmatrix} 0.2016 & 0.17984 \\ 0.1114 & 0.3286 \end{bmatrix}$$

Factors $\delta(A, a)$, $m_S(A)$, $m_B(T, A)$

Elim A:

$$m_A(T) = \sum_a \delta(A, a) \cdot m_S(A) \cdot m_B(T, A)$$

$$= 1 \cdot [1] \cdot m_B \cdot (T, a) + 0 \times [1] \times m_B(T, A)$$

$$= [0.26016, 0.17984]$$

list $m_A(T)$

$[t \quad \neg t]$: $m_A(\cancel{t}) = [0.26016 \quad 0.17984]$

The MAP result implies $P(T \mid a)$ is 0.26016 with $T = 1$.

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4. a)

i. Parameters:

θ_A, $\theta_{B,0}$, $\theta_{B,1}$, $\theta_{C,0}$, $\theta_{C,1}$
$P(A=1)$, $=P(B|A=0)$, $P(B|A=1)$, $P(C|A=0)$, $P(C|A=1)$

$\theta_{D,0,0}$, $\theta_{D,0,1}$, $\theta_{D,1,0}$, $\theta_{D,1,1}$.
$P(D=1, A=0, B=0)$, " ", " "

ii) MLE's:

$\theta_A = \frac{49}{146}$

$\theta_{B,0} = \frac{34+2+32}{97} = \frac{68}{97}$

$\theta_{B,1} = \frac{10+19+14}{49} = \frac{43}{49}$

$\theta_{C,0} = \frac{56}{97}$

$\theta_{C,1} = \frac{19}{49}$

$\theta_{D,0,0} = 4/6.$

$\theta_{D,0,1} = 8/29$

$\theta_{D,1,0} = 21/65$

$\theta_{D,1,1} = 0$

iii) Smoothing:

$\theta_A = \frac{50}{148}$

$\theta_{B,0} = \frac{69}{99}$

$\theta_{B,1} = \frac{44}{51}$

$\theta_{C,0} = \frac{57}{99}$

$\theta_{C,1} = \frac{20}{51}$

$\theta_{D,0,0} = \frac{5}{8}$

$\theta_{D,0,1} = 9/31$

$\theta_{D,1,0} = \frac{22}{67}$

$\theta_{D,1,1} = \frac{1}{48}$

b) Samples:

	A	B	C	D
S_1	1	0	1	?
S_2	1	1	?	0

Need $P(D \mid a, \neg b, c)$, $P(C \mid a, b, \neg d)$

$$P(D \mid a, \neg b, c) = \frac{P(a, \neg b, c, D)}{p(a, \neg b, c)} = \alpha P(a, \neg b, c, D).$$

$$= \alpha \underbrace{P(a) \cdot P(\neg b \mid a)}_{\text{constant, fold into normalization.}} p(c \mid a) \cdot p(D \mid \neg b, c).$$

$$= \beta \; p(D \mid \neg b, c)$$

$\underline{\underline{D = d}}$

$= \beta P(d \mid \neg b, c)$

$\propto \frac{8}{29}$

$D = \neg d$

$= \beta (\neg d \mid \neg b, c)$

$\propto \frac{21}{29}$

weight w/ $D = d$: $\frac{8}{29} \approx 0.28$

weight w/ $D = \neg d$: $\frac{21}{29} \approx 0.724$

Now $P(C \mid a, b, \neg d) = \dfrac{P(a, b, C, \neg d)}{p(a, b, \neg d)} = \alpha P(a, b, C, \neg$

$$= \alpha \underbrace{P(a) P(b \mid a)}_{\text{constant, fold into normalization}} P(C \mid a) \, p(\neg d \mid C, b)$$

$$= \beta \; p(C \mid a) \; p(\neg d \mid C, b)$$

$\underline{C = c}$

$P(C|a) \cdot P(\neg d | c, b)$

$\frac{19}{49} \cdot 1$

0.3877

$\downarrow$

weight: 0.4833

$\underline{C = \neg c}$

$P(\neg c | a) | P(\neg d | \neg c, b)$

$\frac{30}{49} \cdot \frac{44}{65}$

0.4144

$\downarrow$

weight: 0.5166

ratio: 1.0554

ii) M-Step: Dataset

	A	B	C	D
S1	1	0	1	$d: 8/29$ $\neg d: 21/29$
S2	1	1	$c=c: 0.5166$ $c=\neg c: 0.4835$	0

New MLE params:

$\theta_A : \frac{51}{148}$

$\theta_{B|A} = \frac{44}{51}$

$\theta_{B|\neg A} =$ no change

$\theta_{D|B,c} = 0$

$\theta_{D|\neg B,c} = \frac{8 + 0.2754}{30} = 0.2759$

$\theta_{C|A} = \frac{19 + 1 + 0.4833}{51}$

$= 0.4016$

$\theta_{C|\neg A} = n/c$

$\theta_{D|B,\neg c} = \frac{21}{65 + 0.5166}$

$= 0.3205$

$\theta_{D|\neg B,\neg c} = n/c.$

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iii) E-step

$P(D \mid a, \neg b, c)$

$= B\, P(D \mid \neg b, c)$

$D = d \rightarrow D = \neg d$

$0.2759 \qquad 0.724$

$B: P(C \mid a) \cdot p(\neg d \mid C, b):$

$C = c$

$0.4016 \cdot 1$

0.4016

$C = \neg c$

$(0.5984)(1 - 0.3205)$

$= 0.4066.$

weight: 0.4969

weight: 0.50309

(Rounding errors may cause final answers to vary slightly)