COMP 424 – Final Exam Tutorial

Practice Questions for Final Exam

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Exam Format

- Very similar to the Midterm with more MC questions
  - Multiple Choice Questions:
    (definitions & algorithm properties)

    both pre-midterm & post-midterm material

  - Problem Sets:

    Some pre-midterm exercises, but mostly post-midterm material

Today: parts of some big chapters

1. Search (only part of it)

2. Bayesian Networks

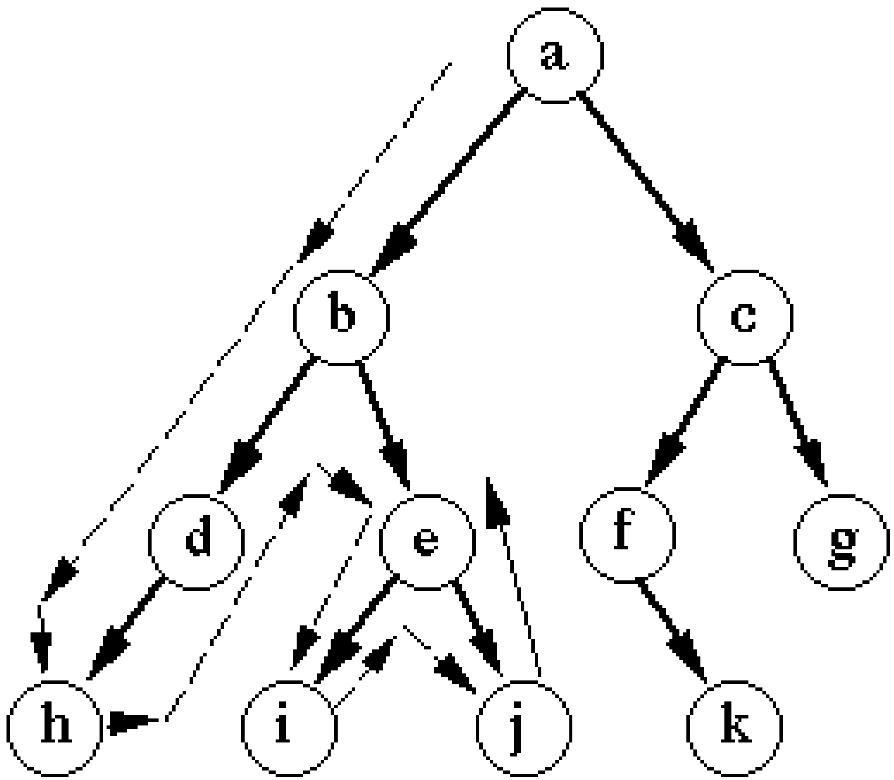
3. Temporal Inference with Hidden Markov Models (HMM)

4. Markov Decision Process (MDP)

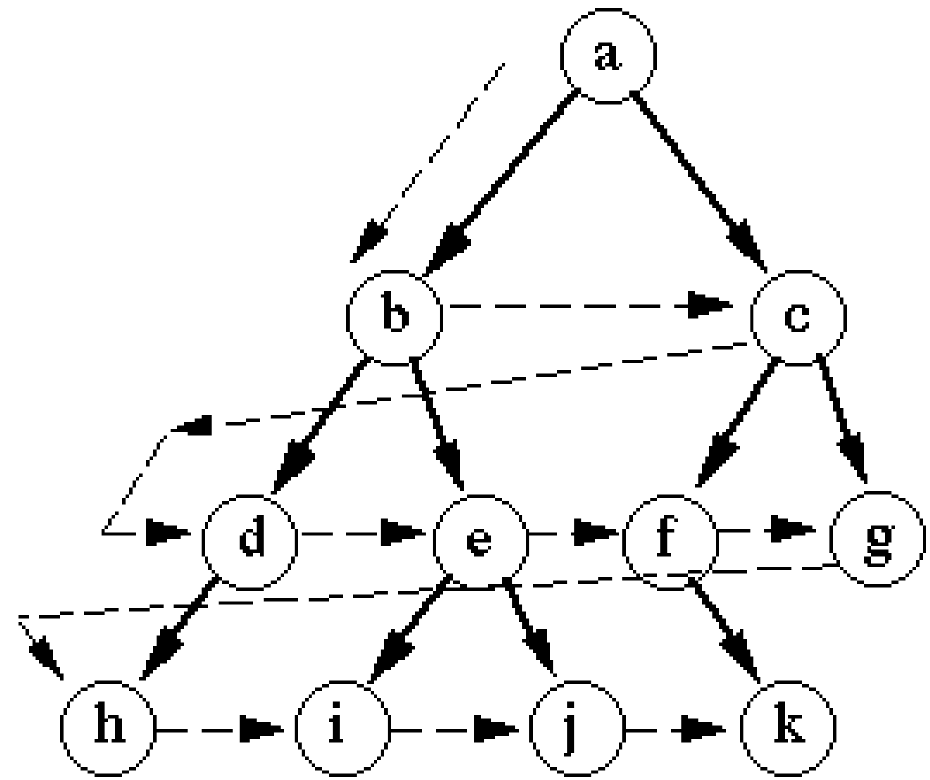
5. Utility

1. Search

(uninformed)



Depth-first search



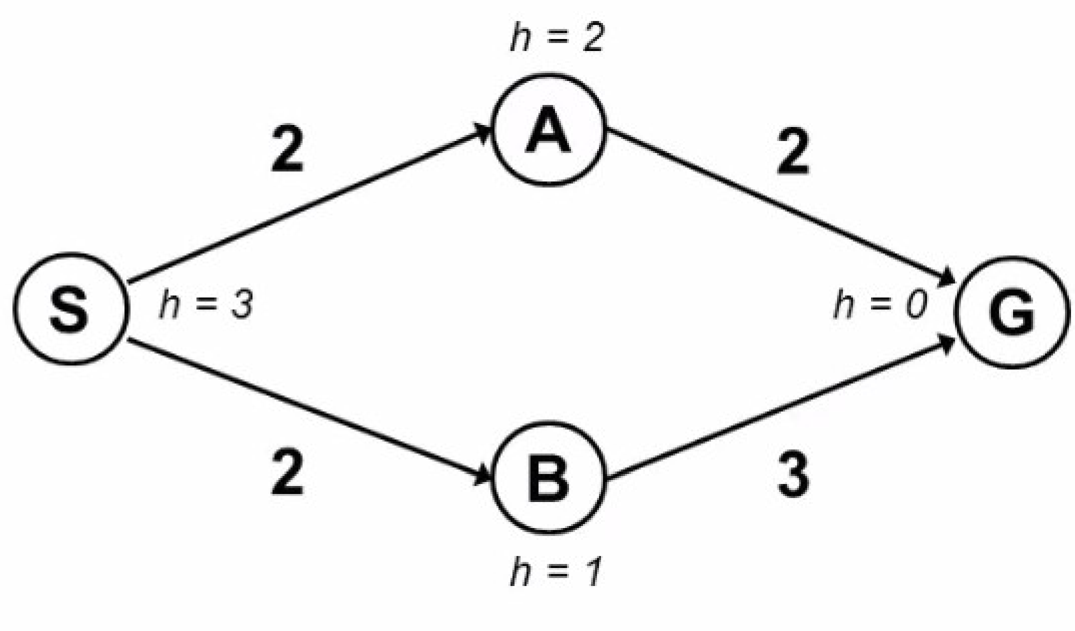
Breadth-first search

1. Search

(informed)

<u>Admissible</u> heuristic:
h(n) <= shortest path from *n* to <u>any</u> goal

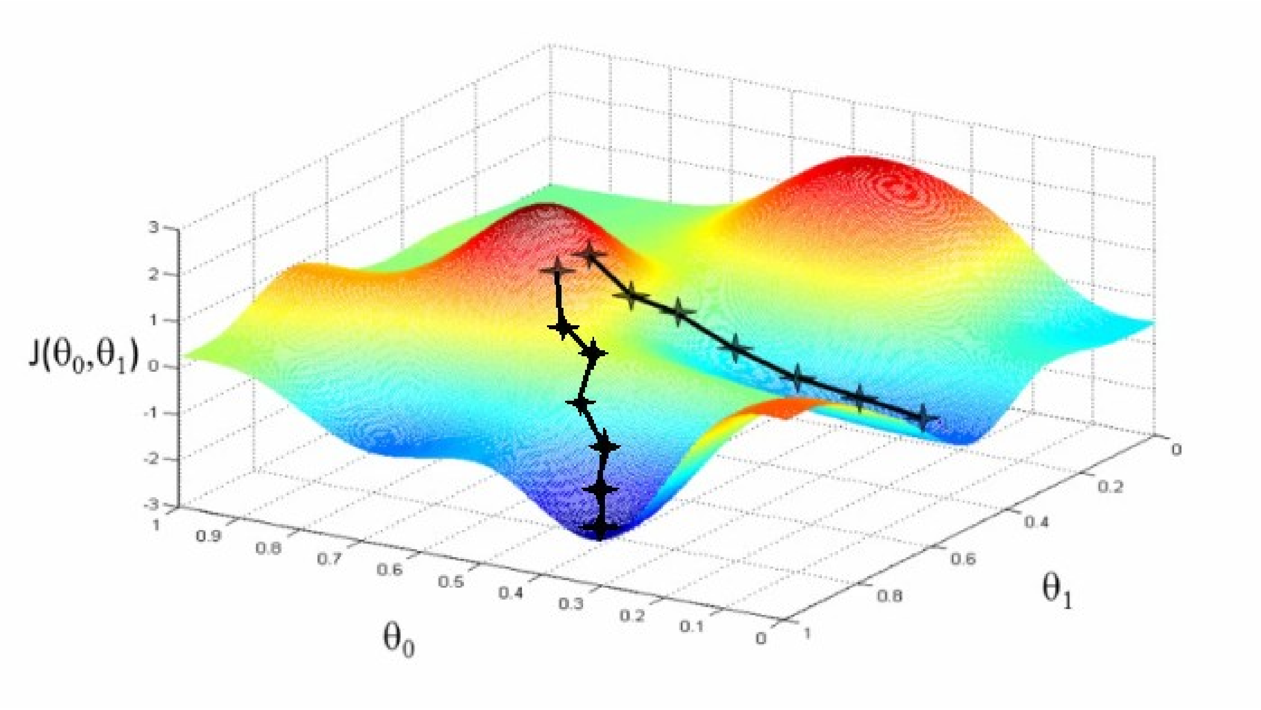
<u>Dominant</u> heuristic:
h2(n) >= *h1(n)* for all *n* AND *h1* & *h2* are admissible



1. Search

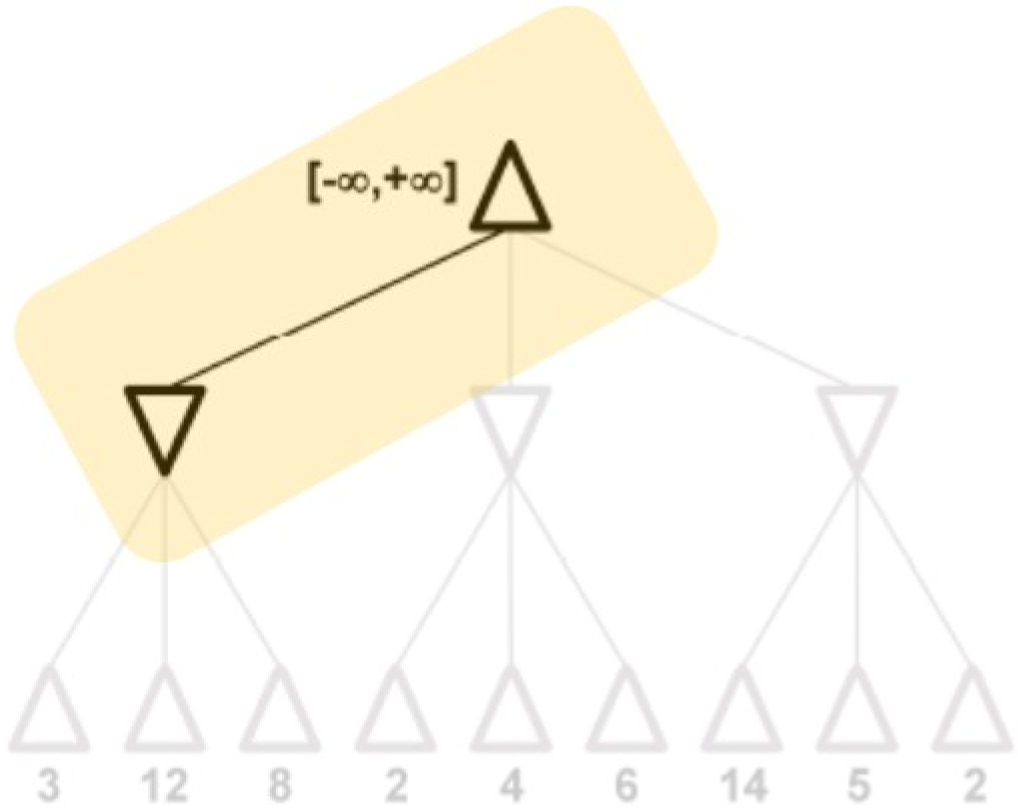
(optimization on continuous space)

Hill Climbing:



Alpha - Beta prunning

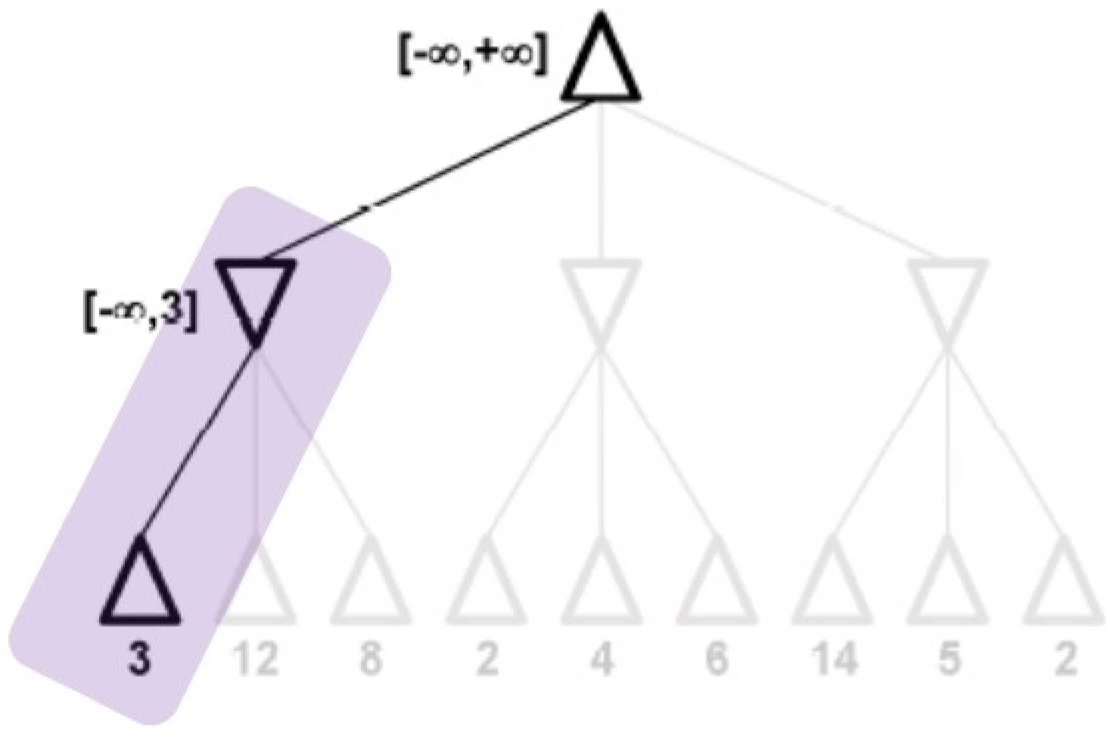
Example



Search the first move for Max
(trying to update α)

Alpha - Beta prunning

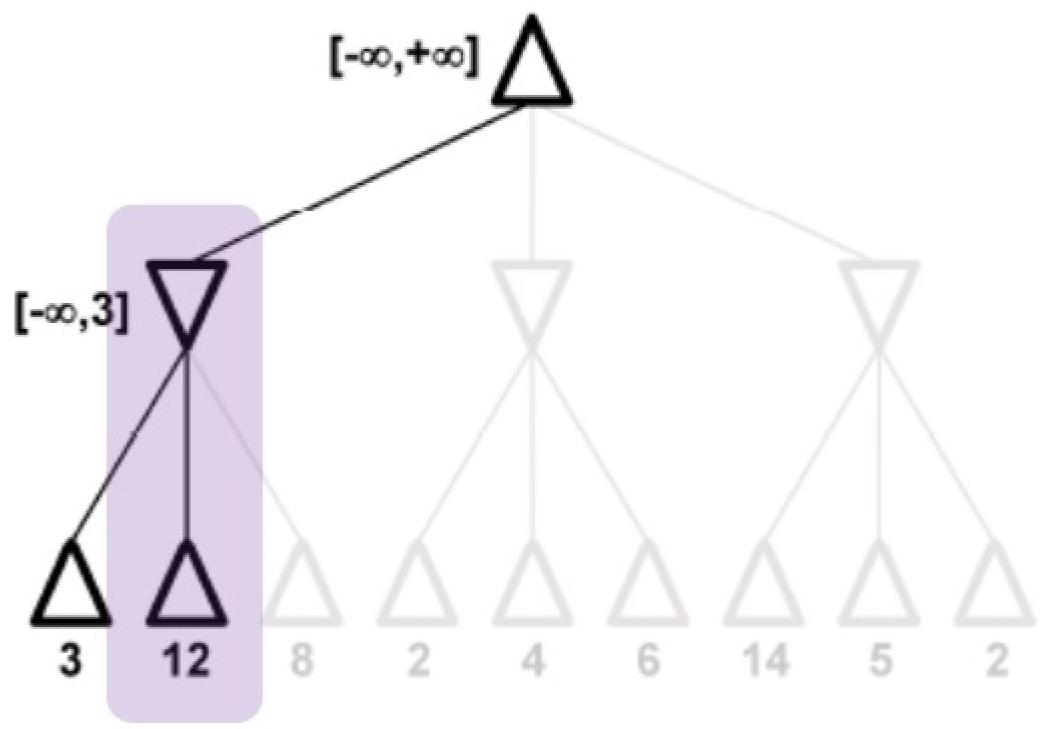
Example



Search the first move for Min
(update β to 3)

Alpha - Beta prunning

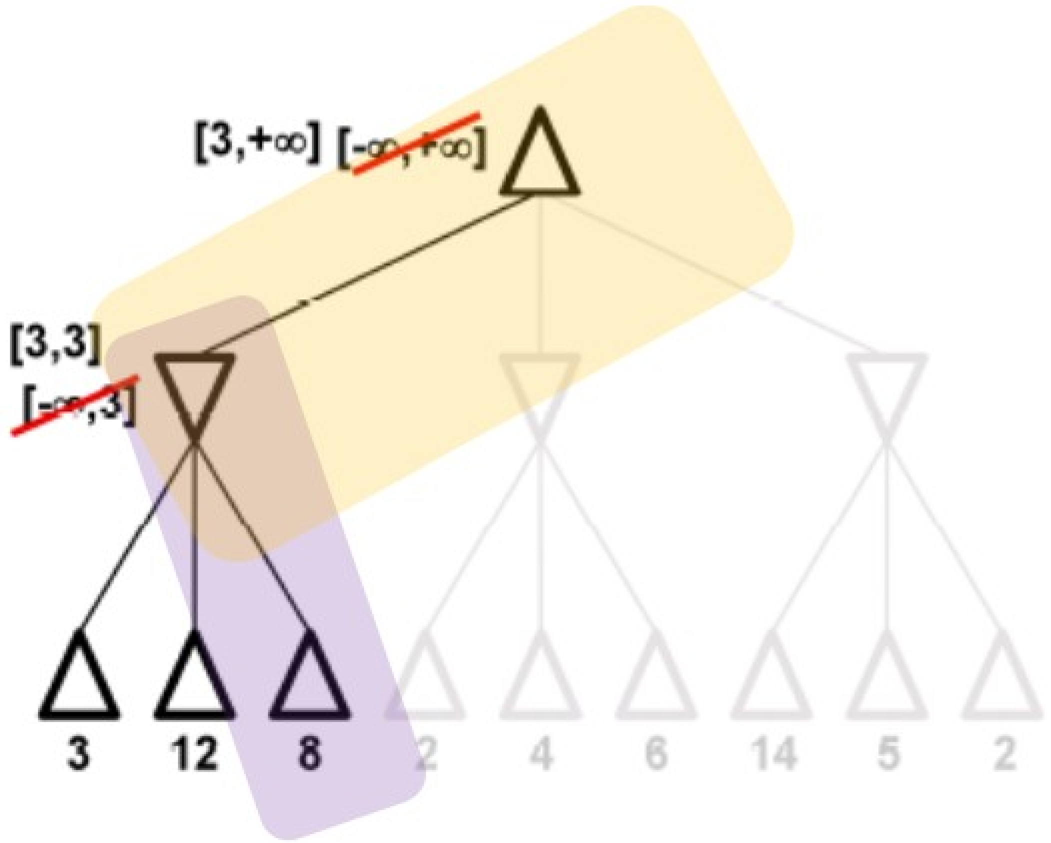
Example



Consider the second move for Min: not better than 3!
(no change to β)

Alpha - Beta prunning

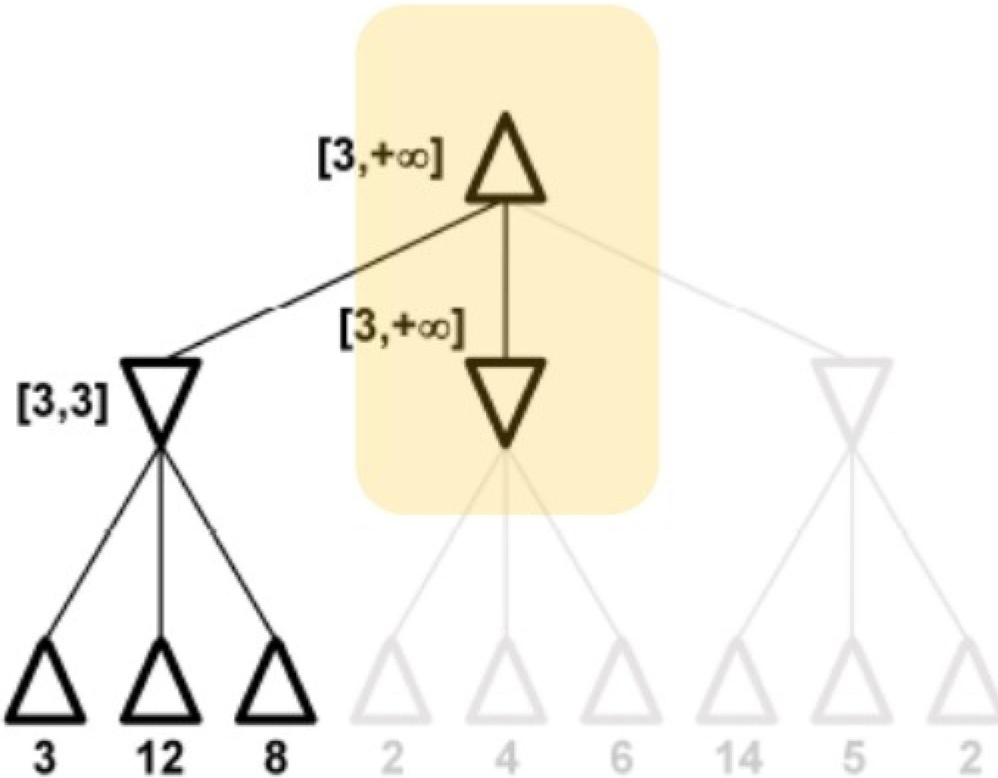
Example



Consider the third move for Min: not better than 3.
Return 3 from min node (write [3,3]).
Update α at top level for Max.

Alpha - Beta prunning

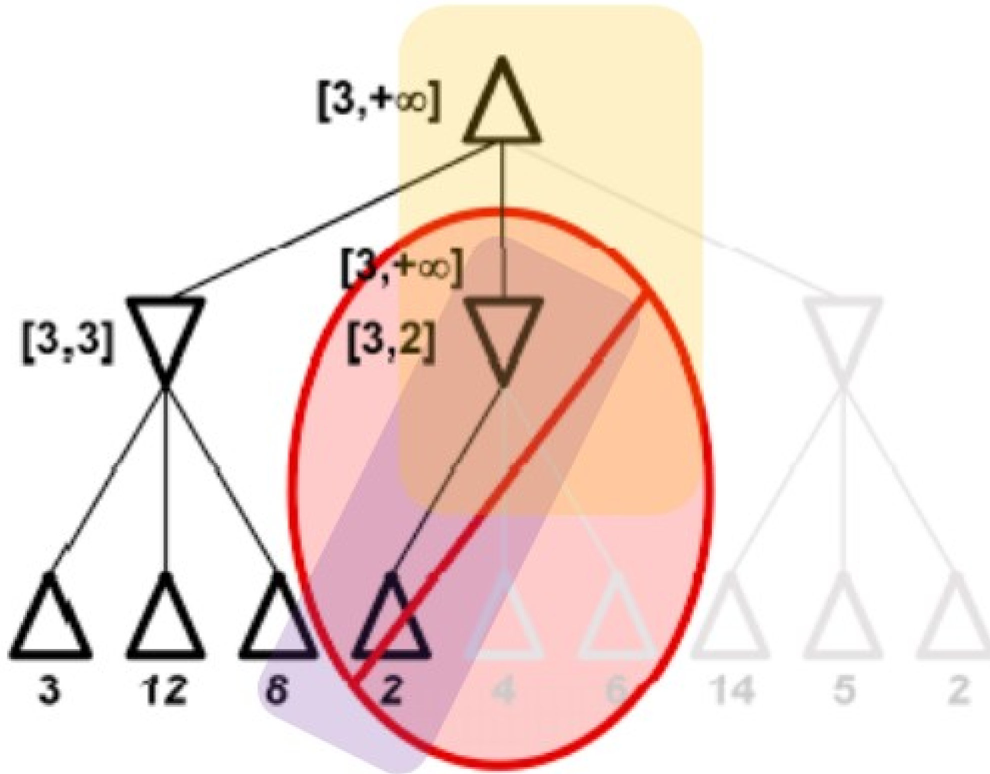
Example



Consider second move for Max, passing in [3, +∞]

Alpha - Beta prunning

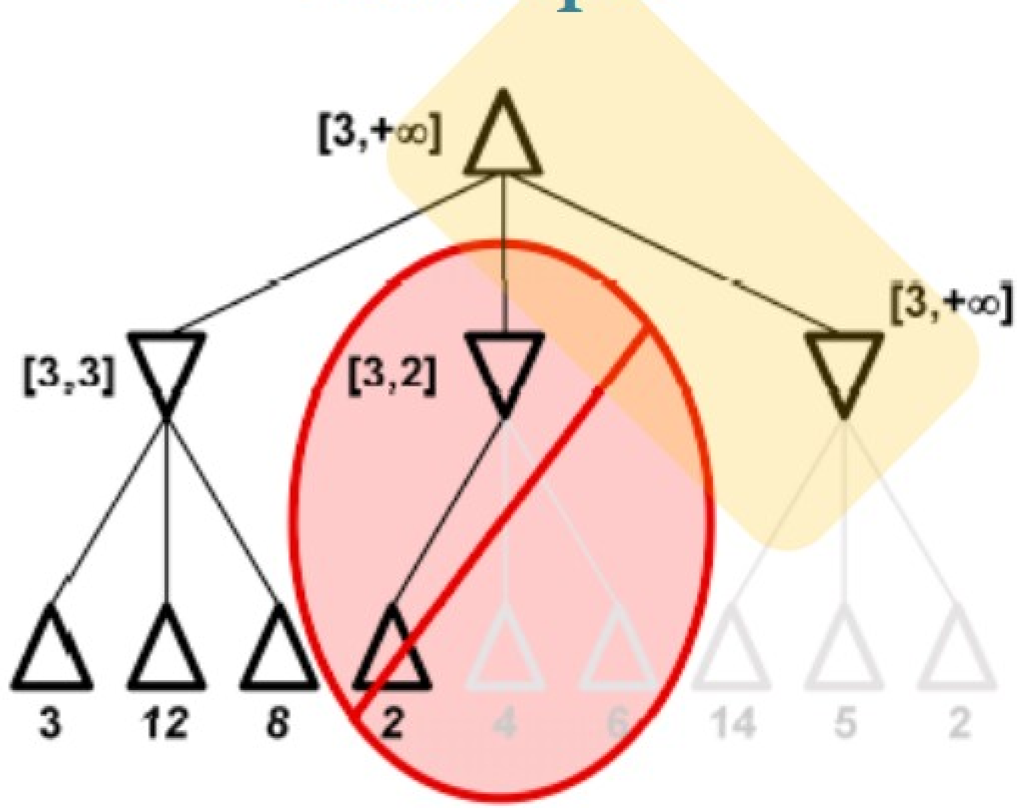
Example



Consider first move of Min
Update β and notice inconsistency [3,2]
Return 2 to top **immediately**, (no change to α, as 3 is better)

Alpha - Beta prunning

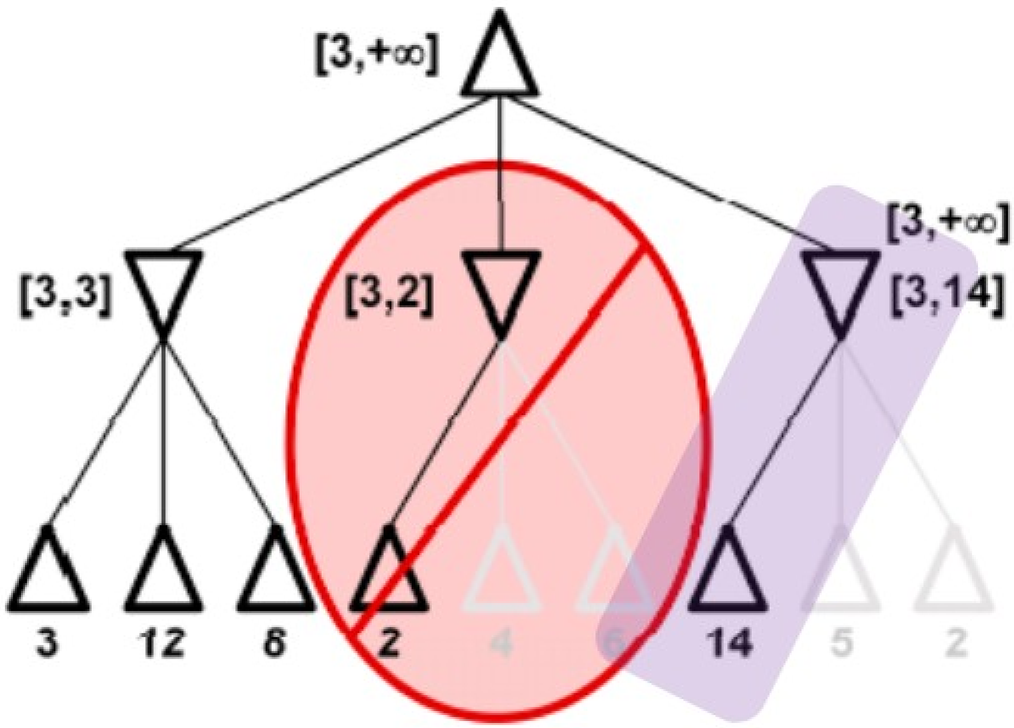
Example



Consider third move of Max, passing in [3, +∞]

Alpha - Beta prunning

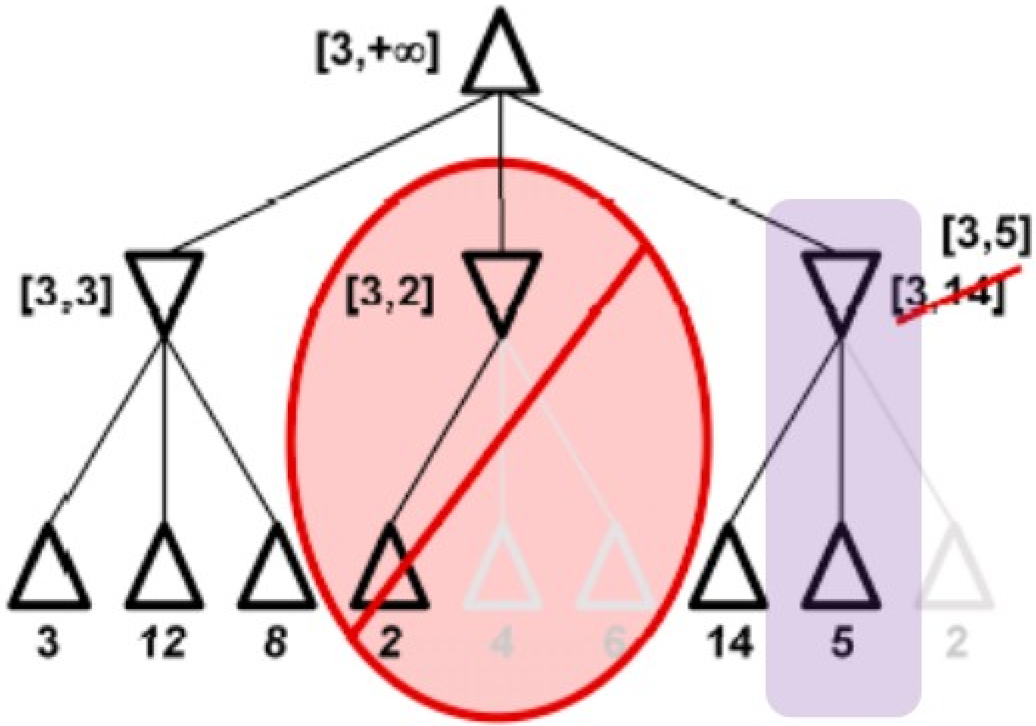
Example



Consider first move of Min
Update β to 14

Alpha - Beta prunning

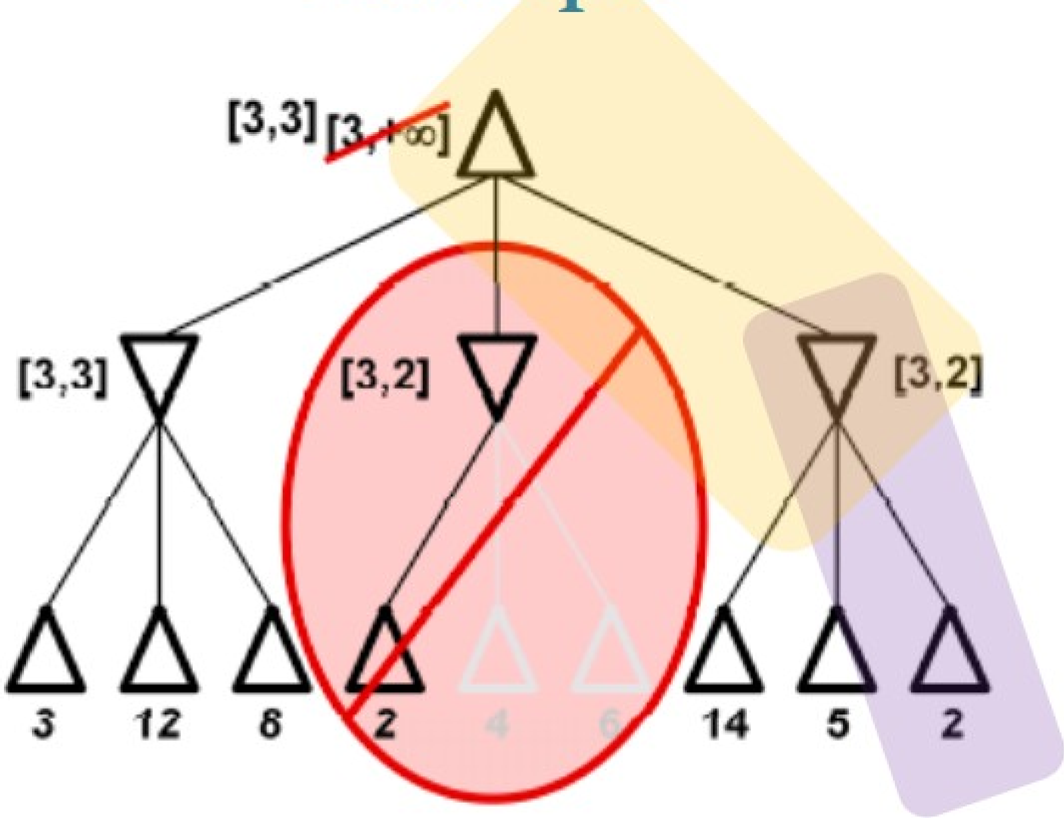
Example



Consider second move of Min
Update β to 5

Alpha - Beta prunning

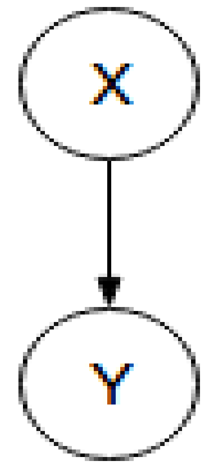
Example



Third move: update β and notice inconsistency [3,2]
Return 2 to top, but this time it doesn't save us any effort
Return 3 at the top level (done!)

2. Bayesian Networks

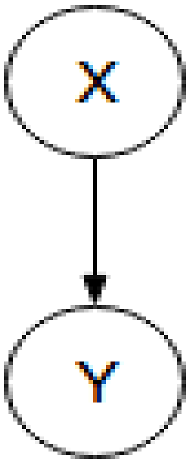
Consider the following graph:



- Joint distribution: Pr(X, Y) =
-

2. Bayesian Networks

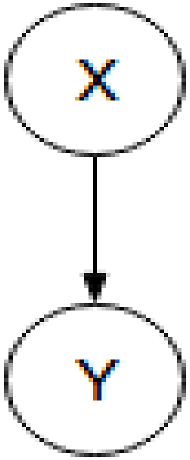
Consider the following graph:



- Joint distribution: Pr(X, Y) = Pr(X) * Pr(Y | X)
- Bayes rule: Pr(X | Y) =

2. Bayesian Networks

Consider the following graph:



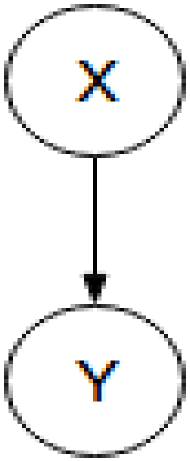
- <u>Joint distribution</u>: Pr(X, Y) = Pr(X) * Pr(Y | X)
- <u>Bayes rule</u>: Pr(X | Y) = Pr(X,Y) / Pr(Y)

  Pr(Y | X) * Pr(X) / Pr(Y)

Posterior = Likelihood * Prior / Normalization cst.

2. Bayesian Networks

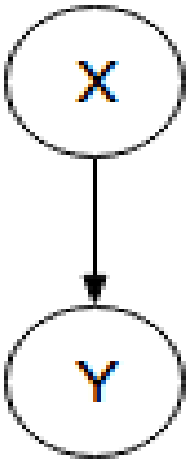
Consider the following graph:



- Enumerate the parameters that must be learned.

2. Bayesian Networks

Consider the following graph:



- Enumerate the parameters that must be learned.

$$\theta_X = P(X)$$

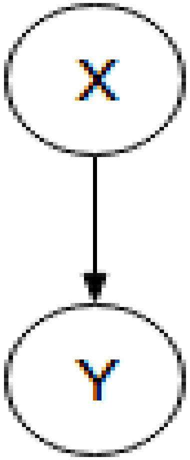
$$\theta_{Y,1} = P(Y|X=1)$$

$$\theta_{Y,0} = P(Y|X=0)$$

2. Bayesian Networks

- Consider the following graph:
- Given samples:

X	Y	# of instances
0	0	1
0	1	2
1	0	3
1	1	4



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- Compute MLE:

2. Bayesian Networks

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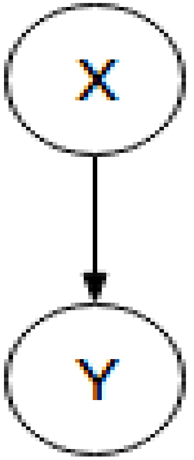


$$\theta_X = P(X)$$

$$\theta_Y = P(Y|X=1)$$

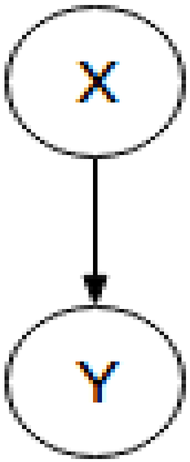
$$\theta_Y = P(Y|X=0)$$

- Compute MLE:
  - P(X=1) = (3+4) / (1+2+3+4) = 7 / 10
  - P(Y=1 | X=1) = (4) / (4+3) = 4 / 7
  - P(Y=1 | X=0) = (2) / (2+1) = 2 / 3

2. Bayesian Networks

- Consider the following graph:
- Given samples:

X	Y	# of instances
0	0	1
0	1	2
1	0	3
1	1	4



$$\theta_X = P(X)$$

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$$\theta_Y = P(Y|X=0)$$

- Give the maximum a posterior estimate for each parameter after applying Laplace smoothing:

2. Bayesian Networks

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- Given samples:

X	Y	# of instances
0	0	1
0	1	2
1	0	3
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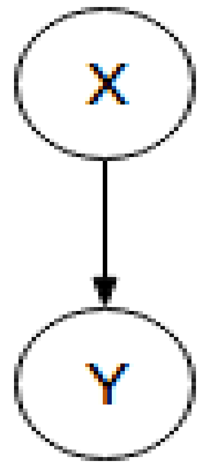


$$\theta_X = P(X)$$

$$\theta_Y = P(Y|X=1)$$

$$\theta_Y = P(Y|X=0)$$

- Give the maximum a posterior estimate for each parameter after applying Laplace smoothing:
  - P(X=1) = (4+3 **+1**) / (1+2+3+4 **+2**) = 8 / 12
  - P(Y=1 | X=1) = (4 **+1**) / (3+4 **+2**) = 5 / 9
  - P(Y=1 | X=0) = (2 **+1**) / (1+2 **+2**) = 3 / 5

2. Bayesian Networks

- Missing Data: same as before but with extra entry: <X=0, Y=?>
- We will apply the (soft) EM algorithm on these instances. Initialize the model using MLE parameter estimates.
- Show the computation of the first E-step:

X	Y
0	0
0	1
0	1
1	0
1	0
1	0
1	1
1	1
1	1
1	1
0	?

2. Bayesian Networks

X	Y
0	0
0	1
0	1
1	0
1	0
1	0
1	1
1	1
1	1
1	1
0	?

- Missing Data: same as before but with extra entry: <X=0, Y=?>
- We will apply the (soft) EM algorithm on these instances. Initialize the model using MLE parameter estimates.
- Show the computation of the first E-step:
  - From before we have:
    P(Y=1 | X=0) = 2 / 3
    so we have weights
      Y=1 : 2 / 3
      Y=0 : 1 / 3

2. Bayesian Networks

- Missing Data: same as before but with extra entry: <X=0, Y=?>
- We will apply the (soft) EM algorithm on these instances. Initialize the model using MLE parameter estimates.
- What are the parameters obtained for the first M-step?

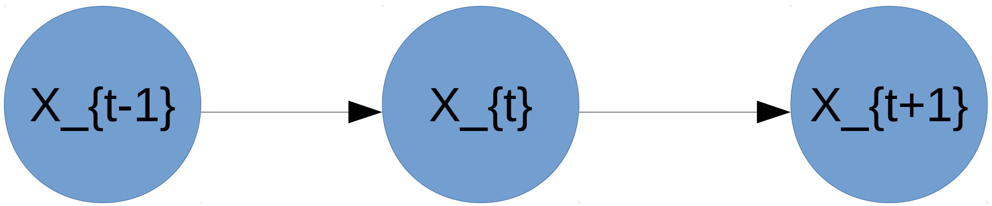
X	Y
0	0
0	1
0	1
1	0
1	0
1	0
1	1
1	1
1	1
1	1
0	?

2. Bayesian Networks

X	Y
0	0
0	1
0	1
1	0
1	0
1	0
1	1
1	1
1	1
1	1
0	?

- Missing Data: same as before but with extra entry: <X=0, Y=?>
- We will apply the (soft) EM algorithm on these instances. Initialize the model using MLE parameter estimates.
- What are the parameters obtained for the first M-step?
  - $P(X=1) = 7 / 11$
  - $P(Y=1 \mid X=1) = 4 / 7$ (same as before)
  - $P(Y=1 \mid X=0) = (2\mathbf{+2/3}) / [(2\mathbf{+2/3})+(1\mathbf{+1/3})] = 2 / 3$ converged!
- Note: for this example, the data does not provide us with more info and has converged in 1 step. No further E-step is required.

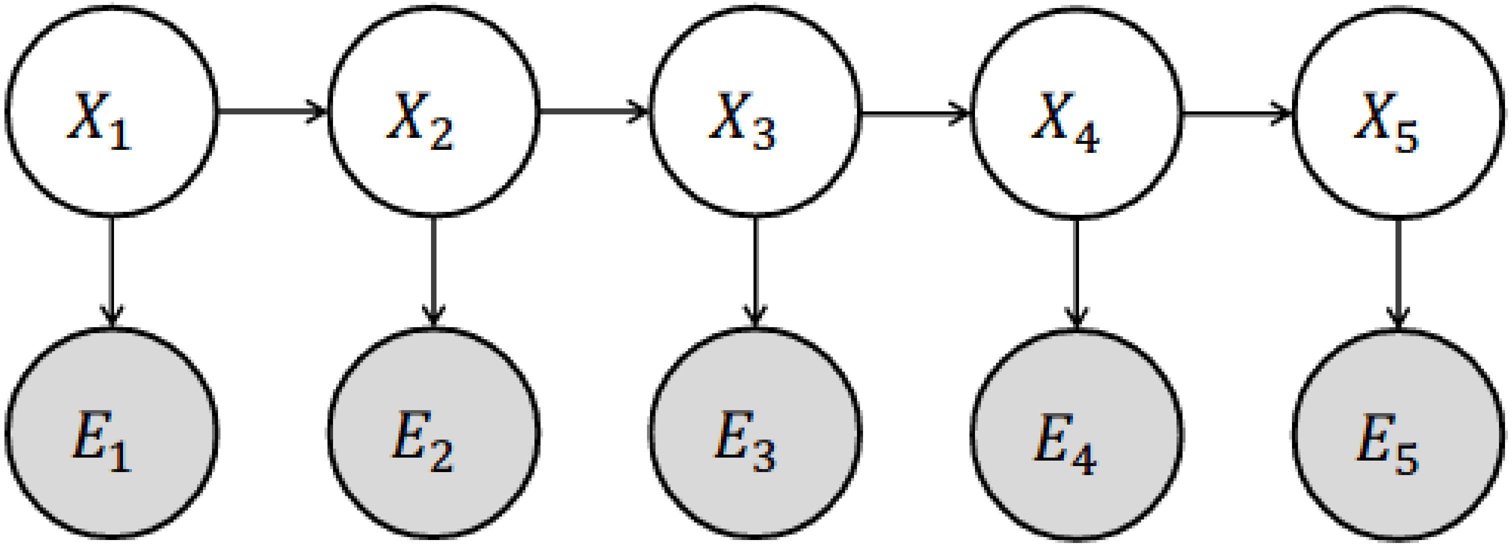
3. Hidden Markov Models

- Encodes time dependence
- Markov assumption:
  - "The future is **independent** of the past **given** the present"
  - $X_{t+1} \perp X_{t-1} | X_t$
  - 

- Stationary process:
  ie: the probabilities don't change over time

3. Hidden Markov Models

- Recall from our discussion of Bayes Nets:



- $P(\boldsymbol{X}, \boldsymbol{E}) = P(X_1) \prod_{t=1}^{T-1} P(X_{t+1}|X_t) \prod_{t=1}^{T} P(E_t|X_t)$

Initial state probability

State transition probabilities

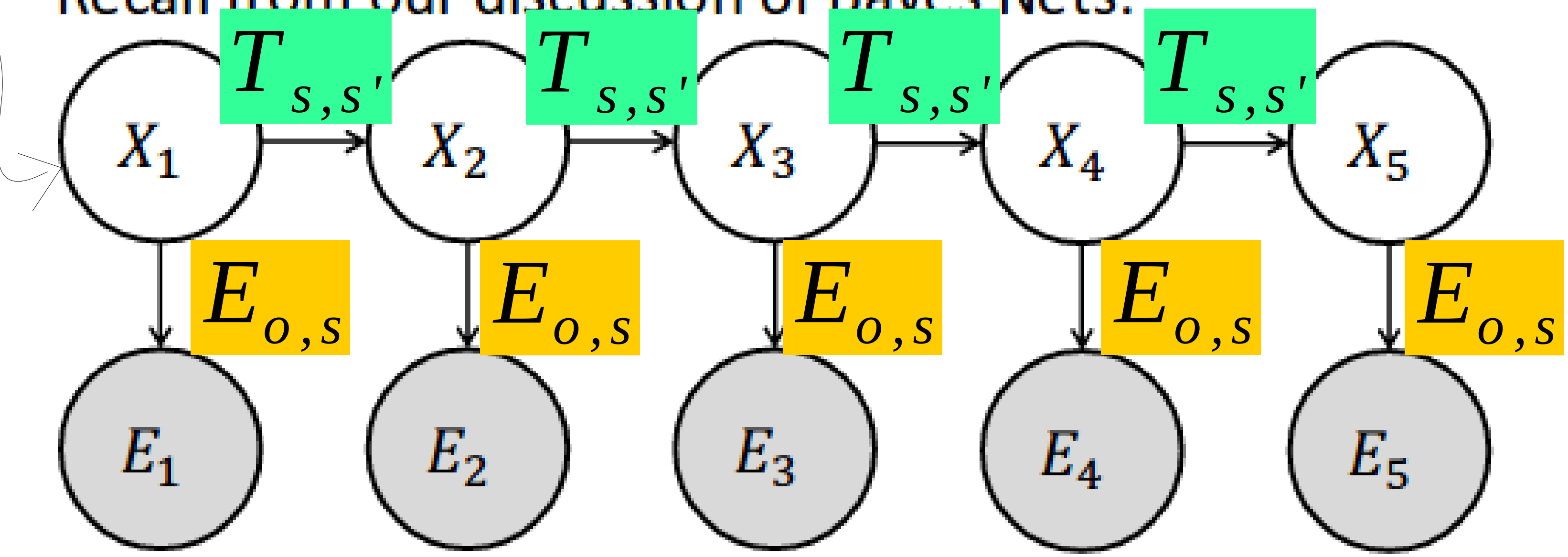
Emission probabilities

3. Hidden Markov Models

Parameters:

π_s

- Recall from our discussion of Bayes Nets:



- $P(\boldsymbol{X}, \boldsymbol{E}) = P(X_1) \prod_{t=1}^{T-1} P(X_{t+1} | X_t) \prod_{t=1}^{T} P(E_t | X_t)$

Initial state probability

π_s

State transition probabilities

$T_{s,s'}$

Emission probabilities

$E_{o,s}$

3. Hidden Markov Models

- "Forward Algorithm": $\alpha_s(t)$
  --> *proba of <u>current state and observations</u> up until now*
  --> $Pr(X_t = s, E_{1:t})$
- "Backward Algorithm": $\beta_s(t)$
  --> *proba of <u>next observations given current state</u>*
  --> $Pr(E_{t+1:T} | X_t = s)$
- "Viterbi Algorithm": $\delta_s(t)$
  --> *most likely <u>sequence of states given set of observations</u>*
  --> forward algo (replace sum by max) & backpointers
  --> $max_{X_{1:t-1}} Pr(X_{1:t-1}, E_{1:t}, X_t = s)$
- "Baum-Welch Algorithm": (also called "forward-backward algorithm")
  --> **E.M. with HMMs**
  --> *Predict <u>parameters & most likely sequence given only observations</u>!*
  --1--> Start with rnd. params.
  --2--> E-step: run *Viterbi* to get sequence of states
  --3--> M-step: update parameters with MLE based on previous seq.
  --4--> repeat 2 & 3

3. Hidden Markov Models

- A student either understands (*Und*), or is confused (*Conf*). The TA can <u>observe</u> that the student is *silent*, or *asking* questions.
- The probability that a student came to the tutorial already understanding the material is 0.3. $\pi_{und} = 0.3$ $\pi_{conf} = 0.7$
-
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-
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3. Hidden Markov Models

- A student either understands (*Und*), or is confused (*Conf*). The TA can <u>observe</u> that the student is *silent*, or *asking* questions.
- The probability that a student came to the tutorial already understanding the material is 0.3. $\pi_{und}=0.3$ $\pi_{conf}=0.7$
- There is a 40% chance that a confused student will start to understand the material.

$T_{conf,und}=Pr(Und|Conf)=0.4$ $T_{conf,conf}=Pr(Conf|Conf)=0.6$

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- There is a 40% chance that a confused student will start to understand the material.

$$T_{conf,und} = Pr(Und|Conf) = 0.4 \quad T_{conf,conf} = Pr(Conf|Conf) = 0.6$$

- If the student already understands, there is a 80% chance that they will continue to understand in the next time step.

$$T_{und,und} = Pr(Und|Und) = 0.8 \quad T_{und,conf} = Pr(Conf|Und) = 0.2$$

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3. Hidden Markov Models

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$$T_{und,und} = Pr(Und|Und) = 0.8 \quad T_{und,conf} = Pr(Conf|Und) = 0.2$$

- The probability of a student who understands the material asking questions is 0.1

$$E_{ask,und} = Pr(Ask|Und) = 0.1 \quad E_{sil,und} = Pr(Sil|Und) = 0.9$$

-

3. Hidden Markov Models

- A student either understands (*Und*), or is confused (*Conf*). The TA can <u>observe</u> that the student is *silent*, or *asking* questions.
- The probability that a student came to the tutorial already understanding the material is 0.3. $\pi_{und} = 0.3$ $\pi_{conf} = 0.7$
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$$T_{und,und} = Pr(Und|Und) = 0.8 \quad T_{und,conf} = Pr(Conf|Und) = 0.2$$

- The probability of a student who understands the material asking questions is 0.1

$$E_{ask,und} = Pr(Ask|Und) = 0.1 \quad E_{sil,und} = Pr(Sil|Und) = 0.9$$

- If they don't understand, it's a 50/50 chance whether they ask questions or stay silent.

$$E_{ask,conf} = Pr(Ask|Conf) = 0.5 \quad E_{sil,conf} = Pr(Sil|Conf) = 0.5$$

3. Hidden Markov Models

$\pi_{und} = 0.3$ $\pi_{conf} = 0.7$

$T_{conf,und} = Pr(Und|Conf) = 0.4$ $T_{conf,conf} = Pr(Conf|Conf) = 0.6$

$T_{und,und} = Pr(Und|Und) = 0.8$ $T_{und,conf} = Pr(Conf|Und) = 0.2$

$E_{ask,und} = Pr(Ask|Und) = 0.1$ $E_{sil,conf} = Pr(Sil|Conf) = 0.5$

$E_{ask,conf} = Pr(Ask|Conf) = 0.5$ $E_{sil,und} = Pr(Sil|Und) = 0.9$

	Silent	**Asking**	**Silent**	**Silent**
Understands	0.27	$\alpha_{und}(2)$	0.0732	0.0683
Confused	$\alpha_{conf}(1)$	0.132	0.0432	$\alpha_{conf}(4)$

3. Hidden Markov Models

$\pi_{und} = 0.3$ $\pi_{conf} = 0.7$

$T_{conf,und} = Pr(Und|Conf) = 0.4$ $T_{conf,conf} = Pr(Conf|Conf) = 0.6$

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$E_{ask,und} = Pr(Ask|Und) = 0.1$ $E_{sil,conf} = Pr(Sil|Conf) = 0.5$

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	Silent	**Asking**	**Silent**	**Silent**
Understands	0.27	$\alpha_{und}(2)$	0.0732	0.0683
Confused	$\alpha_{conf}(1)$	0.132	0.0432	$\alpha_{conf}(4)$

$$\alpha_{conf}(1) = \pi_{conf} * E_{sil,conf} = 0.7 * 0.5 = 0.35$$

3. Hidden Markov Models

$\pi_{und} = 0.3$ $\pi_{conf} = 0.7$

$T_{conf,und} = Pr(Und|Conf) = 0.4$ $T_{conf,conf} = Pr(Conf|Conf) = 0.6$

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	Silent	Asking	Silent	Silent
Understands	0.27	$\alpha_{und}(2)$	0.0732	0.0683
Confused	$\alpha_{conf}(1)$	0.132	0.0432	$\alpha_{conf}(4)$

$$\alpha_{conf}(1) = \pi_{conf} * E_{sil,conf} = 0.7 * 0.5 = 0.35$$

$$\alpha_{und}(2) = [\alpha_{und}(1) * T_{und,und} + \alpha_{conf}(1) * T_{conf,und}] * E_{ask,und}$$
$$= [0.27 * 0.8 + 0.35 * 0.2] * 0.1 = 0.0356$$

3. Hidden Markov Models

$\pi_{und}=0.3$ $\pi_{conf}=0.7$

$T_{conf,und}=Pr(Und|Conf)=0.4$ $T_{conf,conf}=Pr(Conf|Conf)=0.6$

$T_{und,und}=Pr(Und|Und)=0.8$ $T_{und,conf}=Pr(Conf|Und)=0.2$

$E_{ask,und}=Pr(Ask|Und)=0.1$ $E_{sil,conf}=Pr(Sil|Conf)=0.5$

$E_{ask,conf}=Pr(Ask|Conf)=0.5$ $E_{sil,und}=Pr(Sil|Und)=0.9$

	Silent	**Asking**	**Silent**	**Silent**
Understands	0.27	$\alpha_{und}(2)$	0.0732	0.0683
Confused	$\alpha_{conf}(1)$	0.132	0.0432	$\alpha_{conf}(4)$

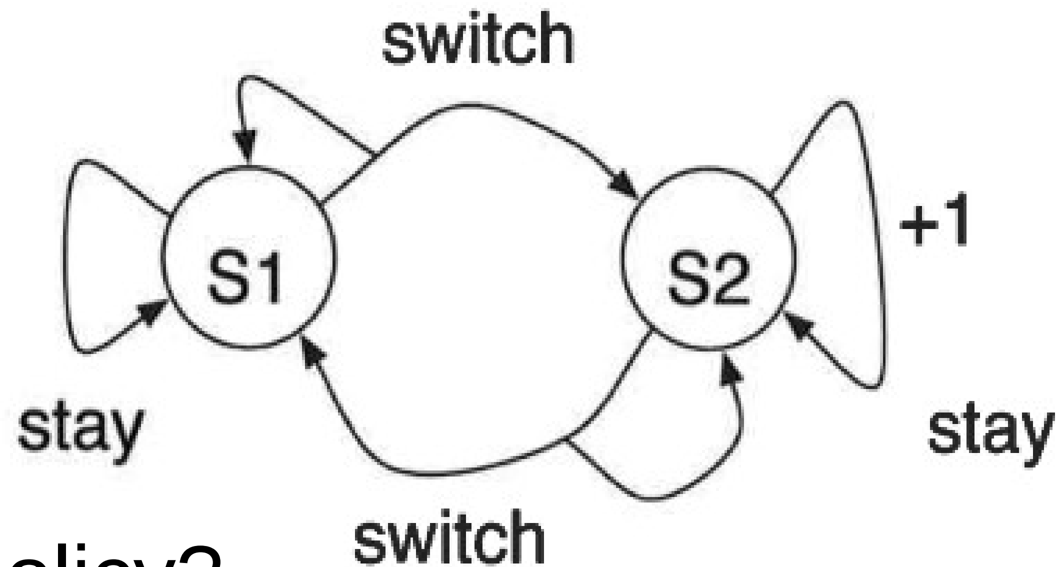
$$\alpha_{conf}(1)=\pi_{conf}*E_{sil,conf}=0.7*0.5=0.35$$

$$\alpha_{und}(2)=[\alpha_{und}(1)*T_{und,und}+\alpha_{conf}(1)*T_{conf,und}]*E_{ask,und}$$
$$=[0.27*0.8+0.35*0.2]*0.1=0.0356$$

$$\alpha_{conf}(4)=[\alpha_{und}(3)*T_{und,conf}+\alpha_{conf}(3)*T_{conf,conf}]*E_{sil,conf}$$
$$=[0.0732*0.2+0.0432*0.6]*0.5=0.02028$$

4. Markov Decision Process

- Consider the MDP below. The switch action only works with propa 0.8. The stay action is deterministic. All rewards are 0 except for performing the stay action in S2. The discount factor is 1/2.



- a) what is the optimal policy?

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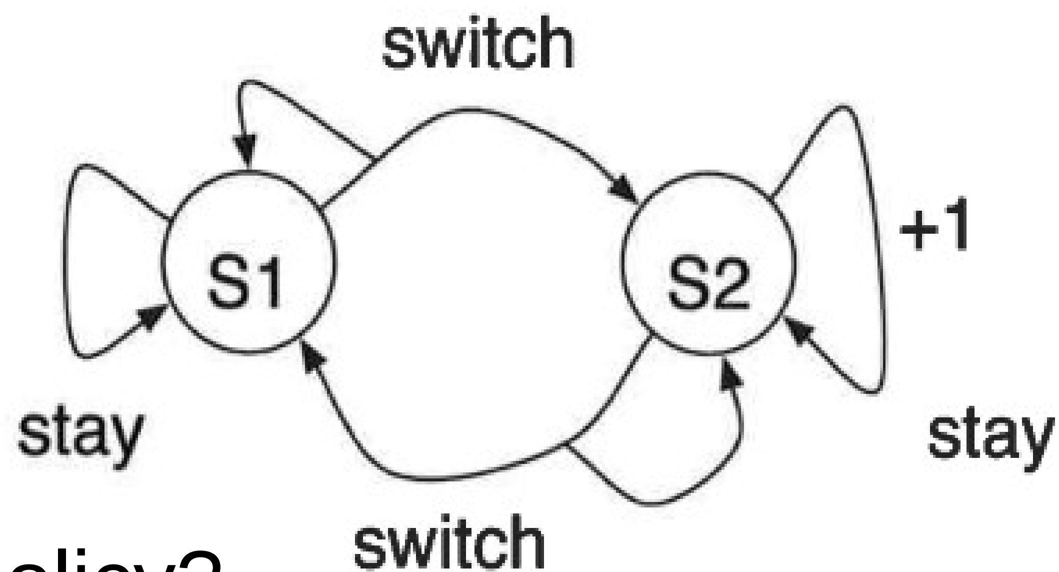
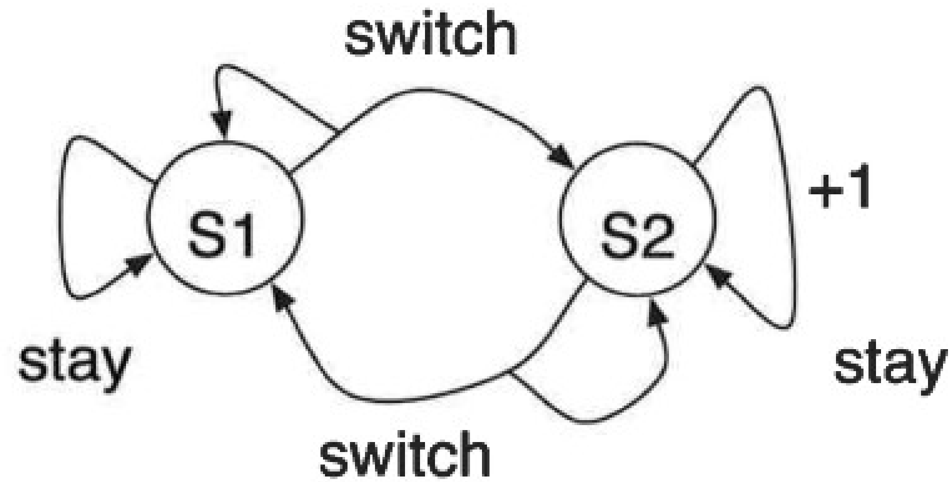


- a) what is the optimal policy?

$$\pi^*(S1) = switch$$

$$\pi^*(S2) = stay$$

4. Markov Decision Process



- T(s, switch, s') = 0.8
  T(s, stay, s) = 1
  R(S1, *a*) = 0 for all *a*
  R(S2, stay) = 1
  R(S2, switch) = 0
  The discount factor (g) is 0.5

  $\pi^*(S2) = stay \quad \pi^*(S1) = switch$

- b) compute the optimal value function

4. Markov Decision Process

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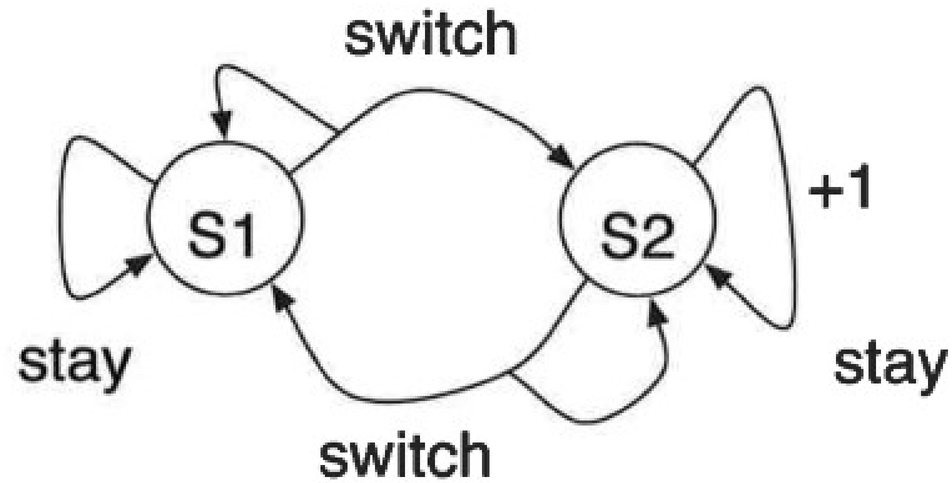

- b) compute the optimal value function

$$V^*(S1) = R(S1, \pi^*(S1)) + \gamma \sum_s [T(S1, \pi^*(S1), s) * V^*(s)]$$
$$V^*(S2) = R(S2, \pi^*(S2)) + \gamma \sum_s [T(S2, \pi^*(S2), s) * V^*(s)]$$

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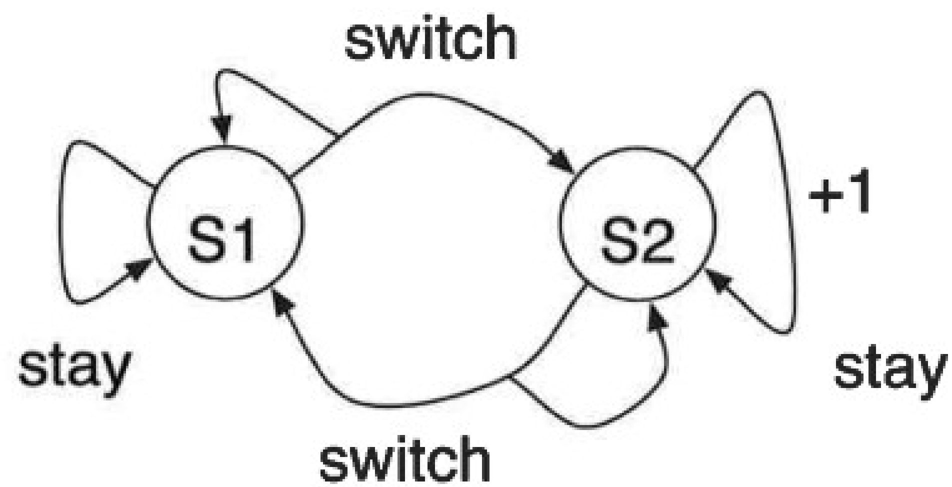

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$$V^*(S2) = R(S2, \pi^*(S2)) + \gamma \sum_s [T(S2, \pi^*(S2), s) * V^*(s)]$$

$$V^*(S2) = 1 + 0.5[0 * V^*(S1) + 1 * V^*(S2)] = 1 + 0.5 * V^*(S2)$$
$$V^*(S2) = 2$$

$$V^*(S1) = 0 + 0.5[0.2 * V^*(S1) + 0.8 * V^*(S2)] = 0.1 * V^*(S1) + 0.4 * 2$$
$$V^*(S1) = 8/9$$

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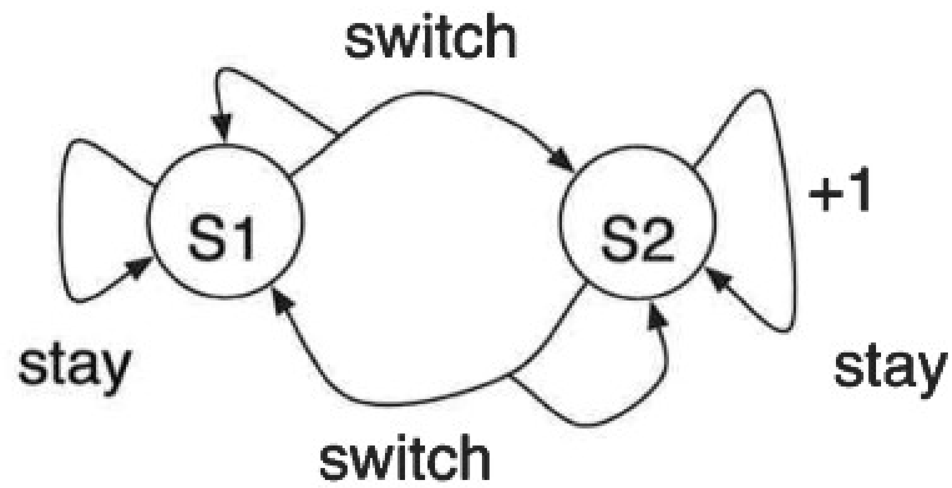

$\pi^*(S2) = stay \quad \pi^*(S1) = switch \qquad V^*(S1) = 8/9 \quad V^*(S2) = 2$

- c) perform the first 2 steps of value iteration (start with initial values of 0)

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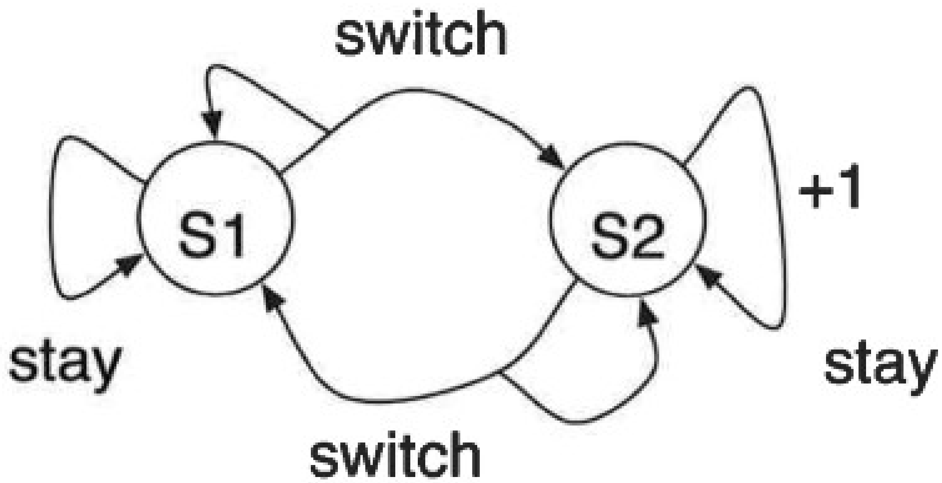

$\pi^*(S2) = stay \quad \pi^*(S1) = switch \qquad V^*(S1) = 8/9 \quad V^*(S2) = 2$

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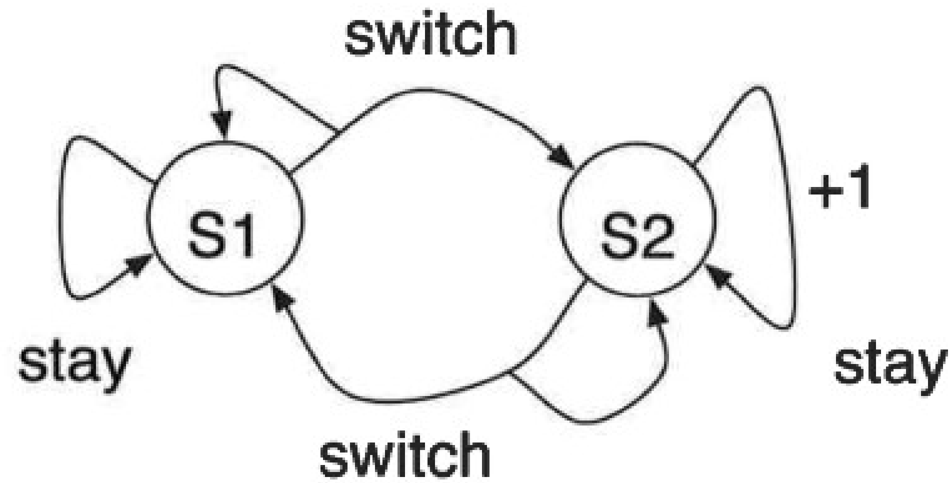

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  S1: a = stay 0 + 0.5 [1*V(S1) + 0*V(S2)] = 0
  a = switch 0 + 0.5 [0.2*V(S1) + 0.8*V(S2)] = 0
  S2: a = stay 1 + 0.5 [0*V(S1) + 1*V(S2)] = 1
  a = switch 0 + 0.5 [0.8*V(S1) + 0.2*V(S2)] = 0

  ---> V(S1) = 0 & V(S2) = 1

4. Markov Decision Process

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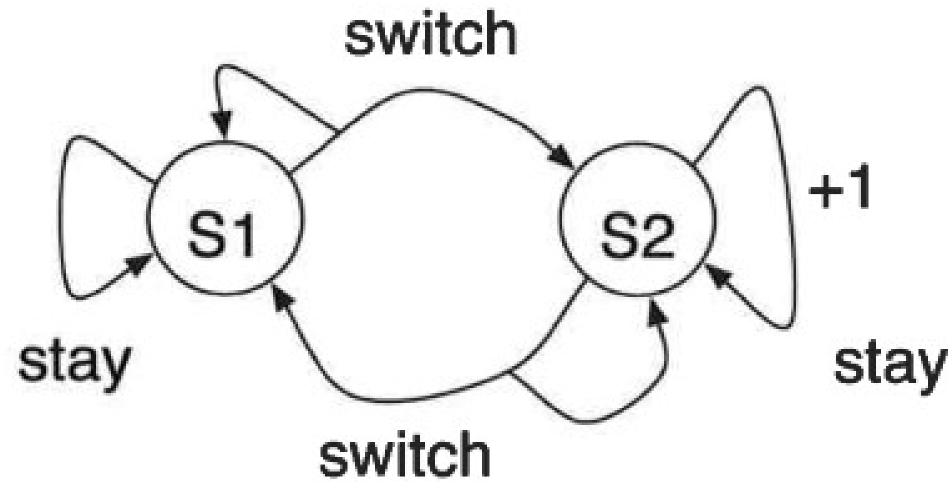
  T(s, stay, s) = 1
  R(S1, *a*) = 0 for all *a*
  R(S2, stay) = 1
  R(S2, switch) = 0
  The discount factor (g) is 0.5



$\pi^*(S2) = stay \quad \pi^*(S1) = switch$ $\quad V^*(S1) = 8/9 \quad V^*(S2) = 2$

- c) perform the first 2 steps of value iteration (start with initial values of 0) $V(S) = max_a [R(S,a) + \gamma \sum_{s'} [T(S,a,s') * V(s')]]$
  ---> V(S1) = 0 & V(S2) = 1

S1: a = stay	0 + 0.5 [1*V(S1) + 0*V(S2)] = 0
a = switch	0 + 0.5 [0.2*V(S1) + 0.8*V(S2)] = 0.4
S2: a = stay	1 + 0.5 [0*V(S1) + 1*V(S2)] = 1.5
a = switch	0 + 0.5 [0.8*V(S1) + 0.2*V(S2)] = 0.1

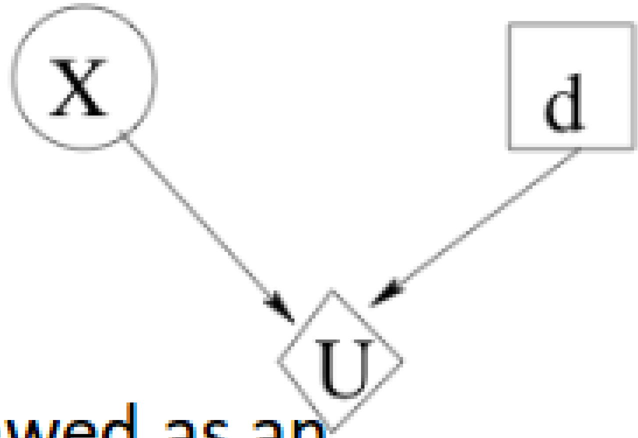
---> V(S1) = 0.4 & V(S2) = 1.5

5. Utility

- Alice wants to take a trip to Chicago, but she has no car.
- A plane ticket costs $400, and the advertised travel time is 2 hours.
- $200 can get her a ticket on an 8 hour train trip.
- Or, she can spend 18 hours on a bus for only $50.
- Every hour she spends on the commute costs her another $20 of wasted opportunity, and there's a 40% chance that there will be traffic, which doubles the time for all three modes of travel.
- <u>(a) Draw a decision graph for this problem</u>

5. Utility

- Represent decision models graphically:
  - Random variables are represented as **oval** nodes.
    - Parameters associated with such nodes are *probabilities*.
  - Decisions (actions) are represented as **rectangles**.
  - Utilities are represented as **diamonds**.
    - Parameters associated with such nodes are *utility values* for all possible values of the parents.



- Restrictions on nodes:
  - Utility nodes have no out-going arcs.
  - Decision nodes have no incoming arcs.
- Computing the optimal action can be viewed as an **inference** task.

5. Utility

- plane ticket costs \$400, travel time is 2 hours.
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- Or, she can spend 18 hours on a bus for only \$50.
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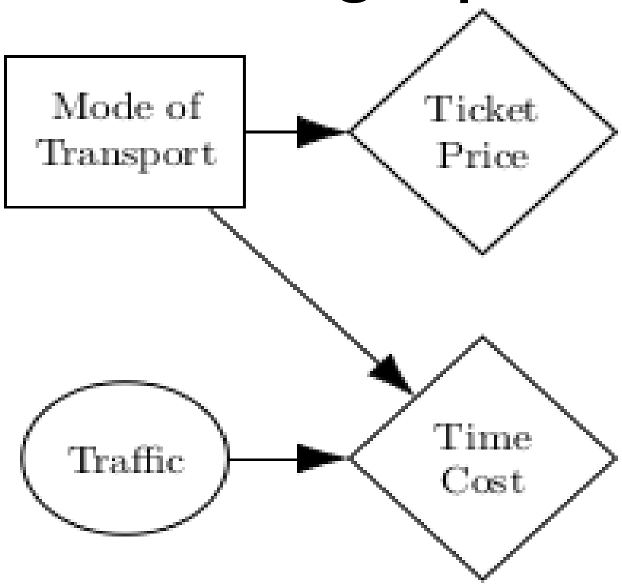
- <u>(a) Draw a decision graph for this problem</u>



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- <u>(b) Which ticket should Alice buy to minimize trip cost?</u>

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- <u>(b) Which ticket should Alice buy to minimize trip cost?</u>

$$\underbrace{\begin{array}{lll} EU(Plane) = & -400 & -20 * [0.6*2 + 0.4*4] = -456 \\ EU(Train) = & -200 & -20 * [0.6*8 + 0.4*16] = -424 \quad \texttt{<---} \\ EU(Bus) = & -50 & -20 * [0.6*18 + 0.4*36] = -554 \end{array}}$$

Ticket

Time

--> *a* = Train* with *EU(a*) = -424*

5. Utility

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- $200 can get her a ticket on an 8 hour train trip.
- Or, she can spend 18 hours on a bus for only $50.
- Every hour costs her $20 of wasted opportunity,
- 40% there will be traffic, which doubles the time.
- (c) How much should she be willing to pay a fortune teller to tell her with 100% certainty whether there will be trafic?

5. Utility

Value of Perfect Information

- Suppose we knew $X=x$, then we would choose a_x* such that:

$$EU(a_x^*|E, X = x) = \max_a \sum_i U(c_i)P(c_i|E, a, X = x)$$

- X is a random variable whose value is unknown, so we must compute expected gain over all possible values:

$$VPI_E(X) = \left(\sum_x P(X = x|E)EU(a_x^*|E, X = x) \right) - EU(a^*|E)$$

This is the value of knowing *X* exactly!

5. Utility

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$$\left.\begin{aligned} EU(Plane|Tr=0) &= -400 - 40 = -440 \\ EU(Train|Tr=0) &= -200 - 160 = -360 \\ EU(Bus|Tr=0) &= -50 - 360 = -410 \end{aligned}\right\} \quad a^*_{Tr=0} = Train,\ EU(a^*_{Tr=0}|Tr=0) = -360$$

$$\left.\begin{aligned} EU(Plane|Tr=1) &= -400 - 80 = -480 \\ EU(Train|Tr=1) &= -200 - 320 = -520 \\ EU(Bus|Tr=1) &= -50 - 720 = -770 \end{aligned}\right\} \quad a^*_{Tr=1} = Plane,\ EU(a^*_{Tr=1}|Tr=1) = -480$$

5. Utility

- <u>(c) How much should she be willing to pay a fortune teller to tell her with 100% certainty whether there will be trafic?</u>

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Recall from (b), *a* = Train* with *EU(a*) = -424*

$$\begin{aligned} VPI(X) &= \left(\sum_x P(Tr=x) EU(a^*_{Tr=x}|Tr=x) \right) - EU(a^*) \\ &= P(Tr=0)EU(a^*_{Tr=0}|Tr=0) + P(Tr=1|E)EU(a^*_{Tr=1}|Tr=1) - EU(a^*) \\ &= (-360 \times 0.6 + -480 \times 0.4) - (-424) = 16 \end{aligned}$$

Good Luck!

Questions ?

~ Nicolas Angelard-Gontier ~
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