A Regression Analysis of the Gender Pay Gap (Technical Report)

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1 Data Wrangling

1.1 Data Import

Importing both the provided sample and the full original PUMS dataset (merged, pre-wrangled, and filtered appropriately in an .R script). The provided sample will be used for analysis, while the full dataset might come in handy for certain visualizations such as maps, which require more data to be meaningful.

```
# Sample of 3000 to be wrangled in RMarkdown
ACSSmallRaw <- readr::read_csv("data/ACS3k.csv")

# Full dataset (already wrangled, see .R script for details)
ACSBig <- readRDS("data/ACSBig.Rds")</pre>
```

1.2 Preparation

Preparing the levels for the factor variables we intend to recode.

```
# Setting factor levels
sex_levels <- c(M = "1",
                "F" = "2")
# will only be used for tallying
race_levels <- c(White = "1",</pre>
                 Black = "2",
                 Native = "3",
                 Native = "4",
                 Native = "5",
                 Asian = 6,
                 Native = "7",
                 Other = "8",
                 Multiple = "9")
# will only be used for tallying
marital_status_levels <- c(Married = "1",</pre>
                            Widowed = "2",
                            Divorced = "3",
                            Separated = "4",
                            Single = "5")
# collapsing degree
degree_levels <- c("Agriculture" = "11",</pre>
                    "Environmental" = "13",
                    "Architecture" = "14",
                    "Ethnic Studies" = "15",
                    "Media and Journalism" = "19",
                    "Communication" = "20",
                    "Computer Science and IT" = "21",
                    "Cosmetology and Gastronomy" = "22",
                    "Education" = "23",
```

```
"Engineering" = "24",
                   "Engineering" = "25",
                   "Languages" = "26",
                   "Consumer Sciences" = "29",
                   "Law and Policy" = "32",
                   "English Literature" = "33",
                   "Liberal Arts" = "34",
                   "Library Science" = "35",
                   "Biological Sciences" = "36",
                   "Mathematics and Statistics" = "37",
                   "Military" = "38",
                   "Interdisciplinary" = "40",
                   "Fitness" = "41",
                   "Philosophy" = "48",
                   "Theology" = "49",
                   "Physical Sciences" = "50",
                   "Physical Sciences" = "51",
                   "Psychology" = "52",
                   "Law and Policy" = "53",
                   "Law and Policy" = "54",
                   "Social Sciences" = "55",
                   "Construction" = "56",
                   "Construction" = "57",
                   "Transportation" = "59",
                   "Arts" = "60",
                   "Medicine" = "61",
                   "Business and Finance" = "62",
                   "History" = "64")
# will be used for filtering
employment_levels <- c("Employed working" = "1",</pre>
                       "Employed not working" = "2",
                       Unemployed = "3",
                       "Military working" = "4",
                       "Military not working" = "5",
                       "Not in labor force" = "6")
region_levels <- c(Northeast = "1",
                   Midwest = "2",
                   South = "3",
                   West = "4",
                   "Puerto Rico" = "9")
state_levels <- c(AL = "1",
                  AK = "2",
                  AZ = "4"
                  AR = "5"
                  CA = "6",
                  CO = "8".
                  CT = "9"
                  DE = "10",
                  DC = "11",
                  FL = "12",
```

```
GA = "13",
                  HI = "15",
                  ID = "16",
                  IL = "17"
                  IN = "18",
                  IA = "19"
                  KS = "20"
                  KY = "21",
                  LA = "22",
                  ME = "23",
                  MD = "24",
                  MA = "25",
                  MI = "26",
                  MN = "27"
                  MS = "28",
                  MO = "29",
                  MT = "30"
                  NE = "31",
                  NV = "32",
                  NH = "33",
                  NJ = "34"
                  NM = "35",
                  NY = "36"
                  NC = "37",
                  ND = "38",
                  OH = "39",
                  0K = "40",
                  OR = "41",
                  PA = "42"
                  RI = "44",
                  SC = "45"
                  SD = "46",
                  TN = "47",
                  TX = "48",
                  UT = "49",
                  VT = "50",
                  VA = "51"
                  WA = "53".
                  WV = "54",
                  WI = "55",
                  WY = "56",
                  "Puerto Rico" = "72")
# collapsing industry
industry_levels <- c("Agriculture, Forestry, Fishing and Hunting" = "11",
                     "Mining, Quarrying, and Oil and Gas Extraction" = "21",
                     "Utilities" = "22",
                     "Construction" = "23",
                     "Manufacturing" = "31",
                     "Manufacturing" = "32",
                     "Manufacturing" = "33",
                     "Manufacturing" = "3M",
                     "Wholesale Trade" = "42",
```

```
"Retail Trade" = "44",
"Retail Trade" = "45",
"Transportation and Warehousing" = "48",
"Transportation and Warehousing" = "49",
"Information" = "51",
"Finance and Insurance" = "52",
"Real Estate and Rental and Leasing" = "53",
"Professional, Scientific, and Technical Services" = "54",
"Management of Companies and Enterprises" = "55",
"Administrative and Support and Waste Management
and Remediation Services" = "56",
"Educational Services" = "61",
"Health Care and Social Assistance" = "62",
"Arts, Entertainment, and Recreation" = "71",
"Accommodation and Food Services" = "72",
"Other Services (except Public Administration)" = "81",
"Public Administration" = "92")
```

1.3 Variable Selection

Selecting the variables of interest (see the Data Dictionary in Appendix A for details), renaming and mutating them appropriately, and filtering based on specific criteria.

```
# Wrangling the data
ACSSmall <- ACSSmallRaw %>%
  # Adjusting income
  mutate(ADJINC.x = ADJINC.x / 10<sup>6</sup>, # adding decimal point to ADJINC
         WAGP = WAGP * ADJINC.x, # adjusting dollar amounts for inflation
         WAGP = round(WAGP)) %>% # rounding to nearest dollar
  # Selecting which variables to keep
  select(SEX, AGEP, CIT, RAC1P, MIL, DIS, # general demographics
         NP, MAR, NRC, # family and household
         SCHL, FOD1P, FOD2P, SCIENGP, # educational background
         ESR, WKHP, NAICSP, # employment
         WAGP, # income
         REGION.x, ST.x) %>% # location
  # Renaming the variables
  rename(sex = SEX,
         age = AGEP,
         citizenship = CIT,
         race = RAC1P,
         military = MIL,
         disabled = DIS,
         people_in_household = NP,
         marital_status = MAR,
         children = NRC,
         education = SCHL,
         degree 1 = FOD1P,
         degree_2 = FOD2P,
```

```
stem_degree = SCIENGP,
       employment = ESR,
       hours_per_week = WKHP,
       industry = NAICSP,
       wage_income = WAGP,
       region = REGION.x,
       state = ST.x) %>%
# Converting entries to the appropriate data types
mutate(sex = as.factor(sex) %>%
         forcats::fct_recode(!!!sex_levels),
       employment = as.factor(employment) %>%
         forcats::fct_recode(!!!employment_levels),
       race = as.factor(race) %>%
         forcats::fct_recode(!!!race_levels),
       marital_status = as.factor(marital_status) %>%
         forcats::fct_recode(!!!marital_status_levels),
       region = as.factor(region) %>%
         forcats::fct_recode(!!!region_levels),
       state = as.factor(state) %>%
         forcats::fct_recode(!!!state_levels),
       # for citizenship, 5 stands for not a citizen
       citizenship = ifelse(citizenship == 5, "No", "Yes") %>%
         as.factor(),
       privilege = ifelse(race %in% c("White", "Asian"),
                               "Yes", "No") %>%
         as.factor(),
       # for military, 4 stands for never enlisted
       military = ifelse(military == 4, "No", "Yes"),
       military = ifelse(is.na(military), "No", military) %>%
         as.factor(),
       ever_married = ifelse(marital_status == "Single", "No", "Yes") %>%
         as.factor(),
       # for education, 22, 23, and 24 stand for graduate degrees
       grad_degree = ifelse(education %in% c(22, 23, 24), "Yes", "No"),
       grad_degree = ifelse(is.na(grad_degree), "No", grad_degree) %>%
         as.factor(),
       # for STEMdegree, 1 stands for a STEM degree
       stem_degree = ifelse(stem_degree == 1, "Yes", "No"),
       stem_degree = ifelse(is.na(stem_degree), "No", stem_degree) %>%
         as.factor(),
```

```
disabled = ifelse(disabled == 1, "Yes", "No") %>%
        as.factor(),
       # Filling empty industry fields with NAs
       industry = ifelse(industry == "", NA, industry),
       # Collapsing industry codes into broad NAICS sectors (only taking the first
       # two digits of the NAICS code, which represent the broader industry sectors)
       industry = substr(industry, start = 1, stop = 2) %>%
        as.factor() %>%
        fct_recode(!!!industry_levels),
       # Collapsing education codes into broader fields (only taking the first
       # two digits of each code, which represent the broader field)
       degree_1 = substr(degree_1, start = 1, stop = 2) %>%
        as.factor() %>%
        fct_recode(!!!degree_levels),
       degree_2 = substr(degree_2, start = 1, stop = 2) %>%
        as.factor() %>%
        fct recode(!!!degree levels),
       # Merging the two degree variables, taking care of NAs and repeated values
       degree = ifelse(is.na(degree_1) | is.na(degree_2),
                       as.character(degree_1),
                       ifelse(as.character(degree_1) == as.character(degree_2),
                              as.character(degree_1),
                              paste(degree_1, degree_2, sep = " and ")))) %>%
# Filtering the individuals of interest
filter(!(is.na(wage_income)) & wage_income > 0, # salary income is positive
       employment %in% c("Employed working",
                         "Employed not working",
                         "Military working"), # employed and/or working
       age >= 18) %>% # only people over 18
# Removing merged variables and those used for filtering or joining
select(-employment, -education, -degree_1, -degree_2) %>%
# Dropping unused factor levels
mutate_if(is.factor, fct_drop)
```

2 Data Exploration

In this section, we will be exploring distributions and associations graphically and numerically, as a preparation for our model fitting and in order to better understand our data.

Let's take a look at the variables in our dataset.

```
names(ACSSmall) %>% cat(sep = "\n")
     sex
     age
     citizenship
     race
     military
     disabled
     people_in_household
     marital_status
     children
     stem_degree
     hours_per_week
     industry
     wage_income
     region
     state
     privilege
     ever_married
     grad_degree
     degree
setequal(names(ACSBig), names(ACSSmall))
```

[1] TRUE

Variables to be used for graphical exploration only (because they have too many levels): state, industry, degree, region.

Variables to be used for our regression: wage_income (NUMERIC RESPONSE), sex, age (NUMERIC), citizenship, privilege, military, disabled, ever_married, children (NUMERIC), stem_degree, grad_degree, hours_per_week (NUMERIC), people_in_household (NUMERIC)

2.1 Univariate Data Exploration

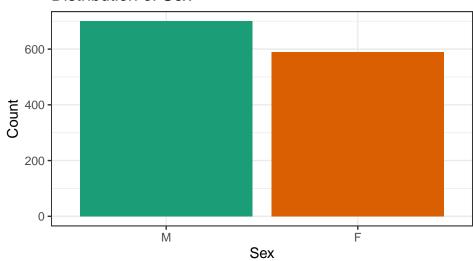
In this section we explore univariate distributions of some of the most important demographic variables, especially those that we aim to include in the regression, to see whether we would benefit from any log transformations. The code is trivial and takes up unnecessary space, so it has been deferred to the appendix (see appendix B - Univariate Data Exploration).

2.1.1 Sex

Table 1: Distribution of Sex

Sex	Tally
Μ	700
F	589

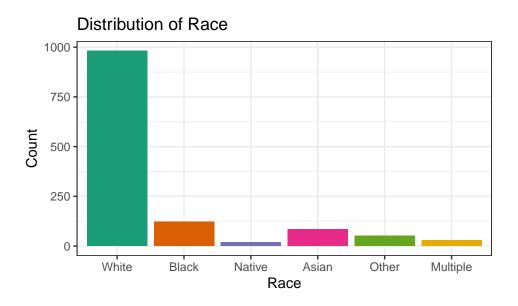
Distribution of Sex



2.1.2 Race

Table 2: Distribution of Race

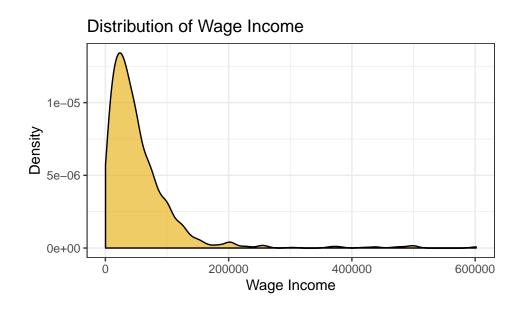
Race	Tally
White	981
Black	122
Native	20
Asian	84
Other	52
Multiple	30



2.1.3 Income

Table 3: Distribution of Wage Income

Min	Q1	Median	Q3	Max	Mean	SD	N	Missing
222	20224	40448	70783	601657	54908.96	62257.94	1289	0

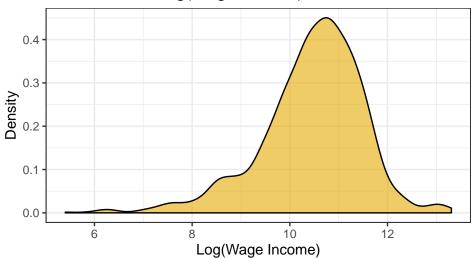


We notice that the distribution of wage income is strongly skewed right, so it would be wise to consider a \log_{10} transformation.

Table 4: Distribution of Log(Wage Income)

	Min	Q1	Median	Q3	Max	Mean	SD	N	Missing
5.	402677	9.914625	10.60777	11.16737	13.30744	10.46028	1.042913	1289	0

Distribution of Log(Wage Income)

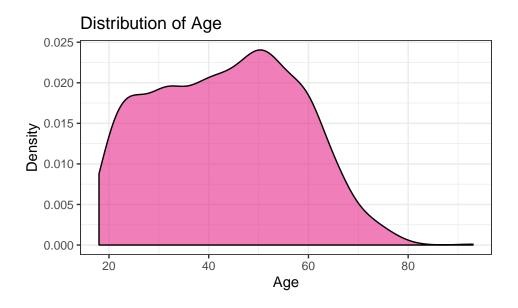


This looks much better, despite a slight left skew. We shall use <code>log_wage_income</code> as our response variable, partly because of skewness, but also because income is better considered on a multiplicative rather than additive scale. In other words, \$1,000 is worth a lot more to a poor person than a millionaire because \$1,000 is a much greater fraction of the poor person's wealth.

2.1.4 Age

Table 5: Distribution of Age

Min	Q1	Median	Q3	Max	Mean	SD	N	Missing
18	31	44	54	93	43.45617	14.28057	1289	0

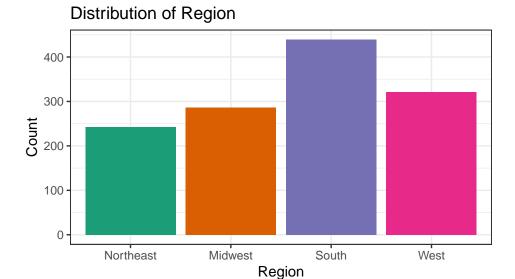


Understandably, the number of individuals in our sample starts to fall dramatically past the age of 60, since that is usually when the sweet release of death is upon us.

2.1.5 Region

Table 6: Distribution of Region

Region	Tally
Northeast	243
Midwest	286
South	439
West	321

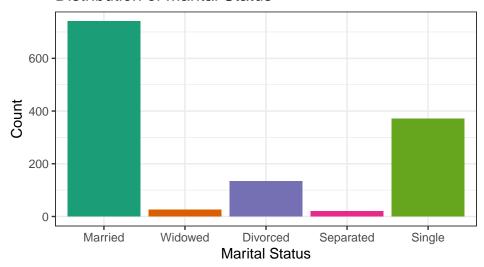


2.1.6 Marital Status

Table 7: Distribution of Marital Status

Marital Status	Tally
Married	740
Widowed	25
Divorced	133
Separated	20
Single	371

Distribution of Marital Status



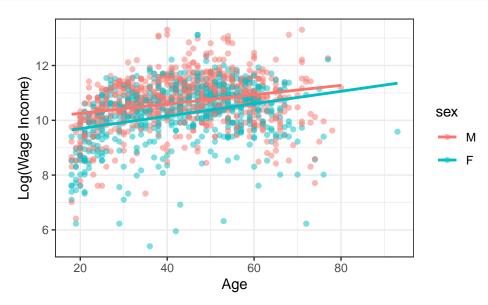
2.2 Multivariate Data Exploration

In this section we take a multivariate approach to exploring our data.

```
ACSSmall %>%
  group_by(privilege, sex) %>%
  summarize(average_wage = mean(wage_income))
```

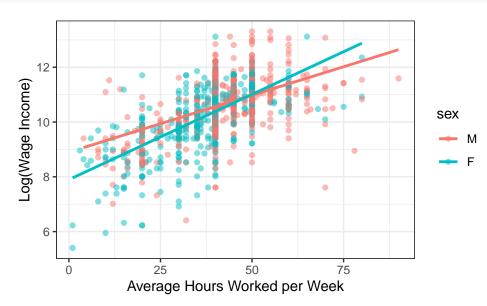
```
# A tibble: 4 x 3
# Groups: privilege [2]
 privilege sex average_wage
 <fct>
           <fct>
                         <dbl>
1 No
                        44661.
2 No
           F
                        35232.
3 Yes
           М
                        68847.
           F
                        44966.
4 Yes
```

```
ACSSmall %>%
  ggplot(aes(x = age, y = log_wage_income, color = sex)) +
  geom_point(alpha = 0.5) +
  geom_smooth(method = "lm", se = FALSE) +
  labs(x = "Age", y = "Log(Wage Income)")
```



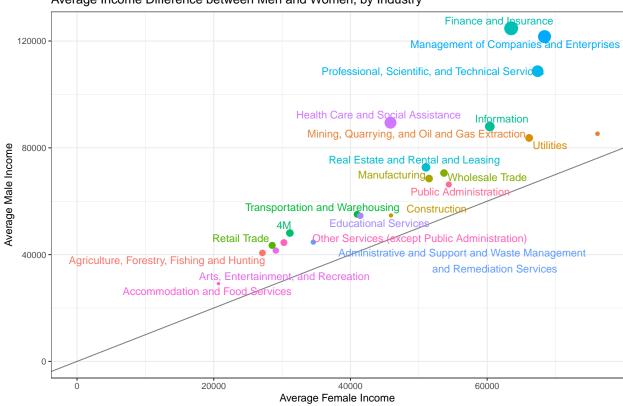
It seems like men earn on average more than women of the same age, but the rate of change of log_wage_income as age increases is relatively the same for the two genders. We will later do a nested F test to see whether the rates of change are actually different, and whether we need two regression lines at all.

```
ACSSmall %>%
ggplot(aes(x = hours_per_week, y = log_wage_income, color = sex)) +
geom_point(alpha = 0.5) +
geom_smooth(method = "lm", se = FALSE) +
labs(x = "Average Hours Worked per Week", y = "Log(Wage Income)")
```



It seems like the rate of change of log_wage_income as age increases is higher for women than for men. We will later do a nested F test to see whether the rates of change are indeed different, and whether we need two regression lines at all.

Average Income Difference between Men and Women, by Industry



3 Modeling

Having completed our variable exploration, we suspect that there is a significant association between wage income and gender. Now let us proceed by fitting a regression model, in order to find out which variables help us account for the variability in income, and more specifically whether gender is indeed a significant predictor thereof.

3.1 Choosing Predictors

It is time to select predictors for our model. We shall use the best subsets procedure in order to find a model that accounts for the most variability in income, while also keeping matters simple.

```
# Creating a dataset to be used for our predictor selection procedure
ACSReg1 <- ACSSmall %>%
    # selecting the potential predictors...
select(sex, age, citizenship, privilege, military, disabled, ever_married, children,
    stem_degree, people_in_household, hours_per_week, grad_degree,
    # ... and the response
    log_wage_income)
```

```
# Using best subsets method
output <- leaps::regsubsets(log_wage_income ~ ., nbest = 1, data = ACSReg1)
with(summary(output), data.frame(adjr2, cp, bic, outmat)) %>%
  arrange(desc(adjr2)) %>%
  rename(R2_adj = adjr2,
         Cp = cp,
         BIC = bic) \%
  mutate(R2_adj = round(R2_adj * 100, 2),
         Cp = round(Cp, 2),
         BIC = round(BIC, 2)) %>%
  t() %>%
  as.data.frame() %>%
  kable(booktabs = TRUE, col.names = c("Model 1", "Model 2", "Model 3",
                                       "Model 4", "Model 5", "Model 6",
                                       "Model 7", "Model 8")) %>%
  kable_styling(latex_options = c("striped", "HOLD_position"), font_size = 9)
```

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
R2_adj	42.67	42.45	42.19	41.50	40.75	39.46	35.68	30.71
Cp	16.64	20.59	25.39	39.43	55.04	82.57	165.02	274.00
BIC	-645.40	-646.60	-646.98	-638.33	-628.34	-607.29	-537.11	-449.36
sexF	*	*	*	*				
age	*	*	*	*	*	*	*	
citizenshipYes								
privilegeYes	*	*	*					
militaryYes								
disabledYes	*	*						
$ever_marriedYes$	*							
children								
$stem_degreeYes$	*	*	*	*	*			
people_in_household								
hours_per_week	*	*	*	*	*	*	*	*
$\operatorname{grad_degreeYes}$	*	*	*	*	*	*		

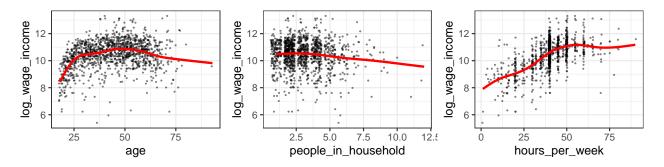
Our adjusted R squared is not bad, at 42.6%. Mallows' Cp is a bit concerning, at 16.6 lowest. Let's see if any of our quantitative variables might benefit from the usage of a polynomial model.

```
p1 <- ggplot(ACSReg1, aes(x = age, y = log_wage_income)) +
    geom_jitter(size = 0.2, alpha = 0.5) +
    geom_smooth(se = FALSE, color = "red")

p2 <- ggplot(ACSReg1, aes(x = people_in_household, y = log_wage_income)) +
    geom_jitter(size = 0.2, alpha = 0.5) +
    geom_smooth(se = FALSE, color = "red")

p3 <- ggplot(ACSReg1, aes(x = hours_per_week, y = log_wage_income)) +
    geom_jitter(size = 0.2, alpha = 0.5) +
    geom_smooth(se = FALSE, color = "red")

cowplot::plot_grid(p1, p2, p3, nrow = 1)</pre>
```



The relationship between log(income) and age is visibly curved, so we might want to consider a quadratic model.

Table 8: Correlation Matrix for Numeric Variables

	log_wage_income	children	people_in_household	age	hours_per_week
log_wage_income	1.000	-0.040	-0.144	0.238	0.555
children	-0.040	1.000	0.744	-0.198	-0.035
people_in_household	-0.144	0.744	1.000	-0.237	-0.107
age	0.238	-0.198	-0.237	1.000	0.025
hours_per_week	0.555	-0.035	-0.107	0.025	1.000

The correlation matrix for our quantitative predictors also shows that people_in_household and children are highly correlated (0.744), so we might as well remove one of them to avoid multicollinearity in our final model. We remove children since it is less correlated with our response, log_wage_income (-0.040), than people_in_household (-0.144).

```
ACSReg2 <- ACSReg1 %>%
  # Adding in squared age as a predictor
mutate(ageSq = age^2) %>%
  # Removing highly correlated predictor
select(-children)
```

Now let's run the best subsets procedure again:

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
R2_adj	49.04	48.76	47.96	47.04	45.90	42.51	37.81	32.31
Cp	10.62	16.53	35.79	58.08	85.85	170.40	288.22	426.61
BIC	-812.46	-811.69	-797.78	-781.27	-760.01	-687.91	-592.83	-489.68
sexF	*	*	*					
age	*	*	*	*	*	*	*	
citizenshipYes	*							
privilegeYes	*	*						
militaryYes								
disabledYes								
$ever_marriedYes$								
$stem_degreeYes$	*	*	*	*				
people_in_household								
hours_per_week	*	*	*	*	*	*	*	*
$grad_degreeYes$	*	*	*	*	*			
ageSq	*	*	*	*	*	*		

The adjusted R squared for the best model has increased from 42.7% to 49%, and Mallows' Cp has decreased to 10.6. The BIC has also decreased significantly from -645 to -812. This looks promising! Let's fit the corresponding model now.

```
Estimate Std. Error t value Pr(>|t|)
(Intercept)
               5.83127098 0.19963808 29.209 < 2e-16 ***
sexF
                                       -5.062 4.75e-07 ***
              -0.21633367
                           0.04273615
age
               0.11171849
                           0.00888981 12.567 < 2e-16 ***
              -0.00109013
                           0.00009944 -10.962 < 2e-16 ***
ageSq
citizenshipYes 0.23469629
                           0.08348647
                                        2.811 0.00501 **
privilegeYes
               0.23062495
                           0.05552735
                                        4.153 3.49e-05 ***
stem_degreeYes
               0.34677111
                           0.06754743
                                        5.134 3.28e-07 ***
hours_per_week
               0.04091504
                           0.00197064 20.762 < 2e-16 ***
grad_degreeYes
               0.41218860
                           0.06428703
                                        6.412 2.02e-10 ***
```

Residual standard error: 0.7445 on 1280 degrees of freedom Multiple R-squared: 0.4935, Adjusted R-squared: 0.4904 F-statistic: 155.9 on 8 and 1280 DF, p-value: < 2.2e-16

This looks good! All of our predictors are significant individually (see p-values for the t-statistics), as is our model as a whole (see p-value for the F-statistic). Let's now play around now using manual forward selection and see if any interaction terms might be of use. We will add the interaction between sex and hours_per_week since it adds the biggest increase in adjusted R squared.

Estimate Std. Error t value Pr(>|t|)

```
(Intercept)
                     6.13847740 0.21082322 29.117 < 2e-16 ***
sexF
                   -0.85632199   0.15507447   -5.522   4.05e-08 ***
                    0.03369606 0.00258105 13.055
hours_per_week
                                                    < 2e-16 ***
                     0.11175199 0.00882996 12.656
                                                    < 2e-16 ***
age
                    -0.00109449 0.00009878 -11.080
                                                    < 2e-16 ***
ageSq
                    0.24002195 0.08293367
                                             2.894 0.00387 **
citizenshipYes
                                             4.246 2.33e-05 ***
privilegeYes
                    0.23423660 0.05515993
stem_degreeYes
                    0.33655526 0.06713488
                                             5.013 6.11e-07 ***
grad_degreeYes
                    0.40182656 0.06389985
                                             6.288 4.39e-10 ***
sexF:hours_per_week  0.01616058  0.00376628
                                             4.291 1.91e-05 ***
```

Residual standard error: 0.7395 on 1279 degrees of freedom Multiple R-squared: 0.5007, Adjusted R-squared: 0.4972 F-statistic: 142.5 on 9 and 1279 DF, p-value: < 2.2e-16

This model looks good! We have 7 predictors. We could cut some for the interest of simplicity without too much of an impact on the adjusted R squared, but 7 predictors is not excessive so we shall leave all of them in.

3.2 Fitting Model

We now fit our final model.

```
Estimate Std. Error t value Pr(>|t|)
(Intercept)
                   6.13847740 0.21082322 29.117 < 2e-16 ***
sexF
                  hours_per_week
                  0.03369606  0.00258105  13.055  < 2e-16 ***
                                               < 2e-16 ***
age
                  0.11175199 0.00882996 12.656
ageSq
                  -0.00109449 0.00009878 -11.080 < 2e-16 ***
citizenshipYes
                  0.24002195 0.08293367
                                         2.894 0.00387 **
privilegeYes
                   0.23423660 0.05515993
                                         4.246 2.33e-05 ***
stem_degreeYes
                   0.33655526 0.06713488
                                         5.013 6.11e-07 ***
grad_degreeYes
                   0.40182656 0.06389985
                                         6.288 4.39e-10 ***
sexF:hours_per_week 0.01616058 0.00376628
                                         4.291 1.91e-05 ***
```

Residual standard error: 0.7395 on 1279 degrees of freedom Multiple R-squared: 0.5007, Adjusted R-squared: 0.4972 F-statistic: 142.5 on 9 and 1279 DF, p-value: < 2.2e-16

3.3 Confidence intervals

```
(100 * (confint(bestModel) %>% exp() - 1))
```

2.5 % 97.5 %

(Intercept)	30539.6520381	69969.64127181
sexF	-68.6686270	-42.42568958
hours_per_week	2.9046354	3.95205787
age	9.9031331	13.77752167
ageSq	-0.1287443	-0.09002982
citizenshipYes	8.0388802	49.58923620
privilegeYes	13.4307947	40.83945980
stem_degreeYes	22.7339330	59.72155732
grad_degreeYes	31.8462807	69.41592614
sexF:hours_per_week	0.8810390	2.38288218

 $(\exp(0.24002195)$ - 1) * 100 $(\exp(0.23423660)$ - 1) * 100 etc.

4 Analysis

4.1 Interpretation

Let's interpret our model. Each of the coefficients are significant, so we interpret.

Since our response is log-transformed, we will give our interpretation in terms of percentage change.

Sex: coefficient of -0.85632199 so if you're a woman (adjusting for the other predictors) that is associated with a $(e^{-0.85632199} - 1) \cdot 100 = -57.52787\%$ lower wages than for men.

 $\log(\mathrm{income\ for\ man}) = \mathrm{sth\ log}(\mathrm{income\ for\ woman}) = \mathrm{sth\ -0.856\ log}(\mathrm{income\ for\ woman}) - \mathrm{log}(\mathrm{income\ for\ man}) = -0.856\ \mathrm{log}(\mathrm{income\ for\ woman}\ /\ \mathrm{income\ for\ man}) = -0.856\ \mathrm{income\ for\ woman}\ /\ \mathrm{income\ for\ man}$ for man = e^(-0.856) so income for woman = income for man * e^(-0.856) = income for man * 0.4247213

For a man, every additional 5 hours worked per week are associated with $(e^{5 \cdot 0.03369606} - 1) \cdot 100 = 18.35049\%$ higher wages. But for women, every additional 5 hours worked per week are only associated with $(e^{5 \cdot (0.03369606 - 0.01616058)} - 1) \cdot 100 = 28.31054\%$ higher wages! Interesting.

4.2 Nested F-test

Now let's come back to our question of interest - does sex matter in predicting wage?

```
Analysis of Variance Table
```

```
Analysis of Variance Table
```

```
privilege + stem_degree + grad_degree
                RSS Df Sum of Sq F
      Res.Df
                                         Pr(>F)
    1 1280 709.51
    2 1279 699.44 1 10.069 18.411 0.00001914 ***
    Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
fullModel <- lm(log_wage_income ~ sex * hours_per_week + sex * age + age + ageSq +
                 citizenship + privilege + stem_degree + grad_degree,
               data = ACSReg2)
reducedModel <- lm(log_wage_income ~ sex * hours_per_week + age + ageSq + citizenship +
                 privilege + stem_degree + grad_degree, data = ACSReg2)
anova(reducedModel, fullModel)
    Analysis of Variance Table
    Model 1: log_wage_income ~ sex * hours_per_week + age + ageSq + citizenship +
        privilege + stem_degree + grad_degree
    Model 2: log_wage_income ~ sex * hours_per_week + sex * age + age + ageSq +
        citizenship + privilege + stem_degree + grad_degree
      Res.Df
                RSS Df Sum of Sq
                                     F Pr(>F)
```

1279 699.44

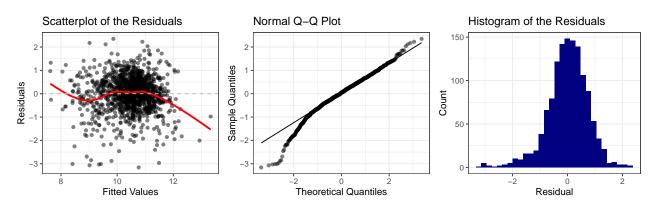
2 1278 699.42 1 0.017468 0.0319 0.8582

5 Assessment

5.1 Conditions

Diagnostic plots to check conditions:

```
diagnostic <- function(model, binwidth) {</pre>
  # Scatterplot of the residuals
 r1 <- ggplot(data = model, aes(x = .fitted, y = .resid)) +
   geom_point(alpha = 0.5) +
   geom_smooth(color = "red", se = FALSE) +
   geom_hline(yintercept = 0, linetype = 2, color = "grey") +
   xlab("Fitted Values") +
   ylab("Residuals") +
   ggtitle("Scatterplot of the Residuals")
  # Normal QQ-plot
  r2 <- ggplot(data = model, aes(sample = .resid)) +
   geom_q(alpha = 0.5) +
   geom_qq_line() +
   xlab("Theoretical Quantiles") +
   ylab("Sample Quantiles") +
   ggtitle("Normal Q-Q Plot")
  # Histogram of the residuals
  r3 <- ggplot(data = model, aes(x = .resid, y = ..count..)) +
   geom_histogram(fill = "navy", binwidth = binwidth) +
   xlab("Residual") +
   ylab("Count") +
   ggtitle("Histogram of the Residuals")
  # Plotting all three side by side
  cowplot::plot_grid(plotlist = list(r1, r2, r3), nrow = 1)
diagnostic(bestModel, binwidth = NULL)
```



Nice model - we like!!

5.2 Cross-validation Correlation

```
# Creating test dataset
ACSTest <- sample(ACSBig %>% mutate(ageSq = age^2), size = 2000)

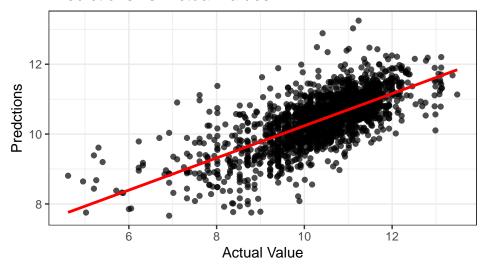
# Computing predictions
ACSTestPred <- ACSTest %>%
   mutate(prediction = predict(bestModel, newdata = .)) %>%
   select(log_wage_income, prediction)

# Finding cross validation correlation
cross_val_cor <- cor(ACSTestPred$log_wage_income, ACSTestPred$prediction)
cross_val_cor</pre>
```

[1] 0.7002889

```
# Plotting actual values against predictions
ggplot(ACSTestPred, aes(x = log_wage_income, y = prediction)) +
  geom_point(alpha = 0.7) +
  geom_smooth(method = "lm", se = FALSE, color = "red") +
  labs(x = "Actual Value", y = "Predctions") +
  ggtitle("Predictions vs. Actual Values")
```

Predictions vs. Actual Values



```
r_squared <- 0.5007
shrinkage <- r_squared - cross_val_cor^2
shrinkage</pre>
```

[1] 0.01029542

The cross validation correlation is not bad - over 70%. Our model is accurate less than half the time This means that our predictive model is expected to perform decently in practice (i.e. when applied to data that was not used in our estimations).

5.3 Logistic Regression

```
ACSLogit <- ACSSmall %>%
 # selecting the potential predictors...
 select(sex, age, citizenship, privilege, military, disabled, ever_married, children,
       stem_degree, people_in_household, hours_per_week, grad_degree,
       # ... and the response
       wage_income) %>%
 # Adding in squared age as a predictor
 mutate(ageSq = age^2) %>%
 # Dichotomizing outcome
 mutate(wealthy = ifelse(wage_income >= 120000, 1, 0) %>%
         as.factor()) %>%
 select(-wage_income)
tally(ACSLogit$wealthy ~ ACSLogit$sex)
                 ACSLogit$sex
    ACSLogit$wealthy M F
                0 626 563
                1 74 26
model1 <- glm(wealthy ~ ., data = ACSLogit, family = binomial(logit),</pre>
           na.action = na.exclude)
msummary(model1)
    Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
    (Intercept)
                    -17.051128 2.621667 -6.504 7.83e-11 ***
                     sexF
                     age
                     citizenshipYes
    privilegeYes
                     0.927774   0.424866   2.184   0.028985 *
                     0.176460 0.384410 0.459 0.646204
    militaryYes
    disabledYes
                     -0.916610 0.752712 -1.218 0.223321
                                       1.410 0.158599
    ever_marriedYes
                      0.598767 0.424717
    children
                      0.228686 0.177027 1.292 0.196421
    stem_degreeYes
                      people_in_household 0.004045 0.132633 0.031 0.975667
                      hours_per_week
    grad_degreeYes
                      ageSq
                     (Dispersion parameter for binomial family taken to be 1)
       Null deviance: 698.75 on 1260 degrees of freedom
    Residual deviance: 511.50 on 1247 degrees of freedom
      (28 observations deleted due to missingness)
    AIC: 539.5
    Number of Fisher Scoring iterations: 7
```

Let's remove what seems most insignificant - citizenship and ever_married. People in household and children are highly correlated so we'll remove one.

Coefficients:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept)
           -17.459310 2.565984 -6.804 1.02e-11 ***
            sexF
             age
privilegeYes
            0.846854 0.412009 2.055 0.03984 *
militaryYes
            disabledYes
            -0.926411 0.751036 -1.234 0.21738
children
            stem_degreeYes
            1.305093 0.275463 4.738 2.16e-06 ***
people_in_household 0.052264 0.125506 0.416 0.67710
hours_per_week
             grad_degreeYes
             1.203622   0.267058   4.507   6.58e-06 ***
            ageSq
```

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 698.75 on 1260 degrees of freedom Residual deviance: 515.14 on 1249 degrees of freedom (28 observations deleted due to missingness)

AIC: 539.14

Number of Fisher Scoring iterations: 7

Coefficients:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept)
              -17.7811173 2.5281560 -7.033 2.02e-12 ***
sexF
               -0.8700505
                        0.2668513 -3.260 0.00111 **
                age
privilegeYes
               0.8603876
                        0.4109600 2.094 0.03630 *
militaryYes
               0.1331174
                        disabledYes
               -0.9481486
                        0.7505415 -1.263 0.20649
               stem_degreeYes
people_in_household 0.1673562
                        0.0742255 2.255 0.02415 *
hours_per_week
                0.0559230
                        0.0117809 4.747 2.07e-06 ***
```

```
grad_degreeYes 1.2442603 0.2649010 4.697 2.64e-06 ***
ageSq -0.0040456 0.0009971 -4.057 4.96e-05 ***

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 698.75 on 1260 degrees of freedom
Residual deviance: 516.59 on 1250 degrees of freedom
(28 observations deleted due to missingness)
AIC: 538.59

Number of Fisher Scoring iterations: 7
```

msummary(model2b)

Coefficients:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept)
           -17.330659
                      2.551685 -6.792 1.11e-11 ***
            -0.875860 0.266524 -3.286 0.00102 **
sexF
age
             privilegeYes
            militaryYes
                      0.382224 0.379 0.70451
            0.144956
                      0.751385 -1.240 0.21500
disabledYes
            -0.931671
children
            0.261894
                      0.103863 2.522 0.01168 *
stem_degreeYes 1.296392
                      0.274425 4.724 2.31e-06 ***
                      0.011656 4.681 2.85e-06 ***
hours_per_week 0.054564
                              4.492 7.06e-06 ***
grad_degreeYes 1.192694
                      0.265523
                      0.001009 -3.914 9.07e-05 ***
            -0.003949
ageSq
```

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 698.75 on 1260 degrees of freedom Residual deviance: 515.31 on 1250 degrees of freedom (28 observations deleted due to missingness) AIC: 537.31

Number of Fisher Scoring iterations: 7

We remove people in household, as well as disabled and military.

Coefficients:

```
Estimate Std. Error z value Pr(>|z|)

(Intercept) -17.448186 2.563235 -6.807 9.96e-12 ***
sexF -0.892400 0.262193 -3.404 0.000665 ***
age 0.437004 0.100443 4.351 1.36e-05 ***
privilegeYes 0.863990 0.412296 2.096 0.036122 *
```

```
0.103254 2.605 0.009185 **
    stem_degreeYes 1.280143
                             0.273442 4.682 2.85e-06 ***
                   0.055034
                             0.011450 4.807 1.54e-06 ***
    hours_per_week
    grad_degreeYes
                   1.207763
                             0.265588
                                      4.548 5.43e-06 ***
                             0.001012 -3.924 8.70e-05 ***
                   -0.003972
    ageSq
    (Dispersion parameter for binomial family taken to be 1)
        Null deviance: 698.75 on 1260 degrees of freedom
    Residual deviance: 517.36 on 1252 degrees of freedom
      (28 observations deleted due to missingness)
    AIC: 535.36
    Number of Fisher Scoring iterations: 7
Try to remove some more?
ACSLogit4 <- ACSLogit3 %>%
 select(-privilege, -children)
model4 <- glm(wealthy ~ ., data = ACSLogit4, family = binomial(logit),</pre>
            na.action = na.exclude)
msummary (model4)
    Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
    (Intercept)
                 -16.5299044 2.4397345 -6.775 1.24e-11 ***
                  sexF
                   age
    stem_degreeYes 1.2269853 0.2715363 4.519 6.22e-06 ***
    hours_per_week 0.0531789 0.0112440 4.730 2.25e-06 ***
    grad_degreeYes
                  1.2959211 0.2641273 4.906 9.28e-07 ***
                   ageSq
    (Dispersion parameter for binomial family taken to be 1)
        Null deviance: 703.32 on 1288 degrees of freedom
    Residual deviance: 530.09 on 1282 degrees of freedom
    AIC: 544.09
    Number of Fisher Scoring iterations: 7
No no, AIC grows. model3 looked good. Interactions? Trying some, no.
model5 <- glm(wealthy ~ . + hours_per_week * stem_degree, data = ACSLogit3, family = binomial(logit),</pre>
            na.action = na.exclude)
msummary(model5)
    Coefficients:
                                 Estimate Std. Error z value Pr(>|z|)
                               -17.117756 2.586012 -6.619 3.61e-11 ***
    (Intercept)
```

children

0.268985

```
-0.925743
                                                0.265403 -3.488 0.000487 ***
     sexF
                                     age
                                     0.890442
                                                0.415740 2.142 0.032208 *
     privilegeYes
     children
                                     0.266621
                                                0.103200
                                                         2.584 0.009779 **
                                               1.253531 -0.080 0.936402
     stem_degreeYes
                                    -0.100023
     hours_per_week
                                     0.046595
                                                0.013732 3.393 0.000691 ***
                                                         4.593 4.37e-06 ***
     grad_degreeYes
                                     1.225565
                                                0.266845
                                    -0.003976
                                                0.001016 -3.913 9.11e-05 ***
     ageSq
     stem_degreeYes:hours_per_week
                                     0.030072
                                                0.026604
                                                           1.130 0.258320
     (Dispersion parameter for binomial family taken to be 1)
         Null deviance: 698.75 on 1260 degrees of freedom
     Residual deviance: 516.03 on 1251 degrees of freedom
       (28 observations deleted due to missingness)
     AIC: 536.03
     Number of Fisher Scoring iterations: 7
# Amy Wagaman's function to calculate coconcordance
getConcordance <- function(model){</pre>
  Con_Dis_Data <- cbind(model$y, model$fitted.values)</pre>
  ones <- Con_Dis_Data[Con_Dis_Data[,1] == 1,]</pre>
  zeros <- Con_Dis_Data[Con_Dis_Data[,1] == 0,]</pre>
  conc <- matrix(0, dim(zeros)[1], dim(ones)[1])</pre>
  disc <- matrix(0, dim(zeros)[1], dim(ones)[1])</pre>
  ties <- matrix(0, dim(zeros)[1], dim(ones)[1])
  for(j in 1:dim(zeros)[1]){
   for(i in 1:dim(ones)[1]){
      if(ones[i,2]>zeros[j,2]){
        conc[j,i]=1
     }else if(ones[i,2]<zeros[j,2]){</pre>
          disc[i,i]=1
     }else if(ones[i,2]==zeros[j,2]){
       ties[j,i]=1
     }
   }
  }
  Pairs <- dim(zeros)[1]*dim(ones)[1]
  PercentConcordance <- (sum(conc)/Pairs)*100
  PercentDiscordance <- (sum(disc)/Pairs)*100
  PercentTied <- (sum(ties)/Pairs)*100
  return(list("Percent Concordance" = PercentConcordance,
              "Percent Discordance" = PercentDiscordance,
              "Percent Tied" = PercentTied,
              "Pairs" = Pairs))
getConcordance(model3)
```

```
$`Percent Concordance`
[1] 86.10508
```

```
$`Percent Discordance`
     [1] 13.86994
     $`Percent Tied`
     [1] 0.02497847
     $Pairs
     [1] 116100
ACSLogit3 <- mutate(ACSLogit3, predProb = predict(model, type = "response"),
                   predComplete = ifelse(predProb > 0.9, 1, 0))
# cross-tally predicted successes with actual success
tally(data = ACSLogit3, wealthy ~ predComplete, margins = TRUE)
           predComplete
     wealthy
              1
      0
            1189
             100
      1
       Total 1289
ADD EMPIRICAL LOGIT PLOTS
Testing
library(lmtest)
lrtest(model3)
     Likelihood ratio test
    Model 1: wealthy ~ sex + age + privilege + children + stem_degree + hours_per_week +
        grad_degree + ageSq
    Model 2: wealthy ~ 1
      #Df LogLik Df Chisq Pr(>Chisq)
     1 9 -258.68
     2 1 -349.37 -8 181.39 < 2.2e-16 ***
     Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
model3_nosex <- glm(wealthy ~ age + privilege + children + stem_degree +
                     hours_per_week + grad_degree + ageSq,
                   data = ACSLogit3, family = binomial(logit),
                   na.action = na.exclude)
lrtest(model3_nosex, model3)
     Likelihood ratio test
    Model 1: wealthy ~ age + privilege + children + stem_degree + hours_per_week +
        grad_degree + ageSq
    Model 2: wealthy ~ sex + age + privilege + children + stem_degree + hours_per_week +
        grad_degree + ageSq
      #Df LogLik Df Chisq Pr(>Chisq)
```

```
1 8 -264.94
2 9 -258.68 1 12.525 0.0004016 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

6 Appendices

6.1 Appendix A - Data Dictionary

A number of variables have been identified as potentially relevant to the issue of the gender pay gap. Optimally, a careful consideration of each of them might provide us with a more precise understanding of the relationship between gender and income. However, only some of the variables below will make it into our final regression models - some will be used for filtering (e.g. **employment**), others are intended for creating meaningful visualizations (e.g. **state**, **industry**), and others might prove to have an insignificant effect on the relationship between gender and income.

In the list below, variables have been grouped under general topics, and we have included their names as they appear in the original data set, their new names as assigned for our analysis, as well as their respective descriptions from the Data Dictionary. For each individual, we will look at:

1. General demographics:

- (a) **SEX** (renamed to **sex**) "Sex" (FACTOR WITH TWO LEVELS)
- (b) \mathbf{AGEP} (renamed to \mathbf{age}) "Age"
- (c) CIT (renamed to citizenship) "Citizenship Status"
- (d) RAC1P (renamed to race) "Recoded detailed race code"
- (e) MIL (renamed to military) "Military service"
- (f) **DIS** (renamed to **disabled**) "Disability recode"

2. Family and household:

- (a) MAR (renamed to married)- "Marital status"
- (b) **NRC** (renamed to **children_no**) "Number of related children in household (unweighted)"

3. Educational background:

- (a) **SCHL** (renamed to **education**) "Educational attainment"
- (b) **FOD1P** (will be merged with **FOD2P** to create **degree**) "Recoded field of degree first entry"
- (c) **FOD2P** (will be merged with **FOD1P** to create **degree**) "Recoded field of degree second entry" ¹
- (d) **SCIENGP** (renamed to **stem_degree**) "Field of Degree Science and Engineering Flag NSF Definition"

4. Employment:

- (a) **ESR** (renamed to **employment**) "Employment status recode"
- (b) **WKHP** (renamed to **hours_per_week**) "Usual hours worked per week past 12 months'

¹e.g. for double majors or dual degrees

(c) **NAICSP** (renamed to **industry**) - "NAICS Industry recode for 2013 and later based on 2012 NAICS codes"

5. Income:

- (a) WAGP (renamed to wage_income) "Wages or salary income past 12 months (use ADJINC to adjust WAGP to constant dollars)"
- (b) **ADJINC** (not renamed, will be used during data wrangling to adjust dollar amounts, then discarded) "Adjustment factor for income and earnings dollar amounts"

6. Location:

- (a) **REGION** (renamed to **region**) "Region code based on 2010 Census definitions"
- (b) ST (renamed to state) "State Code based on 2010 Census definitions"

6.2 Appendix B - Code for Univariate Data Exploration

The code from the Univariate Data Exploration section appears here.

6.2.1 Sex

6.2.2 Race

```
ggtitle("Distribution of Race") +
theme(legend.position = "none") +
scale_fill_brewer(palette = "Dark2")
```

6.2.3 Income

6.2.4 Age

6.2.5 Region

6.2.6 Marital Status