CA Q6 Part 2

Q6 Part(2) (a) In this question we ~~need to show~~ are told that for conductors in the ME's we can set ~~the~~ $j_f^{\perp} = j_f = \sigma E$ and therefore refer to the transvere component as the total ~~component~~ field. we then need to show

$$k \cdot k = 0, \quad k \cdot H = 0$$

$$k \times E = \omega \mu H, \quad k \times H = \omega \varepsilon(\omega) E$$

from the longitudinal & transvere field components, we know

$$\nabla \cdot J = \frac{-\partial \rho}{\partial t}$$

and from the continuity equation we have that

$$J = \sigma E \quad \text{(for conductor)}$$

$$\nabla \cdot (\sigma E) = \frac{-\partial \rho}{\partial t}$$

$$\nabla \cdot (\nabla E) = \frac{-\partial \rho}{\partial t}$$

from the first Maxwell equation $\nabla \cdot E = \rho / \varepsilon_0$

$$\Rightarrow \frac{\partial \rho}{\partial t} < \frac{\sigma \rho}{\varepsilon_0} = 0$$

By solving this we get

$$\rho = \rho_0 e^{-(\sigma/\varepsilon)t}$$

6(part) now from the algebraic form of the MG's
6 part b we know that $\nabla \cdot E = \rho_f / \varepsilon$, likewise, for conductors $\rho_f = 0$, $\Rightarrow \nabla \cdot E = 0$.

if we now substitute $\nabla \cdot (E_0 e^{j(kx-\omega t)}) = 0$

$$\Rightarrow \quad k \cdot E = 0.$$

Similarily $\nabla \cdot H = 0$

$$\Rightarrow \quad H = H_0 e^{j(kx-\omega t)}$$

$$\Rightarrow \boxed{k \cdot H = 0}$$

for the 3rd maxwell equation

$$\nabla \times E = -\mu \, \partial H / \partial t$$

$$\Rightarrow \quad \nabla \times (E_0 e^{j(kx-\omega t)}) = \frac{-\partial}{\partial t} (H_0 e^{-j(kx-\omega t)})$$

$$\Rightarrow \boxed{k \times E = \omega \mu H.}$$

lastly from the fourth maxwell equation

$$\nabla \times H = \sigma E$$

$$k \times H = \omega \varepsilon (\omega E)$$

$$\Rightarrow \boxed{E(\omega) = \varepsilon (1 + i \, \sigma / \varepsilon \omega)}$$

thus we have effectively prooven what was required from the question.

(b) in the second part of this question we know that the complex and frequency dependant

EM constants. Also propagation constants in conductors are complex because of the conduction field.

$$\nabla^2 \varphi - \sigma \mu \frac{\partial \varphi}{\partial t} - \mu_\varepsilon \frac{\partial^2 \varphi}{\partial t^2} = 0$$

If we now substitute $\Phi = \Phi_0 \, e^{i(k r^\circ - \omega t)}$
then we get

$$k^2 = \varepsilon \mu \omega^2 + i \sigma \mu \omega$$

other constants also depend on (k). Hence it is written as $k^2 - (k'^2 - k''^2) + i(k' + ik'')$
and $k^2 - (k'^2 - k''^2) + i2(kk'')$

If we now combine

(1) $k'^2 - k''^2 = \mu \varepsilon \omega^2$ (2) $2k'k'' = \sigma \mu \omega$

Then we get

$$k' = \sqrt{\mu \varepsilon} \cdot \omega \left(\frac{\sqrt{1 + (\sigma/\varepsilon\omega)^2} + 1}{2} \right)^{\frac{1}{2}}$$

$$k'' = \sqrt{\mu \varepsilon} \cdot \omega \left(\frac{\sqrt{1 + (\sigma/\varepsilon\omega)^2} - 1}{2} \right)^{\frac{1}{2}}$$

Similarily we can calculate the others given the quantities.